Journal of Mathematics Instruction, Social Research and Opinion
Vol. 5, No. 4, December 2026, pp. 2751 – 2763, https://doi.org/10.58421/misro.v5i4.904
ISSN 2962-7842

Analysis of Geometric Transformations in User Activities on Smartphone Screens

Abdul Faqih[1], Mona Roszihantika[2], Siti Nurhalizah[3], Zarra Chaira Yara[4], Fadillah Nurhurin[5], Arif Muchyidin[6]
[1,2,3,4,5,6]Universitas Islam Negeri Siber Syekh Nurjati Cirebon, Indonesia.

Article Info

Article history:

Received 2025-12-16
Revised 2025-12-21
Accepted 2026-11-18

Keywords:

Contextual Mathematics Learning
Geogebra
Human–Computer Interaction
Isometry
Similarity Transformation
Touchscreen Gestures

ABSTRACT

Smartphone use relies on gestures and display functions such as scrolling, swiping between pages, rotating the screen, pinch-zooming and mirroring an image, which visibly move, turn, resize or flip on-screen content. Although these operations resemble the basic plane transformations taught in school geometry, the correspondence is seldom made explicit or checked against actual screen content. This study aims to analyse how four common smartphone interactions can be described as translation, rotation, reflection and dilation. A qualitative descriptive design was used, combining a literature review on touchscreen interaction and transformation geometry with a coordinate-based analysis of smartphone screenshots and GeoGebra constructions; the screenshots were checked by pixel correlation and by fitting similarity transformations to matched image features. A mirrored image in a split screen is a reflection about the dividing line (correlation 0.96, compared with 0.88 for a half-turn); page swiping and scrolling displace content by a translation vector; automatic screen rotation turns the content through −90° relative to the device frame and rescales it; and zooming a map is a dilation about a point close to the location marker, with estimated scale factors of about 1.7, 0.85 and 0.43 and rotation below 1°. GeoGebra constructions reproduce each mapping with exact coordinates. All four interactions are similarity transformations, and two-finger gestures can be modelled by a single map $z' = az + b$. The results provide verified, familiar contexts for teaching transformation geometry; the study did not involve learners, and interface animations may add effects that are not pure transformations.

This is an open-access article under the CC BY-SA license.

Corresponding Author:

Siti Nurhalizah
Faculty of Islamic Education and Teacher Training, Syekh Nurjati Cyber State Islamic University Cirebon
Email: sitinurhalizah@mail.syekhnurjati.ac.id

1. INTRODUCTION

Smartphones have changed the way people obtain information and communicate, and they are now part of students' learning environment; their use has been associated with

students' achievement [1]. Smartphone applications make it easier for students to access the information they need, such as subject matter and interactive videos [2]. In mathematics, concrete and digital media have supported learning transformation geometry: a rotation board increased students' activeness in learning rotation [3], and GeoGebra has been used to visualise transformations and improve learning outcomes [4], [5]. Transformation geometry, an important topic in secondary school mathematics, deals with changes in the position, orientation and size of plane figures through translation, reflection, rotation and dilation [6]. At the same time, dragging, swiping, pinching and two-finger rotation have become the common vocabulary for moving, rotating, enlarging and shrinking objects on touch surfaces [7], and children become comfortable performing single-finger touchscreen gestures by about eight years of age [8]. These gestures visibly move, turn, flip and resize on-screen content, so they appear to be natural instances of the four basic transformations. However, this correspondence is rarely made explicit, and it has seldom been checked against what devices actually display, particularly in mathematics education and technological literacy.

Several studies have examined how users perform touchscreen gestures. Martin-Gutierrez and Del Rio Guerra [9] analysed twenty touch gestures performed by individuals with Down syndrome, Dong *et al.* [10] compared surface and motion gestures for mobile augmented reality and proposed the touch–move–release technique, and Yadav *et al.* [8] assessed children's ability to perform touchscreen gestures and to follow prompting techniques in mobile apps. Studies on transformation geometry, by contrast, have focused mainly on learning materials and media, such as an ICT-based module [11], augmented-reality activities accessed with a smartphone [12], and ethnomathematics-based teaching materials [13]. Research on touchscreen gestures and research on transformation geometry learning therefore still run largely in parallel.

Smartphone sensors have been used to measure spatial orientation, with accuracy varying by device [14], and geometric transformations such as scaling, rotation, translation, shearing, and flipping are widely used to augment image data for hand-gesture recognition [15]. Nevertheless, studies that link users' everyday interactions to formal definitions of geometric transformations and verify this link on real screen content remain scarce. This gap leads to this study's research question: how, and how precisely, are the four basic geometric transformations realised when users interact with a smartphone screen?

To answer this question, the study maps common smartphone interactions onto translation, rotation, reflection and dilation by combining literature on human–computer interaction (HCI) and transformation geometry with a coordinate-based analysis of smartphone screenshots and GeoGebra constructions. Unlike a purely conceptual mapping, each example is checked quantitatively: we test whether a mirrored image is a reflection rather than a half-turn, estimate the scale factor and the centre of each map zoom, and compute image coordinates exactly in GeoGebra.

The contribution of the study is threefold: (i) an explicit and verified correspondence between everyday smartphone interactions and the four basic transformations, summarised in coordinate form; (ii) a unifying description of these

interactions as similarity transformations, including a simple model of two-finger gestures; and (iii) a set of familiar, checkable examples that teachers and interface designers can use for contextual geometry learning and for keeping displayed changes consistent with users' actions.

2. METHOD

2.1. Research Design

This study used a qualitative descriptive design, which aims at a comprehensive, low-inference description of a phenomenon in everyday terms [16]. The phenomenon examined is the visible change of on-screen content during common smartphone interactions, interpreted through the definitions of plane geometric transformations. The design combined two strands: (1) a literature review that supplied the definitions and properties of the transformations and the HCI descriptions of touchscreen gestures, and (2) a mathematical analysis of visual examples, namely smartphone screenshots and GeoGebra constructions produced by the authors. The review followed general guidance for literature-based research [17].

2.2. Literature Search and Selection

Literature was searched in Google Scholar, using Publish or Perish to retrieve and screen records, with combinations of keywords such as "transformasi geometri", "geometric transformation", "touchscreen gesture", "smartphone" and "GeoGebra"; the search was conducted on 22 October 2025. Sources were included if they (a) defined or taught translation, reflection, rotation or dilation, or (b) reported empirical or design research on touchscreen or mobile interaction, and if their bibliographic data could be verified. The search yielded 25 articles that met these criteria. Definitions, properties and claims were extracted from these articles and compared with the visual examples. Methodological and mathematical sources (on qualitative description, image analysis and geometry) were consulted separately and are not part of this count.

2.3. Visual Examples and Their Verification

Four interactions were recorded as screenshot sequences on several Android smartphones, including a Xiaomi device running Xiaomi HyperOS 2.0.211.0: a split-screen view showing a photograph and its mirrored copy (Figure 2), swiping between home-screen pages (Figure 2), automatic screen rotation of a photograph (Figure 4) and zooming in Google Maps (Figure 6). For each transformation, a GeoGebra Classic 5 construction was made with a picture of the same object (Figures 1, 4, 6 and 8), and the image coordinates of the anchor points were calculated from the transformation rules given in Section 3.1.

The screenshots were then checked quantitatively. To decide whether the mirrored image in Figure 2 is a reflection or a rotation through 180°, both of which turn a picture upside down, the grey-level intensities of the upper picture were correlated with (a) a vertical flip and (b) a half-turn of the lower picture about the best-fitting axis or centre; the same test was applied to Figure 1. For the map sequence in Figure 6, corresponding points

in consecutive screenshots were detected with the scale-invariant feature transform (SIFT) [18], and a similarity transformation (scale factor k, rotation angle θ and translation) was fitted to them with the random sample consensus (RANSAC) method [19]; the fixed point of the fitted map gives the centre of dilation. Because the first screenshot was embedded at a different size, all screenshots were first brought to a common scale using the interface elements that do not change during zooming. The computations were carried out in Python 3.13 with OpenCV 5.0 and NumPy 2.5, and the code is provided as a Supplementary File.

3. RESULTS AND DISCUSSION

This section first presents the analysis of each transformation, pairing a smartphone example with a GeoGebra construction (Section 3.1), and then discusses the findings in relation to the literature (Section 3.2). Throughout, a geometric transformation is a one-to-one mapping of the plane onto itself, and coordinates refer to the screen frame or the GeoGebra graphics view.

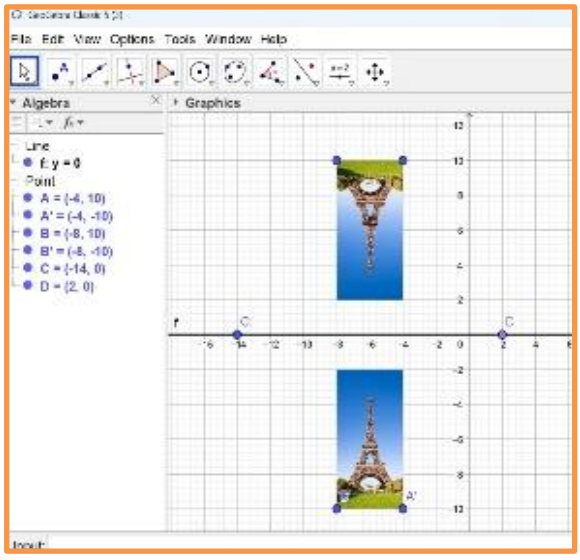
3.1. Results

3.1.1. Reflection

A reflection maps each point to its mirror image in a line: the segment joining a point and its image is perpendicular to the mirror line and bisected by it, so that the point and its image are equidistant from the line [6]. A reflection is an isometry, that is, it preserves distances, angles and size, but it reverses orientation [20]. The reflection in the x-axis is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

Moreover, the determinant of its matrix is -1, indicating a reversal of orientation.



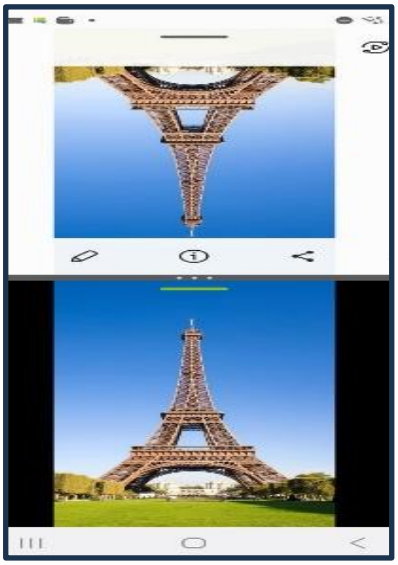


Figure 1. Reflection in the x-axis in GeoGebra (left). Figure 2. A photograph and its vertically mirrored copy in a split-screen view on a smartphone (right).

Figure 1 shows a reflection in the x-axis (the line f: $y = 0$ through $C(-14, 0)$ and $D(2, 0)$) constructed in GeoGebra. The pre-image is a picture of the Eiffel Tower whose upper corners are $A(-4, 10)$ and $B(-8, 10)$; it occupies the rectangle $-8 \le x \le -4$, $2 \le y \le 10$ in quadrant II. By (1), A and B are mapped to $A'(-4, -10)$ and $B'(-8, -10)$, and the whole picture is mapped onto $-8 \le x \le -4$, $-10 \le y \le -2$ in quadrant III. Each point and its image are equidistant from the x-axis, and the two pictures have opposite vertical orientation: in this construction the pre-image is displayed upside down in quadrant II and its image upright in quadrant III.

Figure 2 shows a smartphone in split-screen mode. The lower panel displays a photograph of the Eiffel Tower, and the upper panel displays the same photograph flipped vertically using the phone's built-in gallery app; the divider between the panels acts as the mirror line. Because both a reflection across a horizontal line and a 180° rotation turn a picture upside down, the two possibilities were compared. The upper picture correlates more strongly with a vertical flip of the lower picture ($r = 0.96$) than with a half-turn ($r = 0.88$), and the best-fitting mirror line lies at the divider. The same test applied to Figure 1 gives $r = 0.87$ against 0.70. The smartphone display is therefore a reflection: left and right are preserved while top and bottom are exchanged, and corresponding points, such as the tip of the tower and the base arch, lie at approximately equal distances on either side of the divider. The upper picture is partly cropped by its window, so only the visible part can be compared. Unlike the other examples, this reflection is not produced by a touch gesture, but by a display or editing function that the user activates by touch.

3.1.2. Translation

A translation moves every point of the plane through the same distance in the same direction. Given a directed segment AB, the translation maps each point P to the point P' for which the vector PP' equals the vector AB [21]; in coordinates, the translation by the vector $\mathbf{u} = (a, b)$ is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \tag{2}$$

A translation is an isometry that also preserves orientation, so only the position of a figure changes [20].



Figure 2. Consecutive screenshots taken while swiping to the left between home-screen pages.

Figure 2 shows five consecutive screenshots taken while swiping to the left between home-screen pages. Dragging and panning are the gestures that users choose to move objects and views on touch surfaces [7]. When the page comes to rest, the page and all its icons have been displaced as a rigid unit by a horizontal vector, which is a translation; the same model describes the continuous scrolling of lists and web pages. The intermediate frames, however, show that the launcher adds a decorative transition: in the second and fifth screenshots, the search bar and the icons are visibly tilted and foreshortened. These effects are not part of a translation, so the translation model describes the net displacement of the page rather than every frame of the animation.

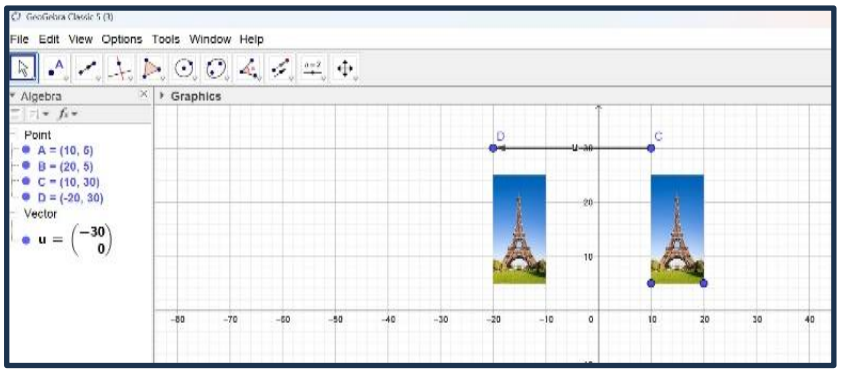


Figure 3. Translation by the vector u = (−30, 0) in GeoGebra.

Figure 3 shows the corresponding GeoGebra construction. The picture of the Eiffel Tower has its lower corners at $A(10, 5)$ and $B(20, 5)$, and the translation vector $\mathbf{u} = D - C = (-30, 0)$ is defined by $C(10, 30)$ and $D(-20, 30)$. By (2), $A' = (-20, 5)$ and $B' = (-10, 5)$: every point moves 30 units in the negative x-direction while its y-coordinate is unchanged. The left picture is therefore the image of the right one; its shape, size and orientation are identical, and only its position has changed.

3.1.3. Rotation

A rotation turns every point of the plane about a fixed centre through a fixed angle [22], [23]. By convention, the angle is positive for a counterclockwise rotation and negative for a clockwise rotation [24]. The rotation about the origin through the angle θ is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{3}$$

It is an isometry that preserves orientation, and its matrix determinant is 1 [20].



Figure 4. Automatic screen rotation of a photograph: initial, intermediate and final frames.

Figure 4 shows three frames recorded while the screen rotated automatically after the phone was turned 90° counterclockwise to landscape. In the first frame, the photograph is in portrait orientation; the second frame shows the system's rotation animation; and in the third frame, the photograph is displayed horizontally within the portrait frame of the screenshot. In the screen's coordinate frame, the content has turned −90°, i.e., clockwise: the top of the tower, which pointed upward, now points to the right. This rotation compensates for the device's opposite rotation, so the content stays upright for the user. The rotated photograph is also rescaled to fit the new screen width, so the complete operation is a rotation followed by a dilation.

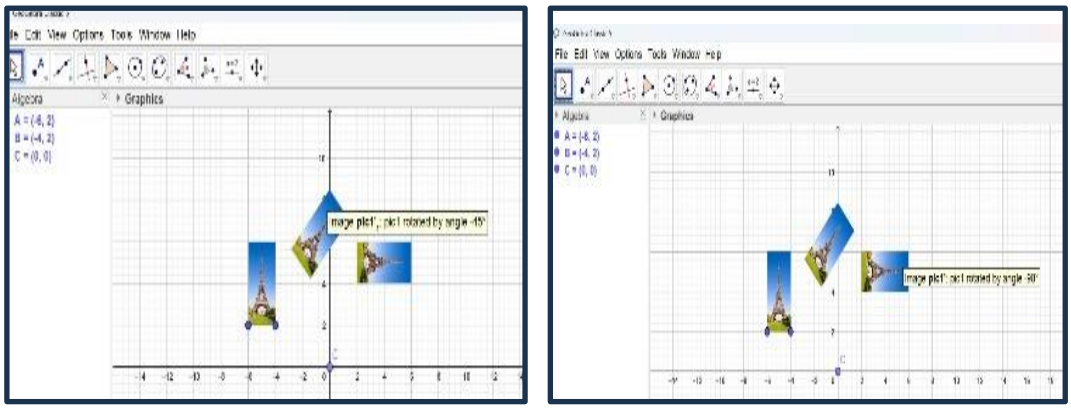


Figure 5. Rotation about $C(0, 0)$ through −45° and −90° in GeoGebra; the two panels show the same construction with the label of the −45° image (left) and of the −90° image (right).

Figure 5 shows rotations of the picture about the origin $C(0, 0)$ in GeoGebra. The lower corners of the pre-image are $A(-6, 2)$ and $B(-4, 2)$. With $\theta = -45°$, (3) gives $A' \approx (-2.83, 5.66)$ and $B' \approx (-1.41, 4.24)$; with $\theta = -90°$, it gives $A' = (2, 6)$ and $B' = (2, 4)$, so that the picture occupies $2 \le x \le 6$, $4 \le y \le 6$ and the top of the tower points in the positive x-direction. These values agree with the positions shown in the graphics view. Writing $\alpha = -\theta$ for the size of a clockwise angle turns (3) into $x' = x \cos \alpha + y \sin \alpha$, $y' = -x \sin \alpha + y \cos \alpha$; this form is valid only when α is measured clockwise. In both cases the distance of every point from C is preserved, and the angle between the pre-image and its image equals $|\theta|$.

3.1.4. Dilation

A dilation with centre O and scale factor k maps each point P to the point P' on the line OP with $OP' = k \cdot OP$; it preserves shape and angles but multiplies lengths by $|k|$ and areas by k^2 [25], [26]. With the centre at the origin, it is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = k \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

For $|k| > 1$, the figure is enlarged and for $0 < |k| < 1$ it is reduced. A dilation is a similarity transformation but not an isometry unless $|k| = 1$ [20].

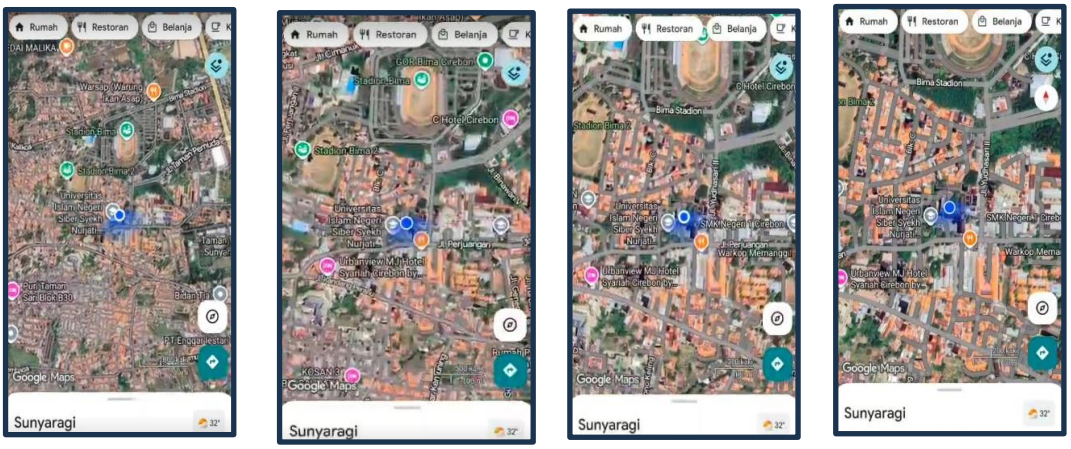


Figure 6. Four consecutive Google Maps screenshots taken while zooming.

Figure 6 shows four Google Maps screenshots taken while zooming with two-finger pinch gestures. Enlarging and reducing a digital image is an image-scaling operation [27]. Fitting a similarity transformation to consecutive screenshots gives $k \approx 1.7$ from the first to the second screenshot (zoom in), $k \approx 0.85$ from the second to the third and $k \approx 0.43$ from the third to the fourth (zoom out), with rotation angles below 1° in each step. The sequence is therefore a zoom-in followed by two zoom-outs rather than a continuous enlargement, and the last screenshot shows a wider area than the first (net $k \approx 0.6$). The fixed points of the fitted maps, i.e. the centres of dilation, lie within about 25 pixels (less than 10% of the screen width) of the blue location marker. This explains why the marker

appears almost stationary while other points move away from it ($k > 1$) or towards it ($k < 1$). The dilation describes the geometry of the map itself: text labels, icons and buttons keep their size, and the application changes the level of detail as the scale changes.

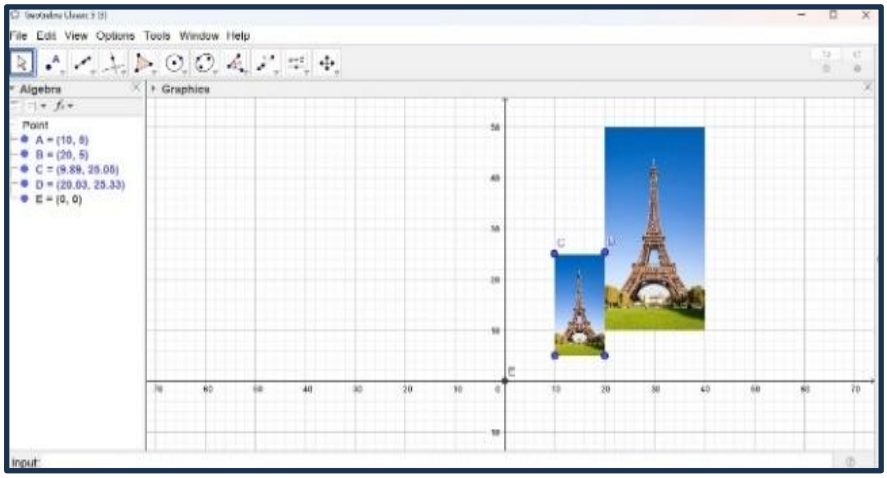


Figure 7. Dilation with centre $E(0, 0)$ and scale factor $k = 2$ in GeoGebra.

Figure 7 shows a dilation with centre $E(0, 0)$ and $k = 2$ in GeoGebra. The pre-image has lower corners $A(10, 5)$ and $B(20, 5)$ and upper corners at approximately $C(9.89, 25.05)$ and $D(20.03, 25.33)$; C and D were placed manually at the corners and therefore deviate slightly from the exact values (10, 25) and (20, 25). By (4), the corners are mapped to $A'(20, 10)$, $B'(40, 10)$, $C' \approx (19.78, 50.10)$ and $D' \approx (40.06, 50.66)$, so the image occupies $20 \leq x \leq 40$, $10 \leq y \leq 50$, in agreement with the enlarged picture. Both side lengths are doubled (10 to 20 and 20 to 40), the area is multiplied by $k^2 = 4$, and the ratio of the sides and all angles are unchanged. Note that D is not the image of C; both are corners of the pre-image.

3.1.5. Summary

Table 1 summarises the four correspondences, the coordinate rule of each transformation, the properties that are preserved or changed, and the qualifications revealed by the analysis of the screenshots.

Table 1. Smartphone interactions and the corresponding geometric transformations

Interaction (figures)	Transformation	Coordinate rule	Preserved	Changed	Qualification
Mirrored copy in a split screen (Fig. 2); GeoGebra (Fig. 1)	Reflection in a line	$(x, y) \mapsto (x, -y)$	Distance, angle, size	Orientation (det = −1)	Display function, not a gesture; $r = 0.96$ vs 0.88 for a half-turn
Swiping between pages, scrolling (Fig. 3); GeoGebra (Fig. 4)	Translation by $\mathbf{u} = (a, b)$	$(x, y) \mapsto (x + a, y + b)$	Distance, angle, size, orientation	Position	Launcher animation adds tilt; model holds for the net displacement
Automatic screen rotation (Fig. 5); GeoGebra (Fig. 6)	Rotation through θ about a centre	$(x, y) \mapsto (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$	Distance, angle, size, orientation	Angular position	$\theta = -90°$ in the screen frame, followed by rescaling
Pinch-zoom on a map (Fig. 7); GeoGebra (Fig. 8)	Dilation with factor k about a centre	$(x, y) \mapsto (kx, ky)$	Shape, angles, ratios	Size (lengths $\times\lvert k\rvert$, area $\times k^2$)	$k \approx 1.7$, 0.85, 0.43; $\lvert\theta\rvert < 1°$; labels not scaled

3.2. Discussion

The results show that each of the four interactions has a precise mathematical counterpart, but they also show where the correspondence needs qualification. Scrolling or swiping, screen rotation and zooming are produced directly by touch gestures or by moving the device, whereas the mirrored image results from a display function. All four transformations belong to the group of similarity transformations of the plane, which preserve shape [20]. Translations, rotations and reflections are isometries; translations and rotations also preserve orientation, whereas reflections reverse it. This is why a mirrored photograph cannot be obtained by any combination of dragging, rotating and zooming, all of which preserve orientation.

This classification gives a compact model of multi-touch manipulation. Identifying the screen with the complex plane, every orientation-preserving similarity can be written as

$$z' = az + b, \quad a = ke^{i\theta}, \quad k > 0 \tag{5}$$

where $|a| = k$ is the scale factor, $\arg a = \theta$ is the angle of rotation, and b is a translation. If two fingers move from p_1 and p_2 to q_1 and q_2, the unique map of this form that keeps the content under both fingers has $a = (q_2 - q_1)/(p_2 - p_1)$ and $b = q_1 - ap_1$. A pure drag ($a = 1$) is a translation, a pure pinch ($\theta = 0$) is a dilation about the fixed point $b/(1 - a)$, and a twist ($k = 1$) is a rotation about the same point; in general, a two-finger gesture combines all three. The centres estimated for Figure 6 are fixed points of exactly this kind, which explains in elementary terms why the centre of a pinch-zoom lies between the fingers rather than necessarily at the centre of the screen.

These findings align with ethnomathematics studies that identify translation, reflection, rotation, and dilation in cultural artefacts such as batik, woven, and ornamental motifs [6], [21]–[23], [25], and extend that line of work from static patterns to dynamic, interactive content. Screen examples differ from cultural artefacts in two respects: the learner can produce and undo the transformation, and the screen can display, or the learner can estimate its parameters (vector, angle, and scale factor). The findings also complement HCI studies that characterise gestures by user agreement and performance [7], [9], [10] by describing in mathematical terms what the gestures do to the displayed content.

For mathematics education, the examples offer contexts that students already know. A teacher can, for instance, ask students to estimate a translation vector from two scrolling screenshots, to decide whether a flipped photograph is a reflection or a half-turn by checking the left–right order of its features, or to estimate the scale factor of a map zoom from the scale bar. Such tasks connect formal notation with familiar actions, in the spirit of contextual and ethnomathematics-based approaches to transformation geometry [13], [26], and GeoGebra provides a convenient way of checking the results [5], [24]. Whether such activities improve conceptual understanding has not been tested in this study.

For interface design, the analysis clarifies which properties users can rely on. A drag that preserves size and orientation, a screen rotation that keeps content upright, and a zoom whose centre stays between the fingers are all cases in which the displayed transformation matches the user's action. Decorative transitions that add tilt or perspective (Figure 2) break this correspondence for a short time; whether this affects usability is an empirical question.

This study has several limitations. First, it is a qualitative descriptive study of a small number of examples recorded by the authors on particular devices and applications; other devices, launchers and map applications may behave differently. Second, the quantitative checks are approximate because pixel correlation and feature-based fitting are affected by image compression, cropping, interface overlays, and the map's changing level of detail; the fitted parameters are estimates. Third, no users or learners were involved, so the study does not measure users' awareness of the transformations or any learning outcome. Fourth, the analysis is limited to two-dimensional transformations, whereas augmented and virtual reality involve three-dimensional interaction [10]. Future studies could test classroom activities based on these examples, systematically compare devices and applications, and extend the analysis to three-dimensional transformations in immersive environments.

4. CONCLUSION

This study examined how everyday smartphone interactions can be described by the basic transformations of plane geometry. Scrolling and swiping between pages displace content by a translation, automatic screen rotation turns content about a centre so that it stays upright, zooming a map enlarges or reduces it by a dilation about a point near the user's fingers, and a mirrored image shown in a split screen is a reflection in the dividing line. Checking the screenshots quantitatively and reconstructing each case in GeoGebra confirmed these correspondences and corrected several intuitive descriptions: a flipped

image must be distinguished from a half-turn, screen rotation is accompanied by rescaling, decorative animations are not pure transformations, and a zoom sequence may combine enlargement and reduction. All four interactions are similarity transformations, and two-finger gestures can be described by a single map that combines translation, rotation and dilation. Transformation geometry is thus embedded in ordinary digital interaction, and the verified examples give teachers familiar contexts for connecting formal notation with students' experience and give designers a criterion for keeping displayed changes consistent with users' actions. Because only a small set of examples was analysed and no learners were involved, classroom studies and extensions to three-dimensional interaction are needed to test the educational value of these contexts.

REFERENCES

[1] D. Kurniawati, "Pengaruh penggunaan gadget terhadap prestasi siswa [The effect of gadget use on student achievement]," *Edukatif: J. Ilmu Pendidik.*, vol. 2, no. 1, pp. 78–84, 2020, doi: 10.31004/edukatif.v2i1.78.

[2] S. Fitri, F. D. Saputra, and M. Taufiq, "Pengaruh penggunaan smartphone terhadap minat belajar siswa SMK Negeri 1 Tasikmalaya [The effect of smartphone use on the learning interest of students of SMK Negeri 1 Tasikmalaya]," *JUPEIS*, vol. 1, no. 3, pp. 1–5, Jul. 2022, doi: 10.55784/jupeis.Vol1.Iss3.65.

[3] C. Ma'rifah and A. Qohar, "Keaktifan siswa pada penerapan penggunaan media pembelajaran papan rotasi (Partasi) materi transformasi geometri [Student activeness in the use of the rotation board (Partasi) learning medium for geometric transformation]," *BRILIANT: J. Riset Konseptual*, vol. 5, no. 4, p. 698, Nov. 2020, doi: 10.28926/briliant.v5i4.525.

[4] A. Sukenti, "Penerapan model pembelajaran berbasis masalah berbantuan aplikasi GeoGebra untuk meningkatkan hasil belajar siswa pada materi transformasi geometri kelas XI IPAS–1 SMA Negeri 7 Medan [Applying GeoGebra-assisted problem-based learning to improve student learning outcomes in geometric transformation in class XI IPAS–1 of SMA Negeri 7 Medan]," *KOLONI*, vol. 2, no. 2, pp. 461–472, Jun. 2023, doi: 10.31004/koloni.v2i2.508.

[5] N. S. Yolanda, Y. Fitri, Refnywidialistuti, D. N. Perdana, V. S. Mulia, and T. Suriani, "Aplikasi GeoGebra dalam pembelajaran transformasi geometri [GeoGebra in the learning of geometric transformation]," *JERKIN*, vol. 4, no. 1, pp. 2240–2243, Jul. 2025, doi: 10.31004/jerkin.v4i1.1964.

[6] D. Yanti and S. Haji, "Studi tentang konsep-konsep transformasi geometri pada kain Besurek Bengkulu [A study of geometric transformation concepts in Besurek fabric of Bengkulu]," *JNPM (J. Nas. Pendidik. Mat.)*, vol. 3, no. 2, p. 265, Sep. 2019, doi: 10.33603/jnpm.v3i2.1744.

[7] J. O. Wobbrock, M. R. Morris, and A. D. Wilson, "User-defined gestures for surface computing," in *Proc. SIGCHI Conf. Hum. Factors Comput. Syst. (CHI '09)*, Boston, MA, USA, 2009, pp. 1083–1092, doi: 10.1145/1518701.1518866.

[8] S. Yadav, P. Chakraborty, A. Kaul, Pooja, B. Gupta, and A. Garg, "Ability of children to perform touchscreen gestures and follow prompting techniques when using mobile apps," *Clin. Exp. Pediatr.*, vol. 63, no. 6, pp. 232–236, Jun. 2020, doi: 10.3345/cep.2019.00997.

[9] J. Martin-Gutierrez and M. S. Del Rio Guerra, "Analysing touchscreen gestures: A study based on individuals with Down syndrome centred on design for all," *Sensors*, vol. 21, no. 4, Art. no. 1328, Feb. 2021, doi: 10.3390/s21041328.

[10] Z. Dong *et al.*, "Touch-move-release: Studies of surface and motion gestures for mobile augmented reality," *Front. Virtual Real.*, vol. 3, Art. no. 927258, Aug. 2022, doi: 10.3389/frvir.2022.927258.

[11] D. Setyadi and A. J. Hapsari, "Development of ICT-based geometry transformation module for high school students," *J. Pendidik. MIPA*, vol. 23, no. 3, pp. 1112–1122, 2022, doi: 10.23960/jpmipa/v23i3.pp1112-1122.

[12] K. Khairiree, "Online learning and augmented reality: Enhancing students learn transformation geometry during the COVID-19 pandemic," in *Proc. 25th Asian Technol. Conf. Math. (ATCM)*, 2020,

pp. 304–313. [Online]. Available: https://atcm.mathandtech.org/EP2020/regular/21846.pdf

[13] D. N. Fitriyah, H. Santoso, and N. Suryadinata, "Bahan ajar transformasi geometri berbasis discovery learning melalui pendekatan etnomatematika [Discovery-learning-based teaching materials for geometric transformation through an ethnomathematics approach]," *J. Elemen*, vol. 4, no. 2, pp. 145–158, Jul. 2018, doi: 10.29408/jel.v4i2.705.

[14] T. Kuhlmann, P. Garaizar, and U.-D. Reips, "Smartphone sensor accuracy varies from device to device in mobile research: The case of spatial orientation," *Behav. Res. Methods*, vol. 53, no. 1, pp. 22–33, Feb. 2021, doi: 10.3758/s13428-020-01404-5.

[15] B.-A. Awaluddin, C.-T. Chao, and J.-S. Chiou, "Investigating effective geometric transformation for image augmentation to improve static hand gestures with a pre-trained convolutional neural network," *Mathematics*, vol. 11, no. 23, Art. no. 4783, Nov. 2023, doi: 10.3390/math11234783.

[16] M. Sandelowski, "Whatever happened to qualitative description?," *Res. Nurs. Health*, vol. 23, no. 4, pp. 334–340, Aug. 2000, doi: 10.1002/1098-240X(200008)23:4<334::AID-NUR9>3.0.CO;2-G.

[17] H. Snyder, "Literature review as a research methodology: An overview and guidelines," *J. Bus. Res.*, vol. 104, pp. 333–339, Nov. 2019, doi: 10.1016/j.jbusres.2019.07.039.

[18] D. G. Lowe, "Distinctive image features from scale-invariant keypoints," *Int. J. Comput. Vis.*, vol. 60, no. 2, pp. 91–110, Nov. 2004, doi: 10.1023/B:VISI.0000029664.99615.94.

[19] M. A. Fischler and R. C. Bolles, "Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography," *Commun. ACM*, vol. 24, no. 6, pp. 381–395, Jun. 1981, doi: 10.1145/358669.358692.

[20] D. A. Brannan, M. F. Esplen, and J. J. Gray, *Geometry*. Cambridge, U.K.: Cambridge Univ. Press, 1999, doi: 10.1017/CBO9780511807503.

[21] U. Hidayati, I. Haidar, and D. Marillah, "Eksplorasi etnomatematika konsep transformasi geometri pada ragam hias Mekongga motif Tabere [Ethnomathematical exploration of geometric transformation concepts in the Tabere ornamental motif of Mekongga]," *JagoMIPA: J. Pendidik. Mat. IPA*, vol. 5, no. 1, pp. 275–285, Mar. 2025, doi: 10.53299/jagomipa.v5i1.1467.

[22] R. Andriyani, R. Rahman, R. Irawati, E. J. Mutaqin, and N. Kamis, "Exploration of batik Jambi on learning transformation geometry," *Prima: J. Pendidik. Mat.*, vol. 7, no. 1, p. 89, Jan. 2023, doi: 10.31000/prima.v7i1.7305.

[23] D. N. Sulisawati, Y. Saputra, and N. Hidayatin, "Eksplorasi konsep rotasi geometris pada motif batik Labako Jember [Exploration of the concept of geometric rotation in the Labako batik motif of Jember]," *Differential: J. Math. Educ.*, vol. 2, no. 2, pp. 233–242, Dec. 2024, doi: 10.32502/differential.v2i2.424.

[24] M. Hanafi, K. N. Wulandari, and R. Wulansari, "Transformasi geometri rotasi berbantuan software GeoGebra [GeoGebra-assisted rotation in transformation geometry]," *Fibonacci: J. Pendidik. Mat. Mat.*, vol. 3, no. 2, pp. 93–102, Dec. 2017, doi: 10.24853/fbc.3.2.93-102.

[25] M. Maryati and R. C. I. Prahmana, "Learning trajectory of dilation and reflection in transformation geometry through the motifs of bamboo woven," *J. Didaktik Mat.*, vol. 8, no. 2, pp. 134–147, Oct. 2021, doi: 10.24815/jdm.v8i2.21283.

[26] F. I. Mufti and T. A. Aziz, "Desain pembelajaran matematika topik transformasi geometri dengan pendekatan realistic mathematics education berbasis etnomatematika [Mathematics learning design for geometric transformation with an ethnomathematics-based realistic mathematics education approach]," *Algoritma: J. Mat. Ilmu Pengetah. Alam Kebumian Angkasa*, vol. 2, no. 4, pp. 115–129, Jul. 2024, doi: 10.62383/algoritma.v2i4.102.

[27] F. Ramadhan, "Implementasi dan analisis image scaling menggunakan bilinear interpolation pada citra kendaraan [Implementation and analysis of image scaling using bilinear interpolation on vehicle images]," *J. Ilmu Komput. Sist. Inf. (JIRSI)*, vol. 4, no. 3, pp. 329–343, Sep. 2025, doi: 10.70340/jirsi.v4i3.235.