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Analysis of Geometric Transformations in Automatic Door System Response with a Motion Sensor

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ABSTRACT

Automatic sliding doors are usually studied as electronic systems, yet their motion offers a concrete context for geometric transformations. This study analyses how translation, reflection, and dilation describe the motion of a two-panel automatic sliding door at the main entrance of a shopping mall in Cirebon, Indonesia. The door was observed in 12 sessions over four days, with approximately 10–15 opening–closing cycles recorded per session, and the panel positions and opening widths were documented on video and mapped onto a Cartesian plane. Each panel moved by a pure horizontal translation along its rail, and the translations of the two panels were reflection-symmetric about the vertical centre line of the doorway. The representative opening width was 30 cm for a single user, 75 cm for two to three users, and 100 cm for a crowd. Taking the single-user opening as the reference aperture, the larger openings correspond to dilation factors k = 2.50 and k = 3.33 about the centre of the door, while the panels themselves remain rigid. Because detection distance and the number of users varied together and the door's control logic was not accessible, these factors describe the observed openings rather than the controller's rule. The door thus illustrates a composition of isometries combined with a change of scale of the aperture and can serve as a contextual resource for teaching geometric transformations.

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1. INTRODUCTION

The development of modern technology has brought significant advances in the automation of public facilities, including automatic door systems widely used in shopping malls, offices, and industrial buildings. According to Febrian et al. [1], the infrared sensor-based automatic door system improves warehouse operational efficiency by enabling automatic object detection, thereby reducing manual response time. Automatic doors are designed to improve efficiency, comfort, and safety by using motion sensors, such as PIR or

ultrasonic, that detect human presence without physical contact. The Automatic Sliding Door has the main components: a human motion sensor and a door mechanical drive motor. Handayani [2], for example, built an automatic door for a faculty office that uses a PIR motion sensor to detect approaching people. The development of these automated systems aligns with the increasing integration of science and technology to meet human needs in the modern era, including the adoption of smart sensors to enhance accessibility in public spaces. Venkataramanan [3] emphasizes the role of body temperature sensors in smart automatic doors to prevent physical contact during the pandemic, thereby enhancing the efficiency and safety of public facilities.

One of the most common types of sensors used in automated door systems is the Passive Infrared (PIR) sensor, a device that detects passive infrared radiation emitted by the human body. As a thermal infrared detector, it responds to changes in this radiation, which, at a body temperature of about 37 °C, lies mainly in the 8–14 μm band [4]. Rabb and Hidayat [5] designed an Arduino-based door in which the PIR sensor output drives a servo motor. Listijorini et al. [6] designed an Arduino-based hands-free door opener that uses an ultrasonic sensor to reduce contact transmission of COVID-19. When humans approach, these PIR sensors emit signals to microcontrollers such as Arduino, which further instruct the servo or DC motor to open the door and close it again after the object moves away. Wijaya et al. [7] used a PIR sensor with a servo motor rotating between 0° and 180° to open a prototype door and reported a working detection range of 1–5 m.

In its application, automatic door systems are not limited to electronic and control elements; they can also be analyzed using the principles of geometric transformations, such as translation, reflection, and dilation. The horizontal movement of the door panels to the right or left represents pure translation, while the symmetry of the two panels' movement about the door's centerline reflects geometric reflections. In addition, adjusting the width of the door opening based on the number of detected visitors can be interpreted as a change of scale (dilation) of the opening. A comparable sliding-door prototype using an ultrasonic sensor and a stepper motor was built by Wijaya, Tafrikhatin, and Prasetyo [8]. The concept makes the automatic door a practical illustration of the application of abstract mathematics in everyday technological engineering. Honestya et al. [9] built an Arduino-based automatic entrance with traditional gate architecture, and Roning et al. [10] built a PIR-triggered automatic sliding-gate prototype driven by a stepper motor. This interdisciplinary relevance enriches mathematics education by enabling real-world applications of technology.

By integrating principles of electronics, mechanics, and mathematics, the study aims to provide a deeper understanding of the operation of automatic door technology and the use of geometric transformations to model and analyze its mechanisms. Natali et al. [11] developed an IoT-based automatic door model in which a visible-light link drives a servo motor and the door status is reported online. In addition, this study contributes to mathematics education while affirming the relevance of applying mathematical theory to modern technology in daily practice. This research is expected to strengthen the connection between mathematical abstraction and practical engineering in the context of automation systems.

One of the sensors that is widely used in the development of automatic door systems is Passive Infrared (PIR). This sensor detects infrared radiation emitted by the human body, enabling it to recognize the presence of moving objects without physical contact. Haris et al. [12] described an automatic door, intended for office and mall entrances, in which a PIR sensor and an Arduino microcontroller drive a servo motor.

In addition to functioning as an automatic access system, sensor-based door technology is also being developed to improve security. Subekti et al. [13] developed an Internet of Things (IoT)-based door security system by utilizing PIR sensors and MC-38 magnetic sensors. The system not only detects the door's movement and condition but also sends image notifications via the Telegram application. The test results show that the system has a relatively fast response time and low error rate, making it effective in improving home security.

The development of automatic door systems is also increasingly optimized through the integration of smart devices, such as Android-based smartphones. Afandi et al. [14] built an Arduino-based portal-gate opener that is operated from a smartphone through an HC-06 Bluetooth module and works at distances of up to 15 m. This system allows users to control the door from a distance with greater flexibility and supports the efficient implementation of home automation. These findings show that collaboration between microcontrollers and mobile devices can improve the ease of use of automated door systems.

In line with that, Roning et al. [10] combined a PIR sensor with an HC-05 Bluetooth module so that a sliding-gate prototype can be opened automatically by the sensor or from an Android smartphone. This approach shows that automatic door systems can be designed adaptively to meet user needs.

Based on these studies, it can be concluded that automatic door systems based on PIR sensors and microcontrollers are not only relevant from a technological perspective but also interesting to study from a mathematical perspective. The regular movement of the door panel can be modelled using geometric transformations, such as translation, reflection, and dilation. Various previous studies have demonstrated the effectiveness of PIR sensors in controlling automatic doors when integrated with microcontrollers and drive motors. Most of these studies focus on electronic aspects, control systems, and hardware performance, such as sensor response speed, detection accuracy, and motor efficiency. However, studies that examine the movement of automatic doors from a mathematical perspective, particularly those that utilize direct observational data and measured door opening widths to construct geometric transformation models, remain relatively limited.

In other words, although automatic doors clearly exhibit regular movement patterns, such as linear displacement, symmetric motion of two panels, and variations in opening width, only a small number of studies have explicitly formulated these phenomena as geometric transformation models grounded in empirical data. This gap constitutes the novelty of the present study, which offers an analysis of geometric transformations (translation, reflection, and dilation) derived directly from observational and measurement data of a commercial two-panel automatic sliding door.

Based on this background, the present study aims to address the following objectives:

1. To identify the types of geometric transformations that emerge in the movement mechanism of sensor-based automatic sliding doors.
2. To model the translation vectors of the door panels based on the measured displacement distances during the opening and closing process.
3. To analyze the axis of symmetry and the reflective properties in the movement of the two automatic door panels.
4. To model the variation in door opening width as a dilation of the door aperture and to estimate the corresponding dilation factors.

By formulating these objectives, this study is expected to clarify the relationship between geometric transformation concepts and their application in modern automated technology, while also reinforcing the relevance of mathematics as an analytical tool in real-world contexts.

2. METHODS

This study used a descriptive observational design that combined field observation with coordinate-based mathematical modelling [15]. Field observation was used to identify and measure the motion of the door panels in their natural setting, and coordinate modelling was used to express the observed patterns as geometric transformations. This combination is consistent with the view that descriptive designs allow researchers to understand the processes and dynamics of a phenomenon holistically [15].

The study was conducted at the main entrance of CSB Mall, Cirebon, which utilizes a two-panel automatic sliding door system equipped with an overhead motion sensor. Commercial sliding doors of this type typically use a radar (microwave) motion detector to trigger opening and an active-infrared presence detector to protect the doorway [16], [17]; the specific sensor model installed at the site could not be inspected. The research objects included the door panels, the movement rails, and the system's response to sensor detection when a single individual, several individuals, or a crowd passed through the entrance. The research location was selected purposively, as the characteristics of the automatic door system at this site allow for repeated observations under relatively consistent conditions [18].

Data were collected through direct observation and documentation. Observations were carried out over four days across three different time intervals, namely morning (09:00–11:00), midday (12:00–14:00), and afternoon (16:00–18:00). One observation session was conducted in each time interval, resulting in a total of twelve observation sessions. Each session involved recording approximately 10 to 15 door-opening and closing cycles, with variations in observed events including single individuals, 2 to 3 individuals, and crowds.

The sensor's detection distance was estimated based on the user's position relative to the door at the moment the system began to respond, using the user's step distance from the marked sensor activation point on the floor. This estimation was verified through a step-by-step analysis of video recordings to ensure the accuracy of sensor activation timing relative to the user's position. The opening width in each cycle was estimated from the same video recordings as the distance between the inner edges of the two panels at the moment of

maximum opening, and the values reported in Table 1 are representative (typical) widths for each user condition.

In addition to field observations, the observed panel positions were modelled on a Cartesian plane with the origin at the bottom of the door's vertical centre line, the x-axis along the floor, and the y-axis along the centre line. The corner points of each panel before and after motion were mapped onto this plane, and the translation, reflection, and dilation relating them were derived analytically, following the approach used to identify transformation concepts in real objects [19]. The dilation factor of the aperture was computed as $k = W/W_0$, where W is the representative opening width for a given user condition and $W_0 = 30$ cm is the representative opening width for a single user.

Data validity was ensured through repeated observations at different times and under different conditions to confirm the consistency of door movement patterns. Furthermore, data triangulation was conducted by comparing direct observation results, field notes, and video documentation [20]. The process of coordinate modeling and the interpretation of geometric transformations were also reviewed with peers to minimize bias and interpretative errors. All validated data were subsequently analyzed to identify and confirm patterns of translation, reflection, and dilation in the automatic door mechanism.

3. RESULTS AND DISCUSSION

Geometry transformation is a branch of mathematics that studies changes in the position, size, or orientation of an object in a coordinate system. In the context of motion-sensor-based automatic door systems, this concept is relevant because the door's movement during opening and closing follows a specific pattern rather than being driven solely by a physical mechanism. Using a mathematical approach, the door's movement can be represented by points, lines, and transformation equations, enabling a more systematic analysis of its movement patterns.

3.1. Results

This part of the results presents findings on the application of geometric transformations to motion-sensor-based automatic door mechanisms, especially during the opening and closing of door panels, represented by translation, reflection, and dilation. Data collected through simulations and field observations show that door movements do not occur randomly, but rather follow a pattern of position changes and sizes that can be mathematically modeled. Thus, these results provide a basis for assessing the extent to which the concept of geometric transformation can be used to describe the automatic door working system in a measurable way and to support further analysis in the next discussion.

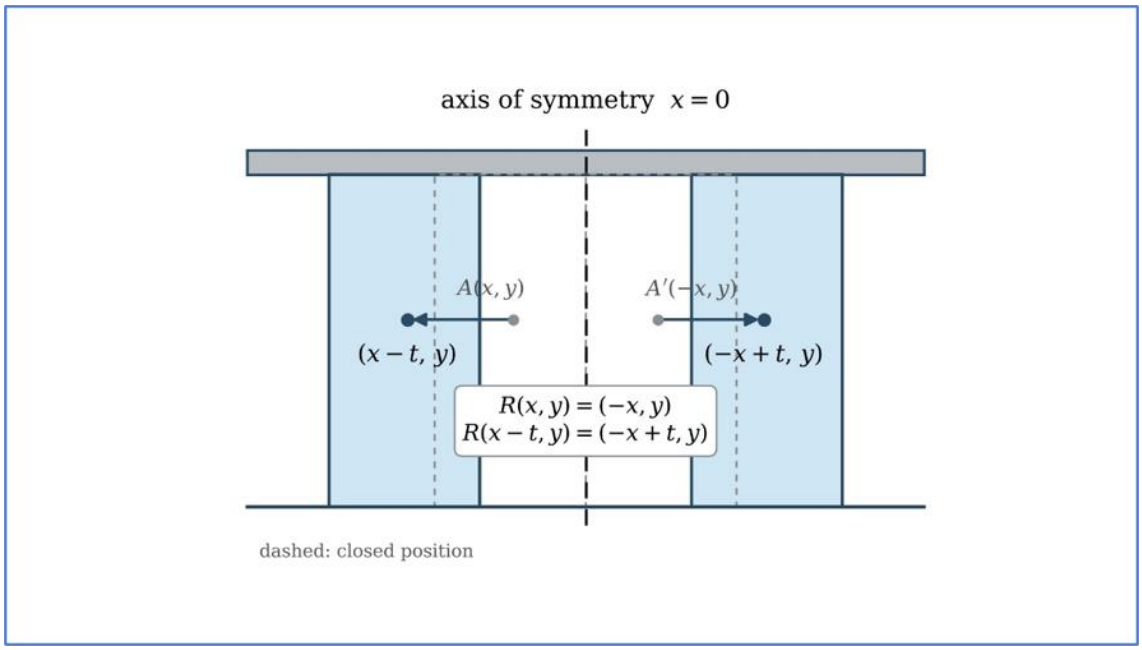


Figure 1. Reflection of the two door panels about the vertical centre line x = 0

The results of observation and analysis of the two-panel automatic sliding door system show that the movement of the two door panels is symmetrical and follows a geometric reflection pattern against the center line of the door. When the door opens, the left panel moves to the left and the right panel moves to the right at the same distance. Instead, when the door closes, the two panels move toward the centerline in a balanced manner.

Analysis of the position of the points in each panel shows that the points in the left panel can be represented by coordinates (x, y), while the corresponding points in the right panel are at (−x, y). This coordinate relationship is consistent throughout the entire door movement process, whether the door is closed, open, or fully open.

In addition, it was found that the movement of the door is not a reflection alone: each panel undergoes a pure translation, and the translations of the two panels are related by reflection about the centre line (Section 3.2). The door drive motor controls the panels' movement symmetrically, keeping the mechanical system stable, in line with the microcontroller-based automatic door design.

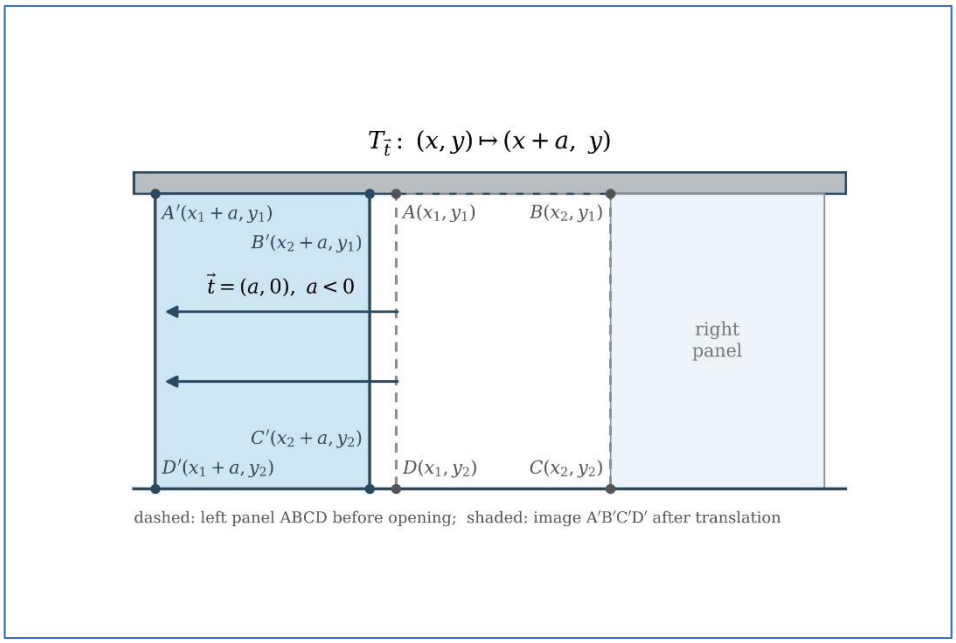


Figure 2. Translation of the left door panel ABCD by the vector t = (a, 0)

Figure 2 illustrates the translation of a door panel. The door opens automatically, without physical contact, when its sensor detects a person. When the sensor detects a person at a certain distance, the integration between the sensor, the door controller, and the drive motor results in a rapid door response.

The translation of the door in the context of geometry can be determined by assigning the corner points of ABCD and modeling the door's movement as a horizontal translation. The use of translation on automatic doors becomes particularly relevant because the door's movement follows a fixed linear trajectory. By mapping the starting points and image points A', B', C', and D', the reader can see how geometry and technology come together in real-life applications.

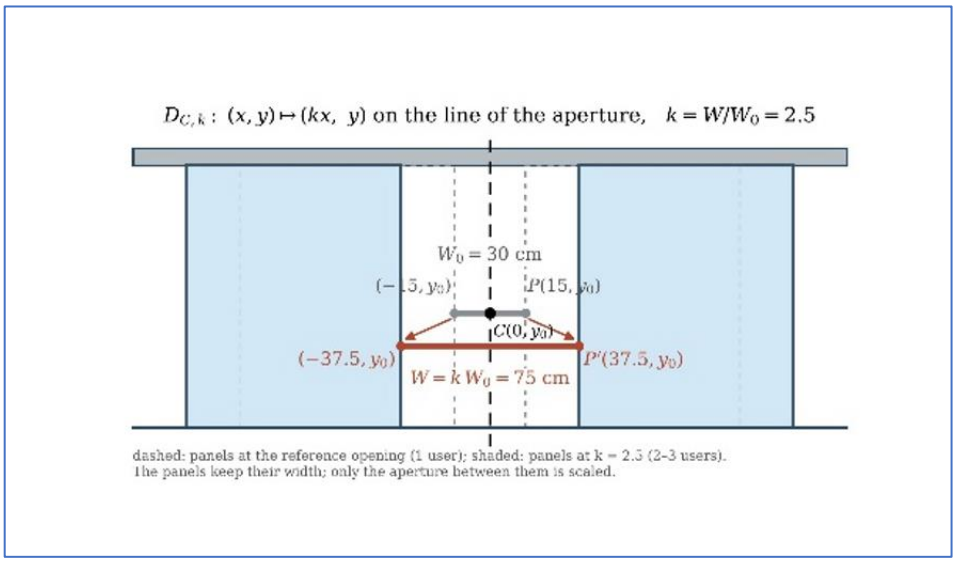


Figure 3. Dilation of the door aperture about its centre C(0, y_0) with factor k = W/W_0

The observations also showed that the door did not always open to the same width. The representative opening widths (Table 1) were 30 cm for a single user, 75 cm for two to three users, and 100 cm for a crowd. Taking the single-user opening as the reference aperture, the larger openings are dilation images of this aperture about the centre of the door, with factors $k = 2.50$ and $k = 3.33$.

In these measurements, the dilation factor increases with the number of users approaching the door. Because the number of users and their detection distance varied together, the data cannot separate the effect of each factor. Dilation is therefore used here as a mathematical description of the observed openings rather than as a model of the controller's internal rule.

This approach shows that geometric transformation is not just an abstract theory in the classroom, but a principle used in modern mechanical systems. This understanding strengthens the argument that the dilation process could serve as a model for explaining variations in automatic door motion in the context of motion-sensor-based technologies.

3.2. Discussion

This part of the discussion interprets the results of the research by relating the findings obtained to the concept of geometry transformation, namely translation, reflection, and dilation, in an automatic door system based on motion sensors. The analysis was carried out to assess the fit between theory and empirical conditions, ensuring that the mathematical modeling used was not only abstract but also reflected the automatic door's working mechanism in real time. Through this approach, the discussion aims to show that the movement patterns of automatic doors can be explained systematically by the principle of geometric transformation, while emphasizing that sensor-based technology has a mathematical basis that can be analyzed and measured, and can be scientifically accounted for.

There are three types of transformations related to the automatic door mechanism, namely translation, reflection, and dilation. Translation describes the linear transfer of the door panel from the initial position to the new position, while reflection occurs when two panels move in opposite directions while maintaining symmetry about the center line. Meanwhile, dilation occurs when the width of the door opening changes with the number of users approaching the door, so that the opening size is not always the same. The three transformations show that the automatic door system can be understood through structured mathematical representations, since translation, reflection, and dilation are forms of geometric transformations that are also found in real objects, such as building design, where changes in the position and size of objects describe translation, reflection, and scale changes systematically [21].

The movement pattern of automatic doors can be explained systematically through the principle of geometric transformation, while emphasizing that sensor-based technology has a mathematical foundation that can be analyzed and scientifically accounted for. The concept of geometric transformations, including translation, reflection, and dilation, applied to real objects has been extensively discussed in ethnomathematical studies, where field observations of physical objects reveal patterns of change in point positions consistent with

geometric transformation theory [21]. In addition, a formal understanding of geometric transformations, including the influence of changes in the position and orientation of objects in Cartesian coordinates, is developed through proofs in transformation geometry, a topic in which students often make errors [22].

Reflection

Reflection is a transformation that maps a point to its image point against a specific line. On a two-panel sliding door, the central line of the door serves as a mirror line, as shown in Figure 1., where the dot on the left panel and its image on the right panel are aligned.
The vertical center line can be modeled as:

$$x = 0$$

So the rules of reflection:

$$(x, y) \rightarrow (-x, y)$$

This visual explanation aligns with the description of the concept of reflection in ethnomathematical research on buildings, where symmetric structures are identified as reflections about a vertical axis [21].
In the context of sliding doors, the left and right panels are mirror images of each other. The important points on the left panel have pairs that can be mathematically calculated using the above reflection rules.

The sliding door picture shows.

- The left panel has a reference point A with the coordinates (x, y)
- The right panel has a image point A' with the coordinates $(-x, y)$
- The center line of the door becomes a symmetrical axis

When the door opens:

- The left panel shifts to the left as far as: $t(x, y) \rightarrow (x - t, y)$
- The right panel shifts to the right as far as: $t(-x, y) \rightarrow (-x + t, y)$

The reflection relationship is preserved, because reflecting the displaced left-panel point gives the displaced right-panel point:

$$R(x - t, y) = (-(x - t), y) = (-x + t, y)$$

Which means the two points remain equally spaced from the center line.

Thus, the image is not just a static illustration, but reflects the dynamic behavior of the automatic door. This coordinate visualization reinforces the finding that reflection occurs consistently during the door movement process.
Reflection on automatic doors is a tangible form of a broader class of geometric transformations. Reflections preserve size and distance, as is clearly seen in the movement of the door panels; such properties are commonly visualised dynamically with GeoGebra in transformation learning [23].
The automatic door movement shows not only reflection but also translation. The left panel is translated to the left, the right panel is translated to the right, but both retain a reflective position. This shows that:

$$T_{(t,0)} = R \circ T_{(-t,0)} \circ R$$

Here R(x, y) = (−x, y). In other words, each panel moves by a pure translation, and the translation of the right panel is the reflection-conjugate of the translation of the left panel; the door is therefore a concrete example of a composition of transformations that can be identified both visually and algebraically.

The reflection movement that appears on an automatic sliding door cannot be explained solely by the concept of geometric transformation; it is also a direct result of the mechanical system that regulates the door's performance. In microcontroller-based automatic doors, the movement of the left and right panels does not occur randomly, but is controlled by an electronic circuit and an actuator device that regulates the direction and speed of the panel's movement. Haris et al. [12] describe an Arduino-based automatic door design using PIR sensors configured so that the motor moves the door panel in a coordinated manner in response to the motion sensor signal. In the observed door, the symmetric motion seen in the video recordings indicates that the drive mechanism moves both panels at the same speed and over the same distance.

This balanced coordination of movements is likely to serve several functions:

1. prevents unbalanced loads on the rails that can cause wear,
2. maintains the stability of the panel movement so that the door does not vibrate or stall,
3. improves sensor response so that doors can open precisely when detecting human presence, and
4. ensures that both panels perform a symmetrical opening or closing movement against the center of the door.

Because of this symmetry, the left and right panels consistently follow a reflection pattern: each panel moves in the opposite direction but maintains the same distance from the door's center line. Thus, the phenomenon of reflection on sliding doors is not only a mathematical concept but also a consequence of mechanical and electronic design aimed at creating stable, precise, and safe movements for users. The findings show that in real applications, mathematical concepts such as reflection have strong links to mechanical engineering and microcontroller control systems used in modern automatic door devices.[12]

Translation

In the Arduino-based prototype described by Haris et al. [12], the PIR sensor works alongside the Arduino as a signal processor, and a servo motor drives the door. In the context of geometry, the movement of the observed sliding door can be analyzed using the concept of a translational transformation, which displaces an object's position without changing its shape, direction, or size. In the context of geometry, automatic door movement is a real-world example of a translational transformation, where the door leaf shifts linearly from the starting position to the open position. Translational analysis of doors can be performed by assigning coordinate points to the door's corners (A, B, C, and D) and determining the positions of their images (A', B', C', and D') as the door moves.

To explain automatic door translation using Figure 2, the door leaf on the left can be modeled as a rectangle with the following four corner points:

a. A : Upper left corner of the door
b. B : upper right corner of the door
c. C : bottom right corner of the door
d. D : bottom left corner of the door

Suppose the initial coordinates of the door before moving are:
a. A(x_1, y_1)
b. B(x_2, y_1)
c. C(x_2, y_2)
d. D(x_1, y_2)

When the sensor detects a person, the door moves horizontally to the left or to the right.

Translation shifts all points of an object by the same distance and in the same direction. If the translation vector is $\vec{t} = (a,b)$ then each point X(x, y) moves to X'(x + a, y + b). In mall automatic doors, the movement of the door only moves in the direction of the horizontal axis, so that:
a. a = horizontal displacement of the panel (cm), with a < 0 for the left panel and a > 0 for the right panel
b. b = 0

Then the translation transformation:

$$X'(x', y') = (x + a, y)$$

If the panel shifts by a, its image is:
a. $\boldsymbol{A'} = (x_1 + a, y_1)$
b. $\boldsymbol{B'} = (x_2 + a, y_1)$
c. $\boldsymbol{C'} = (x_2 + a, y_2)$
d. $\boldsymbol{D'} = (x_1 + a, y_2)$

Thus, the ABCD rectangle moves to A'B'C'D' without changing shape and size, according to the nature of the translation. As a numerical example, each sliding panel is taken as 90 cm wide and 210 cm high, consistent with commercial two-panel sliding doors that provide a full clear opening of about 1.2–1.8 m and a clear height of about 2.1 m [17]. The closed left panel then has corners A(−90, 210), B(0, 210), C(0, 0), and D(−90, 0). For the crowd condition in Table 1 (W = 100 cm), each panel moves by |a| = W/2 = 50 cm, so the left panel is translated by t = (−50, 0) to A′(−140, 210), B′(−50, 210), C′(−50, 0), and D′(−140, 0), while the right panel is translated by (50, 0). Because such a door can open to about 180 cm, the 100 cm opening is the largest observed opening rather than the full opening of the door.

In Figure 2, this is evident when the door opens: the glass plane that originally covers the center of the sliding door disappears from its initial position and shifts to the side as it slides. Automatic door translation is thus a pure horizontal translation.

Mathematical Model of Door Movement Based on Sensor Detection. This model uses the door image and the ABCD coordinate point to clarify the door's x position before and after movement.

Initial condition (closed door). Put the initial coordinates:

$$A(x_1, y_1), B(x_2, y_1), C(x_2, y_2), D(x_1, y_2)$$

In this situation, the sensor has not detected the visitor, so the door is in the following position:

$$P_{closed} = \{A(x_1, y_1), B(x_2, y_1), C(x_2, y_2), D(x_1, y_2)\}$$

When the sensor detects a person, it generates a signal
which triggers the controller to shift the panel horizontally by a.
The door open condition (translation occurs) with the position of the door after translation to:

$$P_{open} = \{A'(x_1 + a, y_1), B'(x_2 + a, y_1), C'(x_2 + a, y_2), D'(x_1 + a, y_2)\}$$

The door movement observed in the video recordings was linear, consistent with this model: the door does not rotate or expand but shifts horizontally along its rails.

Dilation

This section explains how dilation describes the variation in door opening width. A dilation with centre C and factor k > 0 maps a point P to a point P′ on ray CP such that CP′ = k·CP; it changes the size of a figure while preserving its shape. The door panels are rigid and do not change size; what changes is the aperture between the inner edges of the two panels. The aperture is therefore modelled as a horizontal segment at height y_0 whose midpoint C(0, y_0) lies on the door's vertical centre line, and the dilation with centre C and factor k maps its edges (±W_0/2, y_0) to (±kW_0/2, y_0) (Figure 3.). The representative opening widths and the resulting dilation factors are shown in Table 1.

Table 1. Measured Opening Widths and Aperture Dilation Model

User Conditions	Sensor detected distance (m)	Representative opening width W (cm)	Dilation factor k = W/W_0	Aperture edges (±kW_0/2, y_0)	Transformation Explanation
No one	Not detected	0 (closed)	–	(0, y_0)	Initial (closed) state
1 person	2.5 m	30 cm	k = 1.00	(±15, y_0)	Reference aperture (W_0 = 30 cm)
2–3 people	1–1.5 m	75 cm	k = 2.50	(±37.5, y_0)	Enlarged aperture
Crowd	< 1 m	100 cm (largest observed)	k = 3.33	(±50, y_0)	Largest observed aperture

Table 1 shows that the representative opening width increases from 30 cm for a single user to 75 cm for two to three users and 100 cm for a crowd. With W_0 = 30 cm, the corresponding dilation factors are k = 1.00, 2.50, and 3.33. The closed state (W = 0) cannot be obtained from the reference aperture by a dilation with k > 0; it is treated as the initial state from which both panels are translated.

For example, for 2–3 users:

- Centre of dilation: C(0, y_0), the midpoint of the aperture on the door's centre line
- Edge of the reference aperture: P(15, y_0), since $W_0/2$ = 15 cm

The image of P under the dilation is:

$$P'(x', y') = (C_x + k(x - C_x), C_y + k(y - C_y))$$

With k = 2.50 for 2–3 users:

$$P' = (0 + 2.5(15 - 0), y_0 + 2.5(y_0 - y_0)) = (37.5, y_0)$$

Thus the aperture edge moves from 15 cm to 37.5 cm from the centre line, and the opening width becomes 2 × 37.5 = 75 cm, in agreement with Table 1. Each panel is translated by |a| = 37.5 cm and keeps its own width; the change of scale belongs to the aperture, not to the panels. In general, $|a| = kW_0/2 = 15k$ cm, which links the dilation factor of the aperture to the translation of each panel.

This model links the three transformations: each panel undergoes a translation, which is an isometry; the two translations are related by reflection about the centre line; and the resulting aperture is a dilation image of the reference aperture. The dilation factors in Table 1 summarise the observed openings and depend on how the door controller is configured, so they should not be read as a general property of automatic doors. Similar identifications of transformation concepts in real objects have been reported in ethnomathematics studies [19], [21].

4. CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, it can be concluded that the movement mechanism of automatic doors equipped with motion sensors is closely related to geometric transformation concepts, particularly translation, reflection, and dilation. Translation is observed in the horizontal displacement of the door panels from the closed position to the fully open position. Reflection is evident in the symmetric movement of the two panels with respect to the central axis of the doorway, while dilation describes the variation in door opening width as a change of scale of the aperture about the centre of the door, with the panels themselves remaining rigid. These findings indicate that everyday technological phenomena can be mathematically modeled and have the potential to serve as contextual resources for applied mathematics learning.

This study has several limitations that should be acknowledged. First, the research was conducted at a single observation site; therefore, the findings cannot be generalized to all types of automatic doors. Second, the description of door movements relied on

observation and video documentation, without direct access to the automatic door mechanism's internal control logic or system data. Third, there are potential confounding factors, such as security modes, visitor density, or specific technical conditions of the door system, which may influence movement patterns but could not be fully controlled in this study. Fourth, the opening widths in Table 1 are representative values for each user condition, and detection distance and number of users varied together, so the dilation factors describe the observed openings rather than the controller's rule.

Given these limitations, future research is recommended to adopt approaches more closely aligned with experimental studies. The use of software such as GeoGebra or MATLAB can be further developed to evaluate the accuracy of geometric transformation models by comparing simulation results with empirically measured door movements. In addition, future studies may investigate the effectiveness of using automatic door mechanisms as learning media or instructional contexts in improving students' understanding of geometric transformation concepts. Comparative studies between automatic doors that use different sensing technologies, such as radar, active infrared, passive infrared, and ultrasonic sensors, are also suggested to examine differences in the resulting transformation characteristics. Such efforts are expected not only to refine mathematical modeling but also to expand the impact of this research on education and technology.

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