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Procedural Dominance in Students' Reasoning on the Limit Definition: Insights from a Ways of Thinking Framework

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ABSTRACT

This study explores university students' ways of thinking about the limit concept in differential calculus and uncovers how they construct the meaning of the limit definition through their reasoning. Data were collected from 28 students in mathematics education at a university in West Java, Indonesia, through written tasks and semi-structured interviews. Thematic analysis identified dominant reasoning patterns, revealing that 71% of students exhibited procedural reasoning, 18% demonstrated conceptual reasoning, and 11% displayed formal reasoning. Based on a systematic data reduction process, these patterns were categorized into procedural, conceptual, and formal ways of thinking, and six representative participants were purposively selected for in-depth analysis. The findings show that students predominantly exhibited procedural reasoning, relying heavily on algorithmic manipulation and symbolic recall rather than conceptual understanding or formal justification. This pattern indicates that many students have not yet internalized the formal meaning of the limit definition, resulting in mechanical rather than reflective reasoning. These findings highlight the need for instructional designs that promote conceptual reflection and formal reasoning in calculus learning, enabling students to move beyond procedural competence toward a more integrated understanding of the limit concept.

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1. INTRODUCTION

The concept of the limit is a foundational pillar of differential calculus and central to students' understanding of continuity, derivatives, and integrals. It introduces learners to the rigor and precision of mathematical reasoning that distinguishes advanced mathematics from elementary computation [1]. Despite its importance, numerous studies have shown
25 that students often struggle to comprehend the formal definition of the limit, typically resorting to procedural strategies that focus on symbolic manipulation rather than

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conceptual understanding [2], [3]. The concept of a limit is fundamental to establishing the formal underpinnings of calculus. It allows for a precise definition of continuity and differentiability and serves as the basis for integral calculus [4], [5]. The understanding of limits requires students to reconcile intuitive notions, such as approaching a value, with formal mathematical definitions involving $\varepsilon - \delta$ reasoning [6]. Prior research indicates that students frequently rely on heuristic or informal approaches, such as substitution or numeric approximation, rather than engaging with the formal logical structure of the limit definition [7], [8], [9]. These tendencies highlight the gap between procedural competence and conceptual understanding, demonstrating that mastery of symbolic techniques alone does not guarantee meaningful comprehension of the limit concept. To address this gap, the Ways of Thinking (WoT) framework from Harel provides a valuable lens for examining students' cognitive processes in mathematics [10]. WoT distinguishes between procedural, conceptual, and formal reasoning, emphasizing how learners construct and interpret mathematical meaning [11], [12]. Procedural thinking involves following algorithmic steps without necessarily understanding underlying concepts, while conceptual thinking focuses on relational understanding and meaning-making. Formal thinking extends conceptual reasoning to rigorous, logically consistent frameworks. Applying the WoT framework to calculus allows researchers to analyze not only what students can do but also how they reason and make sense of formal mathematical concepts [13], [14].

The purpose of this research is to explore university students' ways of thinking about the definition of the limit through evidence drawn from written tasks and semi-structured interviews. The focus is on how students approach, interpret, and reason about the formal definition of the limit in calculus. By employing thematic analysis, this study seeks to reveal dominant patterns in students' reasoning and provide insights into the nature of their understanding. The findings indicate that students predominantly use procedural thinking, underscoring the need for pedagogical strategies that encourage conceptual engagement and the development of formal reasoning in calculus. Substantial research has investigated students' understanding of the limit concept, often focusing on common misconceptions, procedural errors, and/or difficulties in applying $\varepsilon - \delta$ definitions [15], [16]. The prominent example concerns students' difficulty with the $\varepsilon - \delta$ definition, where many fail to interpret ε and δ as expressing arbitrary degrees of closeness. For instance, when asked to prove that $\lim_{x\to 3} 2x + 1 = 7$ using the $\varepsilon - \delta$ definition, students commonly treat ε and δ as fixed numerical values or attempt direct substitution rather than construct an appropriate δ that depends logically on ε. These persistent difficulties indicate a gap between procedural competence and meaningful comprehension of the limit concept, suggesting the need for deeper insight into how students construct and justify their reasoning. These studies consistently show that students tend to rely on mechanical calculations and heuristic methods rather than engaging in deep conceptual reasoning. However, most prior studies have primarily documented what misconceptions or errors occur, rather than exploring how students think and construct meaning when grappling with the formal definition [17], [18].

In addition, most of the existing research emphasizes either written task analysis or interview data alone, which limits a holistic understanding of students' cognitive processes [19], [20], [21]. This study addresses these gaps by combining written task responses and semi-structured interviews to examine students' ways of thinking using Harel's *Ways of Thinking* framework. By systematically analyzing the reasoning patterns of 28 students in mathematics education and conducting in-depth case analyses of six representative participants, this research provides both breadth and depth in understanding students' cognitive processes. Accordingly, this study is guided by the research question: How do mathematics education students construct, justify, and organize their mathematical reasoning when solving conceptual and procedural tasks, as interpreted through Harel's Ways of Thinking framework? This question directs the investigation toward uncovering: 1) the dominant ways of thinking students demonstrate when reasoning about the limit definition; and 2) how procedural, conceptual, and formal reasoning manifest in their explanations.

2. METHOD

2.1. Research Design

This study employed a qualitative descriptive design to explore university students' ways of thinking about the definition of the limit in differential calculus. The qualitative approach was chosen to capture the nuances of students' reasoning processes and to examine how they construct meaning beyond procedural proficiency [22], [23]. The study is grounded in Harel's *Ways of Thinking (WoT)* framework, which distinguishes procedural, conceptual, and formal reasoning patterns.

2.2. Participant Selection

The participants consisted of 28 undergraduate students enrolled in a Mathematics Education program at a public university in West Java, Indonesia. All participants had previously completed an introductory calculus course. They were selected using purposive sampling to represent a range of academic performance and to ensure a diversity of reasoning approaches.

2.3. Data Collection Instruments

Data were collected using two instruments: 1) written tasks, in which students were asked to explain the meaning of the limit and solve problems involving $\varepsilon - \delta$ definitions; 2) Semi-structured interviews, in which follow-up interviews were conducted to explore students' reasoning behind their written responses, probing how they interpreted and understood the formal definition of the limit.

2.4. Data Analysis

Data were analyzed using thematic analysis [24]. The analysis process included: 1) initial coding to identifying meaningful units in students' written and verbal responses; 2) categorization to grouping codes according to Harel's WoT framework into procedural, conceptual, and formal reasoning; 3) data reduction and selection which six participants were purposively chosen as representative cases to provide in-depth illustrations of reasoning patterns, based on clarity, consistency, and representativeness of the identified ways of thinking.

2.5. Credibility and Trustworthiness

Credibility was ensured through data triangulation, comparing written tasks and interview responses, and by having colleagues review the categorization to enhance the validity and reliability of the analysis [25].

2.6. Ethical Considerations

The study adhered to recognized ethical principles for human-subject research. All participants received a clear explanation of the study's aims, and their voluntary involvement was emphasized. They were informed that their contributions would be used solely for academic purposes and that they retained the right to withdraw at any stage without any repercussions. Throughout the research process, confidentiality was maintained, and participants' rights were fully respected.

3. RESULTS AND DISCUSSION

3.1. Results

Thematic analysis of the written tasks and interviews revealed that students' understanding of the limit definition was predominantly procedural. Across 28 participants, the majority relied on algorithmic manipulation, symbolic substitution, and memorized formulas to solve limit problems, with limited engagement in conceptual or formal reasoning. Conceptual explanations, such as connecting the $\varepsilon - \delta$ definition to the intuitive notion of approaching a value, were rare. From the 28 students, most demonstrated procedural reasoning (71%), followed by conceptual (18%) and formal (11%)." Data on participants' thinking characteristics are shown in Table 1.

Table 1. Characteristics of Ways of Thinking

Ways of thinking	Number of participants	Characteristics
Procedural	20	Reliance on formulas, symbolic manipulation, and step-by-step procedures
Conceptual	5	Attempt to explain the meaning, relate the definition to intuitive understanding.
Formal	3	Attempt to use ε–δ reasoning logically, but often incomplete

The table above illustrates the distribution of students' approaches to limit definitions in differential calculus. Of the 28 participants, the majority (20 students) demonstrated procedural thinking. The main characteristic of this group is a reliance on formulas, symbolic manipulation, and procedural steps. They tended to follow taught algorithms without truly understanding the formal meaning of limits. This phenomenon aligns with previous findings that many students prioritize mechanical procedures over conceptual or formal understanding [25], [26], [27]. Five students demonstrated conceptual thinking, characterized by attempts to explain the meaning of limits and relate them to intuition or functional patterns. Although they began to understand the relationship between function values and limit points, their ability to consistently express formal definitions remained limited. This group demonstrated conceptual awareness but still needed scaffolding to transition to formal thinking. Only three students adopted formal thinking, attempting to use $\varepsilon - \delta$ reasoning logically.

Although this formal logic exists, its implementation is often incomplete or imprecise, indicating that formal thinking remains a rare skill among students. This confirms that procedural mastery does not automatically translate into formal understanding, so pedagogical interventions need to be directed at encouraging the transition from procedural to conceptual and formal understanding. Overall, this distribution underscores the procedural dominance in students' understanding of limit definitions, while also providing a basis for learning strategies that emphasize reflection,
13 conceptual meaning, and formal reasoning. Furthermore, six participants were selected for in-depth analysis as representative cases based on clarity, consistency, and exemplification of procedural dominance. The results of the analysis of these categories are as follows: 1) P15 and P23 relied heavily on substitution and direct computation. When asked to explain the definition, the response was limited to plugging in numbers to see the limit; 2) P8 showed step-by-step manipulation of formulas but attempted to connect to intuition with minimal success; 3) P4 and P27 applied procedural rules correctly; recognized the concept of approaching but could not formalize it rigorously; and 4) P19 followed the algorithmic
9 steps correctly but was confused when asked to explain the meaning of ε–δ in words. These four cases illustrate procedural dominance in students' reasoning while providing insight into variations in procedural approaches among students.

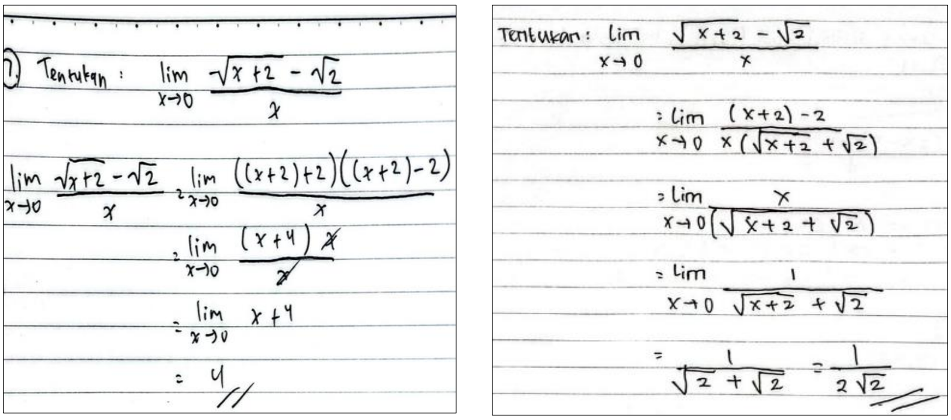

Figure 1. Characteristic of procedural reasoning

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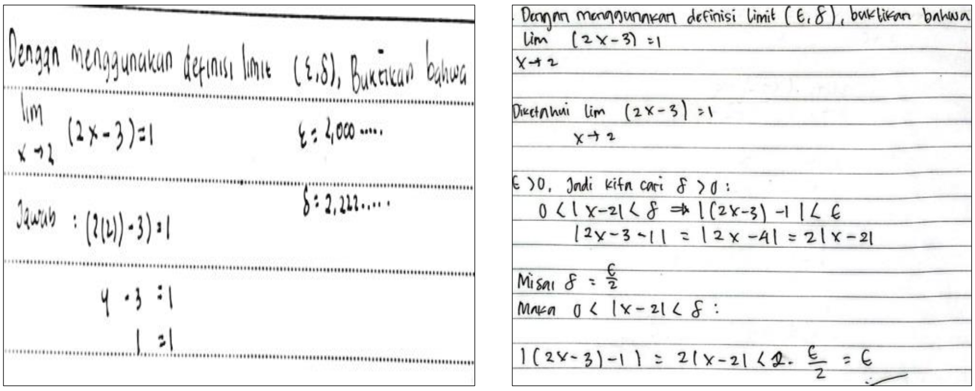

Figure 2. Characteristic of conceptual and formal reasoning

Based on Figures 1 and 2, several participants exhibit characteristics of procedural reasoning. Participants were given tasks in the form of questions, namely: 1) determining the value of $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$; and 2) determining the value of $\lim_{x\to 2}(2x-3)=1$. In the first task, P15 and P23 showed the solution process with an algebraic manipulation procedure so that $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$ where $x \neq 0$. P15 and P23 could not show that the function at the point $(0, L)$ is actually undefined, so that $\varepsilon - \delta$ is needed to find out to what extent the function is defined. Next, in the second task, P8 shows that the function $\lim_{x\to 2}(2x-3)$ has $L = 1$ with $\varepsilon = 2{,}000\ldots$ and $\delta = 2{,}222\ldots$ where the ways of thinking show that P8 understands that ε and δ is the smallest positive numbers from the neighborhood of L and $x \to 2$ even though the results obtained are still intuitive. Next, P19 shows $\lim_{x\to 2}(2x-3) = 1$ with the definition of the precision limit using $\varepsilon - \delta$, namely $\forall\, \varepsilon > 0\ \exists\, \delta > 0 \ni 0 < |x-c| < \delta \to |f(x)-L| < \varepsilon$. P19 shows the solution, namely $|(2x-3)-1| < \varepsilon$ such that $|(2x-4)| < \varepsilon$ such that $|2(x-2)| < \varepsilon$, and so $|x-2| < \frac{\varepsilon}{2}$. Therefore, it can be concluded that $0 < |x-2| < \frac{\varepsilon}{2}$ where $\frac{\varepsilon}{2} = \delta$. However, P19 cannot show examples of the values of ε and δ, cannot explain the logical operator $\forall\, \varepsilon > 0\ \exists\, \delta > 0$, and cannot explain the relationship between ε and δ. The following also shows examples of responses from participants presented in Table 2.

The analysis presented in Table 2 reveals a clear dominance of procedural ways of thinking among students in understanding the definition of the limit. Their reasoning typically centered on applying substitution, algebraic simplification, or L'Hospital's rule to obtain numerical results. This procedural orientation reflects a surface-level engagement with the concept—students tend to treat the limit as a computational task rather than as an idea involving functional behavior or approximation. In contrast, a smaller group of students (five participants) exhibited a conceptual way of thinking, characterized by efforts to explain the meaning of limits in terms of function behavior and proximity of values. These students were able to articulate that the limit describes the value a function approaches as the input nears a certain point, often using graphs or tables as intuitive representations.

Table 2. Interpretation of student interview results based on the ways of thinking category

Ways of thinking	Characteristics	Example student response	Dominant indicators
Procedural	Reliance on step-by-step algorithms, symbolic manipulation, and minimal conceptual understanding	"Limits are usually found by substitution of values. For example, to find the limit of $f(x)$ when $x \to 2$, simply plug 2 into the function. If that does not work, use L'Hospital's rule." (P15)	Algorithmic steps, symbolic computation
Conceptual	Understanding relationships and meaning, connecting limit to function behavior, or intuition, with less formal rigor	"A limit is the value that a function approaches as x gets closer to a certain point. Sometimes we use graphs or tables to see the pattern of function values." (P8)	Intuitive reasoning, relational understanding
Formal	Use of $\varepsilon - \delta$ definitions, logical reasoning, a formal and rigorous approach	"The limit of $f(x)$ when $x \to 2 = L$ is $0 < \lvert x - c \rvert < \delta \to \lvert f(x) - L \rvert < \varepsilon$ (P19)	Using $\varepsilon - \delta$, logical, formal, consistent reasoning; behind symbols and procedures.

Although their explanations lack formal rigor, this group represents an important transitional stage between procedural manipulation and formal reasoning, where understanding begins to shift toward relational and meaning-based perspectives. Only a few students (three participants) demonstrated evidence of a formal way of thinking, attempting to express the limit through the $\varepsilon - \delta$ definition. Their responses indicated emerging logical reasoning, such as relating the closeness of x to a with the closeness of $f(x)$ to L. However, their explanations were often incomplete, suggesting that while they have been introduced to formal reasoning, their grasp of its logical structure remains partial. Overall, the results indicate a hierarchical pattern of understanding among students, ranging from procedural to formal reasoning.

The predominance of procedural thinking underscores the need for instructional interventions that move beyond mechanical computation, fostering environments that promote conceptual engagement and formal abstraction. Encouraging students to reflect on the meaning behind their procedures and to connect symbolic manipulation with formal definitions may help bridge the gap between procedural fluency and genuine mathematical understanding. So, calculus learning strategies need to emphasize reflection, conceptual understanding, and formal reasoning.

1 https://doi.org/10.58421/misro.v4i4.776

3.2. Discussion

The findings of this study reveal that university students predominantly employ procedural thinking when engaging with the formal definition of the limit in differential calculus. This aligns with previous research, which found that students often rely on algorithmic manipulation and symbolic calculation rather than engaging in conceptual or formal reasoning [28], [29], [30].

Dominance of Procedural Thinking

Despite completing prior calculus courses, many students demonstrated limited internalization of the $\varepsilon - \delta$ definition, confirming that procedural competence does not guarantee deep understanding of mathematical concepts [31], [32], [33]. The analysis of six representative participants highlights subtle variations within procedural thinking, such as attempts to connect steps to intuition, yet remaining confined to mechanical processes. This suggests that even students with strong procedural skills may struggle to transition toward conceptual and formal reasoning [34], [35], [36]. Using Harel's Ways of Thinking framework, this study underscores the cognitive patterns that underpin procedural dominance and illuminates how students' approaches shape their understanding of mathematical rigor. These findings underscore the need for pedagogical strategies that foster deeper conceptual engagement and the development of formal reasoning skills, supporting students' transition from procedural competence toward meaningful mathematical understanding.

Furthermore, the results highlight the persistent gap between students' intuitive understanding of limits and their ability to reason formally within the ε–δ framework. Many students perceive the formal definition as a procedural formula to memorize rather than as a logical structure describing the relationship between two quantities. This disconnect reflects a broader challenge in calculus instruction where formal rigor is often introduced prematurely, without sufficient grounding in conceptual meaning. From a *perspective of thinking, the dominance of procedural reasoning suggests that students' cognitive development in learning tends to stabilize at the operational level, requiring explicit scaffolding to reach a* structural understanding [37].

Scarcity of Formal Reasoning and Its Implications

Considering these findings, instructional design should place greater emphasis on bridging intuitive, conceptual, and formal understandings of the limit. Activities that promote reflective reasoning, such as guided explorations, graph-based interpretations, or conceptual linking tasks, may help students reconstruct their understanding beyond rote symbolic manipulation [38], [39], [40]. The *Ways of Thinking* (WoT) framework provides educators with a valuable diagnostic tool for identifying students' reasoning levels and designing targeted interventions that promote cognitive growth. By integrating procedural fluency with conceptual reflection and formal reasoning, calculus instruction can better support students in developing a more coherent and meaningful understanding of mathematical limits. The reasoning procedures tend to dominate students' understanding of limits, primarily due to systemic features of calculus instruction. First, the curriculum of many undergraduate programs emphasizes procedural fluency, with learning objectives centered on solving standard computational problems rather than constructing or analyzing mathematical meaning. This creates a learning environment where algorithms and formulas are prioritized over the exploration of context. Second, assessment designs typically prioritize speed and accuracy in symbolic manipulation, signaling to students that mastering routine procedures is sufficient for academic success. Examinations rarely require students to understand the definition of $\varepsilon - \delta$ or articulate conceptual reasoning, reinforcing the perception that computation is not a crucial understanding. Third, teachers' instructional patterns often model procedures step by step, providing applied examples and algorithmic templates for students to copy. While efficient, this approach inadvertently conveys that mathematics is primarily about following rules rather than understanding ideas. Collectively, these curricular, assessment, and instructional structures shape students' overall learning toward procedural dominance and hinder the development of conceptual and formal reasoning in calculus.

4. CONCLUSION

This study found that university students' understanding of the limit definition is primarily procedural, with limited conceptual and formal reasoning. By applying Harel's Ways of Thinking framework, the study provides empirical insight into the cognitive transition from procedural to formal understanding in calculus learning. Thematic analysis of written tasks and semi-structured interviews revealed a predominance of procedural thinking, characterized by reliance on algorithmic manipulation and symbolic recall. Conceptual and formal reasoning were minimal, indicating limited internalization of the formal meaning of the limit. Several limitations of this study should be noted: 1) scope of participants: although 28 students were analyzed, in-depth focus was on six representative cases, which may limit generalizability; 2) data sources: reliance on written tasks and semi-structured interviews may not capture all dimensions of students' cognitive processes; 3) analytical approach: thematic analysis provides insight into patterns of reasoning but does not fully reveal interpretive or reflective aspects of students' understanding. Building on the findings and limitations, future studies are recommended to: 1) adopt a hermeneutic approach to explore how students construct and interpret meaning of the limit concept in their own cognitive and experiential context; 2) investigate instructional interventions that integrate conceptual and formal reasoning tasks to reduce procedural reliance; 3) expand participant diversity across institutions and educational backgrounds to examine broader patterns of reasoning in calculus learning.

Future research should employ a hermeneutic approach to explore how students construct and interpret the meaning of limits within their cognitive and experiential horizons. Additionally, instructional interventions that integrate conceptual and formal reasoning tasks should be developed to help students transcend procedural dependency. Expanding the diversity of participants across different institutional and educational contexts may also yield broader insights into the patterns of mathematical reasoning that shape students' understanding of calculus concepts. Ultimately, this study contributes to the growing body of research on students' mathematical thinking by providing evidence of how procedural dominance persists in the learning of limits and by emphasizing the pedagogical importance of fostering conceptual and formal reasoning in higher mathematics education.

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REFERENCES

[1] Yoon, H., Bae, Y., Lim, W., & Kwon, O. N. A story of the national calculus curriculum: how culture, research, and policy compete and compromise in shaping the calculus curriculum in South Korea. *ZDM – Mathematics Education*, *53*(3), 663–677, 2021.

[2] Gucler, B. Examining the discourse on the limit concept in a beginning-level calculus classroom. *Educational Studies in Mathematics*, *82*(3), 439–453. https:// doi. org/ 10. 1007/ s10649- 012- 9438-2, 2013.

[3] Gucler, B. The role of symbols in mathematical communication: The case of the limit notation. *Research in Mathematics Education*, *16*(3), 251–268, 2014, https:// doi. org/ 10. 1080/ 14794 802.

[4] Bezuidenhout, J. Limits and continuity: Some conceptions of first-year students. *International Journal of Mathematical Education in Science and Technology.*, *32*(4), 487–500, 2010.

[5] Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. Understanding the limit concept: Beginning with a coordinated process scheme. *Journal of Mathematical Behavior*, *15*(2), 167–192, 1996.

[6] Keene, K. A., Hall, W., & Duca, A. Sequence limits in calculus: Using design research and building on intuition to support instruction. *ZDM Mathematics Education*, *46*, 561–574, 2014.

[7] Kidron, I., & Tall, D. The role of embodiment and symbolism in the potential and actual infinity of the limit process. *Educational Studies in Mathematics*, 2015

[8] Sierpinska, A. Humanities students and epistemological obstacles related to limits. *Educational Studies in Mathematics*, *18*, 371–397, 1987.

[9] Hitt, F., & Dufour, S. Introduction to calculus through an open-ended task in the context of speed: representations and actions by students in action. *ZDM–Mathematics education*, *53*(3), 635-647, 2021.

[10] Harel, G. What is mathematics? A pedagogical answer to a philosophical question. *Proof and other dilemmas: Mathematics and philosophy*, 265-290, 2008.

[11] Harel, G., & Sowder, L. Advanced mathematical-thinking at any age: Its nature and its development. In *Advanced Mathematical Thinking* (pp. 27-50). Routledge, 2013.

[12] Koichu, B., Harel, G., & Manaster, A. Ways of thinking associated with mathematics teachers' problem posing in the context of division of fractions. *Instructional Science*, *41*(4), 681-698, 2013.

[13] Harel, G. Two dual assertions: The first on learning and the second on teaching (or vice versa). The American Mathematical Monthly, 105(6), 497-507, 1998.

[14] Lesh, R., & Harel, G. Problem solving, modeling, and local conceptual development. *Mathematical thinking and learning*, *5*(2-3), 157-189, 2003.

[15] Tall, D., & Vinner, S. Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151–169, 2018.

[16] Kim, D., Kang, H., & Lee, H. Two different epistemologies about limit concepts. International Education Studies, 8(3), 138–145, 2015.

[17] Usman, U., Hasbi, M., & Muslem, A. Profile of Pre-Service Math Teachers' Conception about the Definition of Limit Functions Based on Mathematical Ability. *Jurnal Pendidikan Progresif*, *14*(2), 704-

712, 2024.

[18] Sulastri, R., Suryadi, D., Prabawanto, S., Cahya, E., Siagian, M. D., & Tamur, M. Prospective mathematics teachers' concept image on the limit of a function. In *Journal of Physics: Conference Series* (Vol. 1882, No. 1, p. 012068). IOP Publishing, 2021.

[19] Sulastri, R., Suryadi, D., Prabawanto, S., & Cahya, E. Epistemological Obstacles on Limit and Functions Concepts: A Phenomenological Study in Online Learning. *Mathematics teaching research journal*, *14*(5), 84-106, 2022.

[20] Winarso, W., & Toheri, T. A case study of misconceptions students in the learning of mathematics; The concept limit function in high school. *Jurnal Riset Pendidikan Matematika*, *4*(1), 120–127, 2017.

[21] Saleh, S., Budayasa, I. K., & Lukito, A. The profile in understanding the concept of function limit of the prospective teachers students based on difference in mathematical skill. In *AIP Conference Proceedings* (Vol. 3038, No. 1, p. 020037). AIP Publishing LLC, 2025.

[22] Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. Approaches to qualitative research in mathematics education. *Advances in Mathematics Education, DOI*, *10*, 978-94, 2015.

[23] Sharma, S. Qualitative approaches in mathematics education research: Challenges and possible solutions. *Education Journal*, *2*(2), 50-57, 2013.

[24] Simon, M. A. Analyzing qualitative data in mathematics education. In *Designing, conducting, and publishing quality research in mathematics education* (pp. 111-122). Cham: Springer International Publishing, 2019.

[25] Kim, D. J., & Lim, W. The relative interdependency of colloquial and mathematical discourses regarding the notion and calculations of limit: An evidence-based cross-cultural study. *International Journal of Science and Mathematics Education*, *16*(8), 1561-1579, 2018.

[26] Fernández-Plaza, J. A., & Simpson, A. Three concepts or one? Students' understanding of basic limit concepts. Educational Studies in Mathematics, 93(3), 315-332, 2016.

[27] Roh, K. H. An empirical study of students' understanding of a logical structure in the definition of limit via the ε-strip activity. Educational Studies in Mathematics, 73(3), 263-279, 2010.

[28] Fernández, E. THE STUDENTS'TAKE ON THE EPSILON-DELTA DEFINITION OF A LIMIT. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *14*(1), 43-54, 2004.

[29] Mahadewsing, R., Getrouw, D., & Calor, S. M. Prior knowledge of a calculus course: The impact of prior knowledge on students' errors. *International Electronic Journal of Mathematics Education*, *19*(3), em0786, 2024.

[30] R. Abdul-Wasiu, I. Brilliant Essuman, D. Offei Kwakye, and I. Alhassan, "Descriptive Survey of Preservice Mathematics Teachers' Perceived Knowledge and Use of Teaching and Learning Materials", *J.Math.Instr.Soc.Res.Opin.*, vol. 3, no. 2, pp. 251–264, 2024.

[31] Boero, P. Analyzing the transition to epsilon-delta Calculus: A case study. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education*, pp. 93-99, 2015.

[32] Caglayan, G. Math majors' visual proofs in a dynamic environment: The case of limit of a function and the ϵ–δ approach. *International journal of mathematical education in science and technology*, *46*(6), 797-823, 2015.

[33] Adiredja, A. P. The pancake story and the epsilon–delta definition. *Primus*, *31*(6), 662-677, 2021.

[34] Al-Mutawah, M. A., Thomas, R., Eid, A., Mahmoud, E. Y., & Fateel, M. J. Conceptual understanding, procedural knowledge and problem-solving skills in mathematics: High school graduates work analysis and standpoints. *International journal of education and practice*, *7*(3), 258-273, 2019.

[35] Braithwaite, D. W., & Sprague, L. Conceptual knowledge, procedural knowledge, and metacognition in routine and nonroutine problem solving. *Cognitive Science*, *45*(10), e13048, 2021.

[36] Carpenter, T. P. Conceptual knowledge as a foundation for procedural knowledge. In *Conceptual and procedural knowledge*, Routledge, (pp. 113-132), 2013.

[37] Morin, S., Suryadi, D., Prabowanto, S., & Sulistiyo, D. Junior high school students in solving mathematical ill-structured problems: Analyzing using Harel theory. *Journal of Engineering Science and Technology*, *20*(3), 1-8, 2025.

[38] Gurl, T. J., Markinson, M. P., & Artzt, A. F. Using ChatGPT as a lesson planning assistant with preservice secondary mathematics teachers. *Digital Experiences in Mathematics Education*, *11*(1), 114-

139, 2025.

[39] Blažek, J., & Pech, P. Solving a problem with GeoGebra current possibilities and limits of CAS tools. *Journal of Automated Reasoning*, *69*(3), 20, 2025.

[40] Zhang, Y., Wang, P., Jia, W., Zhang, A., & Chen, G. Dynamic visualization by GeoGebra for mathematics learning: a meta-analysis of 20 years of research. *Journal of Research on Technology in Education*, *57*(2), 437-458, 2025.