Journal of Mathematics Instruction, Social Research and Opinion
Vol. 4, No. 4, December 2025, pp. 1051 – 1064, https://doi.org/10.58421/misro.v4i4.698
ISSN 2962-7842

Implementing Bruner's Theory in Teaching Angles on a Circle to Enhance Problem-Solving Skills

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Article Info

Article history:

Received 2025-08-31
Revised 2025-09-27
Accepted 2025-10-03

Keywords:

Angle on a circle
Bruner's theory
Mathematical problem-solving
Mathematics education

ABSTRACT

Bruner's theory is particularly notable for its application in mathematics, as it outlines three steps in the teaching process: enactive, iconic, and symbolic. This research aims to implement Bruner's theory in teaching Angles on circles to improve problem-solving skills. The research method is a quasi-experiment with a posttest-only control group design. Using cluster random sampling, two classes from one of the junior high schools in Jakarta were selected as samples: the first as an experimental class of 30 students, who learned using Bruner's theory, and the second as a control class of 31 students, who learned with a conventional approach. The instrument is a mathematical problem-solving test in the form of an essay test of four questions. The questions are given at the end of the lesson, and the instruments have been validated through content validity and empirical validity. The results show that, through t-test analyses, it was found that students' problem-solving skills, who learn using Bruner's theory, are higher than those of students who learn using conventional methods. It demonstrates that teaching using Bruner's theory is effective in improving students' problem-solving skills, particularly in the Area of Angles on circles.

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1. INTRODUCTION

Mathematics, as a significant component of the curriculum, plays a fundamental role in shaping logical, analytical, and systematic reasoning skills. The nature of this discipline is rooted in human thought processes involving ideas, mechanisms, and logic, so that mathematics learning at all levels of education emphasizes problem-solving as a fundamental ability needed in mathematics [1], [2]. However, the main obstacle often faced by teachers is the low competence of students in understanding problems, as evidenced by errors in the solution process due to a lack of familiarity with complex mathematical concepts [3].

Problem-solving skills play a significant role in the curriculum due to their close connection to real-life experiences and their urgency in addressing increasingly complex issues in the modern era. According to Wagner & Group, seven skills are essential in the modern era, with critical thinking and problem-solving skills [4]. Both in academia and in everyday life, these skills help students analyze complex problems, develop innovative solutions, and make informed decisions. In line with this, NCTM has long emphasized the importance of problem-solving as the core of mathematics education, to equip students with the tools to understand and apply mathematical concepts effectively and contextually [5].

Considering the fundamental role of problem-solving as the foundation of mathematics, the development of this ability is a significant concern in the learning process. As an indicator of mathematical ability, problem-solving is viewed as a higher-order thinking skill [6]. Dörner & Funke describe it as the ability to analyse problems, determine the causes and priorities of a problem, select various solution options, and make decisions in implementing the selected solution [7]. In line with this, Bariyyah explains problem-solving as the skill of recognising problems, exploring and selecting solution strategies, and making informed decisions [8]. According to experts, problem-solving skills encompass a comprehensive set of abilities that include identifying problems, analysing and exploring potential solutions, making informed decisions, and implementing those solutions. Thus, mathematical problem-solving requires the integration of understanding, reasoning, representation, and communication, so that students can find the right solution and interpret the results within the context of the problems they face.

However, several studies reveal that students' mathematical problem-solving skills are still at a low level, requiring attention. This condition is confirmed by the results of the 2023 TIMSS (Trends in International Mathematics and Science Study) survey conducted by the IEA (International for the Evaluation of Educational Achievement). In the survey, Indonesia ranked 35th with an average score of 411, significantly below the international average of 467 among 46 participating countries [9]. These findings confirm that students in Indonesia still lag behind international standards in solving mathematical problems, a reality consistent with various school-level findings [10].

In this study, the indicators used to measure students' ability in solving mathematical problems are: 1) identifying known elements, questions, and sufficient elements needed to solve the problem; 2) formulating the problem into a mathematical model; 3) identifying which knowledge can be used in solving the problem; 4) identifying calculation errors, formula usage errors, checking the compatibility between what has been found and what is asked, and being able to explain the correctness of the answer [11].

The concept of angles on circles is an essential topic in the eighth-grade math curriculum, included in geometry, and has a close relationship with its application in everyday life; therefore, its mastery is crucial. The importance of this topic is also evident from its presence in both the 2013 curriculum and the independent curriculum, both of which place circles as one of the main topics at the junior high school level.

The angle of a circle appears to be a reasonably simple problem to solve. However, in reality, many students still struggle to master the material. The main difficulty that arises is related to the lack of basic understanding of the geometry principles that are the basis of angles on a circle [12]. This finding aligns with Mifetu's research, which identified students' systematic difficulties in solving problems related to circle geometry. [13]. Related to the lack of basic understanding, Avğın and Ergen found that although students' math motivation was high, their problem-solving skills were still at the Needs Improvement level, so more effective learning strategies were needed to improve problem-solving skills [14]. Based on these problems, the application of Bruner's theory is considered relevant to overcome students' learning gap on the topic of circular angles because it allows students' concept understanding to be formed gradually from concrete experience to abstract representation [15], and improve critical thinking and problem-solving skills [16], [17]. The characteristics of angles on a circle material that can be visualised through direct activities, described as expressed in symbols and formulas, make it suitable to test the effectiveness of Bruner's theory.

The implementation of Bruner's learning theory is considered to enhance student engagement during the learning process and improve student academic achievement [18]. Teachers play a crucial role in selecting effective learning strategies, providing comprehensive infrastructure, and creating an ideal learning environment [19], [20], [21]. The application of Bruner’s learning theory is crucial for developing mathematical problem-solving skills. Bruner's learning theory emphasises the importance of direct experience, discovery, and understanding of concepts. Jerome S. Bruner argues that the learning process will achieve more optimal results if students are actively involved in constructing knowledge through interaction with their environment, where each field of knowledge can be presented in three stages of representation, namely enactive (learning by doing actions), iconic (learning through images and visual media), and symbolic (learning using abstract symbols) [22], [23], [24], [25].

Bruner expanded the dimension of constructivism through discovery in the learning process and cognitive structure, with the view that students can understand complex concepts through a process of independent discovery to build more abstract and meaningful knowledge [26], [27], [28] using interactive media. The process of understanding knowledge concepts can be supported by interactive media used in learning. The use of interactive media in Bruner's theory through a curriculum approach has the potential to increase students' understanding of concepts, interest, and motivation to learn, while enriching their learning experience [29], [30].

In line with this, Safitri's research, et al., which utilises multimedia-based learning media, can enhance students' understanding of mathematical concepts [31]. Interactive media-assisted learning, as explored in Padirayon et al.'s research, can help students build new knowledge based on their prior experiences [32]. The development of learning tools also emphasises the importance of contextual and structured learning design to enhance student learning effectiveness [33], by utilising props and the surrounding environment as learning media [34]. Therefore, the use of these media needs to be aligned with the learning objectives and the needs of students.

Based on the literature review, the researcher found that the United States applies Bruner's theory in helping prospective teachers understand concepts more deeply [35]. On the other hand, South Korea's junior secondary school mathematics curriculum is based on the theory of constructivism, which emphasises problem-solving, whereas Malaysia is still gradually shifting its traditional test-oriented approach towards constructivism [36]. However, no research has been found that applies explicitly to Bruner's theory to the topic of angles on a circle. Therefore, this research aims to address the gap by comprehensively examining the application of Bruner's theory in learning angles in a circle, utilising interactive media as a supporting tool to emphasise the mastery of concepts through a meaningful discovery process.

2. METHOD

The research method used is a quasi-experiment with a posttest control group design [37]. This research was conducted in one of the public schools in East Jakarta in the 2025/2026 academic year. The study sample was selected using the random cluster sampling technique. Class VIII-B was the control class (31 students), which learned through conventional teaching, and Class VIII-D was the experimental class (30 students), which learned using Bruner's learning theory.

The instrument used in the study was a test of mathematical problems in essay form, consisting of up to four questions. The instrument was given after students finished their study. Table 1 presents the test indicators.

Table 1. Problem-Solving Ability Instrument Grid

Problem-Solving Aspects	Problem Solving Indicators
Understanding the problem	Identifying known elements, questions, and sufficient elements needed to solve the problem
Planning problem-solving	Formulating problems into mathematical models
Implementing problem-solving plans	Identifying which knowledge can be used to solve the problem
Reviewing	Identifying calculation errors, formula usage errors, checking the compatibility between the findings and what is being asked, and being able to explain the correctness of the answer

Before the instrument was used, it was validated through a content validity test by eight expert validators, including three mathematics education lecturers, four junior high school teachers, and one senior high school teacher in Indonesia. Next, instrument validation through empirical validity aims to determine the extent to which the items can measure the instrument indicators using the Pearson Product-Moment formula, as follows.

$$r_{xy} = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{n\sum X^2 - (\sum X)^2}.\sqrt{n\sum Y^2 - (Y)^2}} \quad (1)$$

Information:

r_{xy} : Product-Moment Correlation Coefficient

N : Number of respondents

X : Item Score

Y : Total score

After that, measure the reliability of the instrument to determine the reliability of the test results using the Alpha Cronbach formula as follows.

$$r_{11} = \left(\frac{n}{n-1}\right)\left(1 - \frac{\sum S_i^2}{{S_t}^2}\right) \quad (2)$$

Information:
r_{11} : Test reliability coefficient
n : The number of questions
$\sum S_i^2$: The sum of the score variances of each item
${S_t}^2$: Variance total

Then, the differentiating power analysis is carried out to distinguish between high-ability students and low-ability students using the following formula.

$$DP = \frac{\overline{X_A} - \overline{X_B}}{SMI} \quad (3)$$

Information:
P : Differentiating power
X_A : Upper group average
X_B : Lower group average
SMI : Ideal maximum score

After that, the level of difficulty of the instrument was analysed using the following formula.

$$r_{11} = \left(\frac{n}{n-1}\right)\left(1 - \frac{\sum S_i^2}{{S_t}^2}\right) \quad (4)$$

Information:
IK : Difficulty Index
$\bar{X}$: Average
SMI : Ideal maximum score

The results of the instrument analysis recapitulation are presented in Table 2.

Table 2. Instrument Analysis Recapitulation Results

Question Number	Validity	Reliability	Discriminatory Power	Level of Difficulty	Explanation
1	Valid		Fair	Easy	To use
2	Valid	0,8798	Fair	Moderate	To use
3	Valid	(High)	Good	Moderate	To use
4	Valid		Fair	Difficult	To use

Table 2 shows that all questions on this instrument can be used. The learning process was conducted four times to provide treatment. After the learning process ended, a final test was administered. Furthermore, the data were analyzed using a t-test because the purpose of the study was to determine whether there was a significant difference between the learning outcomes of students who received the treatment and the untreated class. T-tests on test instruments are used to ensure that the items can distinguish between students with high and low abilities.

3. RESULTS AND DISCUSSION

3.1. Results

After researchers applied teaching in the experimental class using Bruner's theory and in the control class using conventional teaching, Table 3 shows the descriptive statistics for the posttest results.

Table 3. Mathematics Problem-Solving Skills

Statistic	Mathematical Problem-Solving Skills	
	Control	Experiment
N	31	30
Mean	47,98	61,32
Median	54,17	63,54
Variance	195,16	304,62
Std. Deviation	13,97	17,45
Maximum	71	100
Minimum	16,67	20,8

From Table 3, it is evident that there are differences in students' abilities between the two classes in solving mathematical problems. The mean value of the experimental class is superior to that of the control class, indicating that the application of Bruner's theory in the experimental class yields better results. The median value of the experimental class is also greater than that of the control class, indicating a general trend of higher student abilities in the experimental class. While the highest score in both classes is achieved by the experimental class, with a score of 100, and the lowest score is achieved by the control class, with a score of 16.67, this indicates that the experimental class students achieved higher scores than those in the control class. In terms of data distribution, the variance value between the two classes differs by 109.46, and the standard deviation differs by 3.81, indicating that the experimental class has a wider data distribution. This suggests a more pronounced ability difference among students in the experimental class.

The prerequisite analysis for this research was conducted by first testing the assumptions and then testing the hypothesis. The purpose of this assumption test is to ensure that the data used has met the required prerequisites so that statistical analysis can be carried out precisely and accurately. The conjecture/hypothesis testing was carried out to determine whether there was a difference in the average ability of students in solving mathematical problems between the two classes.

The assumption tests applied are the data distribution test and the variance equality test, using SPSS software. Both tests were conducted first to verify the assumptions regarding the test results of students' abilities in the two sample classes. Table 4 displays the results of the post-test data distribution test.

Table 4. Data Distribution Test Results

Group	Shapiro-Wilk		
	Statistic	Df	Sig.
Experiment	.970	30	.549
Control	.918	30	.024

With a significance threshold of 5% or $\alpha = 0.05$, Table 5 indicates that the significance values of both classes are higher than $\alpha = 0.05$. This suggests that the values of students' ability to solve math problems from both classes are normally distributed. The following contains the results of the variance equality test.

Table 5. Variance Homogeneity Test Results

Factor	Levene Statistic	df1	df2	Sig.
Based on the mean	.101	1	59	.752

Based on Table 5, the variance equality test yielded a significance value higher than the $\alpha = 0.05$ threshold. This finding suggests that the variance in students' ability to solve mathematical problems between the two classes is homogeneous.

All the results of the prerequisite analysis, which include normality and homogeneity tests, indicate that both groups typically have distributed and homogeneous populations. Therefore, the t-test was used to test the hypothesis, and the calculation was performed using SPSS software, as shown in the following table.

Table 6. Hypothesis results

		t-test for Equality of Means				
		T	df	Sig. (2-tailed)	Means Difference	Std. Error Difference
Score	Equal variances assumed	3.350	59	.001	13.562	4.048

Based on Table 6, testing the conjecture/hypothesis results in sig. (2-tailed) = 0.001 so that the significance value is 0.001/2 = 0.0005. The null hypothesis (H_0) is rejected because the sig. (0,0005) < α (0,05). The average ability of students in solving math problems who implement Bruner's learning theory is superior to that of students who follow conventional teaching methods. This indicates that the application of Bruner's learning theory has an impact on students' ability to solve mathematical problems related to angles on a circle.

3.2. Discussion

Based on the t-test, we conclude that the mathematical problem-solving ability of students who learn using Bruner's theory is higher than that of students who learn using conventional learning. Regarding the posttest results based on indicators, we can identify which indicators are higher.

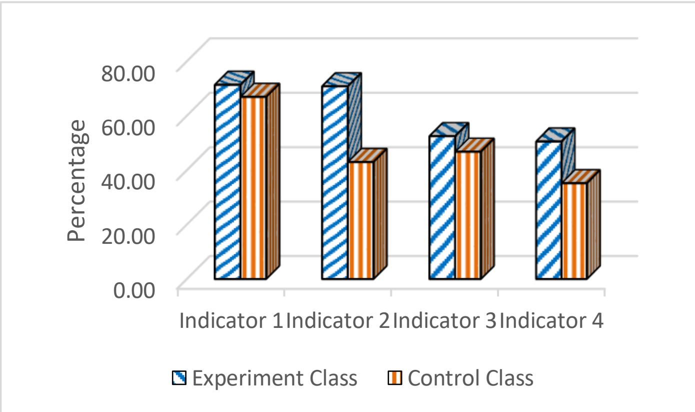

Figure 1. Percentage of Student Achievement Based on Indicators of Mathematical Problem-Solving Skills

Based on Figure 1, it appears that the overall percentage of achievement of mathematical problem-solving ability in the experimental class is higher than the control class because the application of Bruner's theory emphasises student-centred learning through three stages of representation, namely enactive, iconic, and symbolic, so that students are more active in building concept understanding and can transfer knowledge into problem-solving. It can be seen that indicator 1 in both classes is relatively higher because understanding the problem is an easier initial stage, especially in the experimental class, where the enactive stage helps students effectively identify relevant information from the problem. However, in indicators one and two, there was a decrease in the control class because conventional learning did not emphasize the strategy of making plans.

There is a significant difference in indicator two between the experimental and control classes because students in the experimental class receive more assistance in formulating mathematical models through the iconic stage, which facilitates the visualisation of the problem. In contrast, control class students tend to struggle because, during the learning process, they employ the lecture method and provide direct solutions without formulating mathematical models in a structured manner.

The percentage in the experimental class also experienced a fairly sharp decline, as seen in indicators two and three. Although students could formulate the problem well, some still encountered difficulties and imprudence in choosing and developing problem-solving strategies, resulting in an inappropriate solution plan and incorrect answers. In contrast, the control class experienced an increase from indicator 2 to indicator three because even though they were less focused on mathematical modeling, students were accustomed to memorizing the solution procedure so that it was more helpful to the implementation stage of problem solving, but still produced wrong answers due to control class students' errors in understanding the problem so that they carried out inappropriate solution plans and obtained wrong answers.

Indicator 4 in both classes yielded the lowest results because the stage of checking the solution requires more complex metacognitive skills; some students concluded without rechecking the solution, and there was no identification of the errors that occurred. In applying Bruner's theory, although students have progressed through the enactive and iconic stages, they still require further practice to reflect on and evaluate the truth of their answers systematically. This finding indicates that Bruner's theory is effective in enhancing conceptual understanding and the ability to formulate problems; however, further strengthening is still needed in the aspects of strategy and evaluation of answers, so that students' problem-solving skills are more evenly distributed across all indicators.

Application of Bruner's Theory in Learning Angle on a Circle

The results indicate that the application of Bruner's theory has an impact on students' ability to solve mathematical problems related to angles on a circle. The order of the roles of mutually sustainable steps in Bruner's theory, as it relates to students' ability to solve mathematical problems during a lesson on the angle of a circle, can be observed in the learning process.

The first step is: enactive. In this step, students are asked to manipulate concrete objects. Students are given circles, threads, arc rulers, and pins to carry out the enactive step. In the implementation of the circle media activity, students put a point with a pin in the centre of the circle, then they are asked to put four different pins on the circle by forming three angles that have been determined on the worksheet, namely ∠60°, ∠90°, and ∠120°, with threads connected to each pin and an arc ruler to facilitate the work. This activity is illustrated in Figure 2.

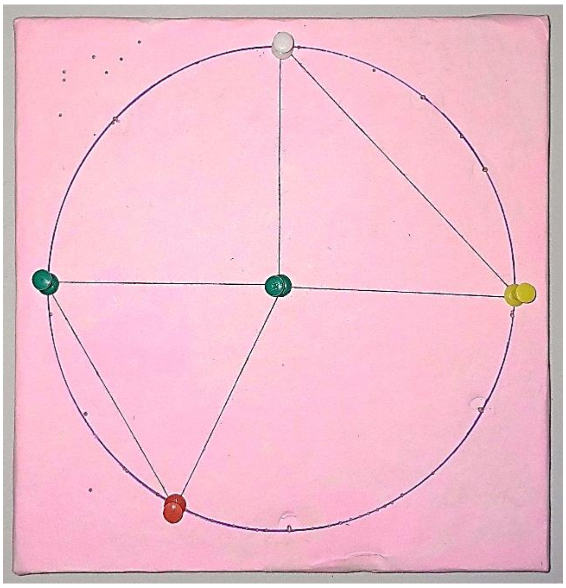

Figure 2. Circle media in the enactive step

From the activity using the circle media above, students are asked to measure the circumference of the circle and the arc length of each corner using a thread, which will be measured using a ruler. Furthermore, students are asked to record the results obtained in the table provided, as shown in Figure 3.

Sector	Circumference	Arc Length	Angle Measure
Sector I	62,8	21 cm	120°
Sector II	62,8	16 cm	90°
Sector III	62,8	10,5 cm	60°

Figure 3. Enactive step

Figure 3 shows that students learn the relationship between angle magnitude and arc length indirectly. This indicates that through enactive activities, students can gain a deeper understanding.

The second step: iconic, students are asked to describe the problem situation in the form of pictures or diagrams that facilitate analysis. The teacher asks questions to review the knowledge gained from students' experiences in the previous step. Afterwards, the teacher guides students to visually represent their understanding, such as by drawing a circle with markings indicating the angle magnitude and arc length on the student worksheet. This can help train students' skills in organising information related to a problem. This activity is illustrated in Figure 4.

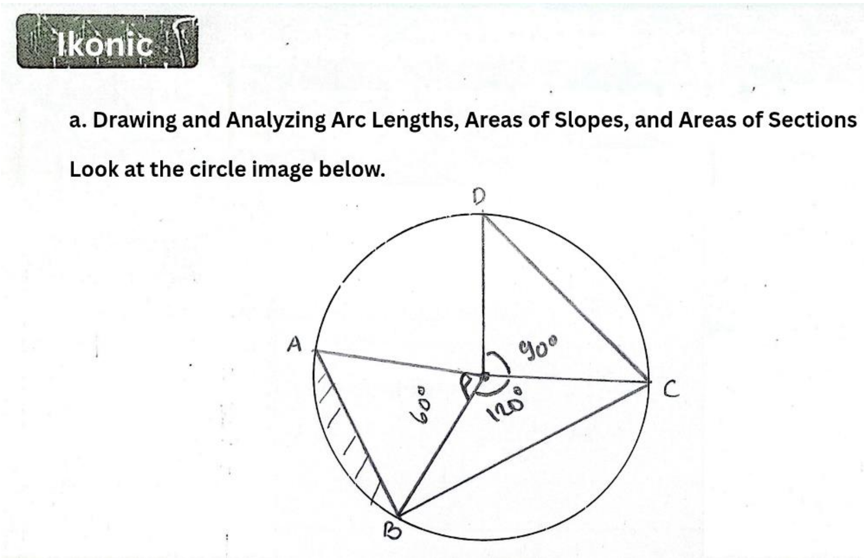

Figure 4. Iconic step

Figure 4 shows that students can represent the activities carried out in the form of a picture and discuss the relationships between arc length and sector, as well as the relationship between sector and segment, based on the information they have understood in the previous stage. The purpose of this step is to understand information obtained from previous direct experience through visual representations, such as pictures or diagrams, so that students can develop their cognitive skills.

The third step: symbolic, students are asked to provide formative feedback and generalisation statements based on the patterns found. Based on the information students have understood in the previous stage, they are asked to determine the formulas for arc length, the area of the sector, and the area of the circular segment. Regarding one of the student results in the symbolic step of the worksheet, it is illustrated in Figure 5.

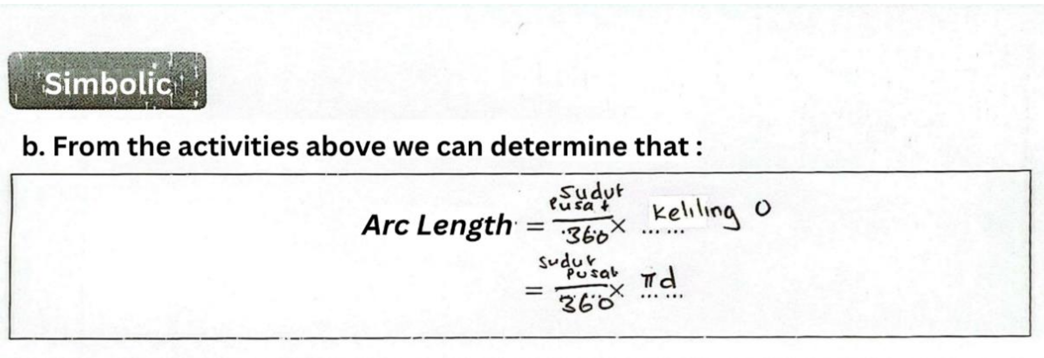

Figure 5. Symbolic step

Figure 5 shows that students can determine the arc length formula by comparing the central angle to the total angles of the circle, then multiplying the result of the comparison by the total circumference of the circle. Meanwhile, when determining the formula for the area of a sector, students compare the central angle of the sector to the total angle of the circle and then multiply the result of the comparison by the total area of the circle. The area of the circle segment is the part of the circle bounded by the arc and the chord, so in determining the formula, students subtract the area of the sector from the area of the isosceles triangle formed by the two radii and the chord.

Figure 5 shows that students can identify the mathematical formula used based on the information they obtained in the previous step. This prepares students to solve increasingly complex problems by generalising a concept and performing calculations efficiently. It is hoped that by applying Bruner's learning theory in mathematics teaching, students will be able to practice solving mathematical problems, thereby developing their problem-solving skills.

This aligns with Wen and Gombo, which suggests that students should rely on their own ability to discover the generalization behind a particular mathematical operation/law through the exploration of mathematical learning problems [38], [39]. Such exploration encourages students to discover learning content, where they actively participate in the teaching process. In line with research conducted by Mabhoza and Ching, constructivism-based learning strategies can improve the quality of students' mathematics learning, active involvement, and the ability of students' critical thinking and problem-solving skills because the gradual transition from concrete to abstract representations is more effective and able to strengthen students' conceptual understanding while facilitating the transfer of knowledge to new contexts [40], [41]. This approach shifts the position of students from passive recipients to active discoverers, thereby significantly improving their mathematical problem-solving capacity.

The description explains that the application of Bruner's learning theory in mathematics learning puts students in conditions that encourage them to be directly involved in problem solving, demanding understanding of knowledge concepts and identifying relevant information from a problem, students' ability to manipulate concrete objects using leaves in understanding addition and subtraction operations in real terms, representing them in images, and operating mathematical symbols, as well as the ability to solve problems efficiently, in line with what is expressed [42], [43]. Thus, students are directed to: skills to identify known information; skills to formulate the problem into a mathematical model; skills to use relevant knowledge in solving the problem; skills to find errors in calculations or application of formulas, assess the suitability of the results to the question, and provide reasons for the correctness of the resulting solution. Mastery of these indicators suggests that students who learn through Bruner's theory are more effective at solving mathematical problems than those who learn through conventional methods.

In formulating the problem into a mathematical model, this indicator shows the greatest improvement because this stage is directly supported by the principle of representation in Bruner's theory. In the enactive stage, students interact with concrete objects, allowing them to understand the problem in real terms. Furthermore, through the iconic stage, students represent these experiences in the form of pictures or visual illustrations that help connect concrete phenomena with mathematical concepts. Finally, in the symbolic stage, students transform the visual representation into a formal mathematical equation or model.

The integration of these three stages enables students to build a bridge from real experience to mathematical abstraction gradually. Therefore, experimental class students have a stronger and more structured representation foundation than control class students who learn with conventional methods. In other words, Bruner's theory can help students understand the meaning of the symbols and formulas used, rather than just memorising them.

Based on this description and the results of hypothesis testing, it is clear that the use of Bruner's theory affects students' ability to solve mathematical problems related to circular angle material. This finding reveals a significant difference in mathematical problem-solving ability between the group of students whose learning was based on Bruner's theory and the group whose learning employed conventional methods.

4. CONCLUSION

Based on the study's results, the application of Bruner's learning theory in learning angles on a circle material proved effective and had a meaningful, positive influence on the development of students' mathematical problem-solving skills between the experimental and control groups. The enactive stage, which involves manipulating concrete objects, successfully deepened students' conceptual understanding and laid a strong foundation for the success of the next stage, namely, iconic and symbolic representations. By using learning methods that align with the characteristics of the material and the needs of students, students' mathematical abilities can be optimised, as the right learning strategy can facilitate the process of knowledge construction, develop high-level thinking skills, and encourage active student involvement in problem-solving.

This research uses available media as a means of learning, which still utilises traditional media in the form of circles, threads, and rulers. However, the learning process is relatively longer in practice, and students' calculations tend to be less accurate due to the manual processing of information. Therefore, the use of technology-based media, such as *GeoGebra, Cabri Geometry, or Microsoft Mathematics, should be considered in future research to support concept visualisation*, accelerate the calculation process, and improve students' mathematical problem-solving skills more systematically and interactively. For further research, it is recommended that Bruner's theory be implemented in mathematics learning to measure various other mathematical abilities at a more diverse educational level.

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