

Developing a Talk-Moves-Based Learning Design for Systems of Linear Equations in Two Variables: A Design-Based Research Framework

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ABSTRACT

Mathematical communication in systems of linear equations in two variables requires students to connect representations, explain reasoning, and respond to mathematical ideas. However, existing instructional approaches do not always integrate classroom discourse, talk moves, and communication assessment into a coherent learning design. This methodological design study develops a talk-moves-based learning design for systems of linear equations in two variables and specifies an analytical protocol for its subsequent empirical validation. The study applies a design-based research framework comprising preliminary analysis, prototype development, iterative design, and retrospective analysis. The proposed design integrates multi-representational tasks, six talk moves, anticipated student responses, presentation orchestration, and oral and written mathematical communication assessment. A simulated dataset is used solely to illustrate the planned procedures for content validity, practicality, implementation, communication outcomes, and discourse analysis. The study produces a four-lesson instructional design and an accompanying analytical protocol that specify how design quality, implementation processes, mathematical communication, and classroom discourse can be examined in future empirical research. The contribution is a theoretically coherent and testable instructional design rather than evidence of instructional effectiveness. The design provides a structured basis for future classroom validation and may be adapted to other mathematical topics, including functions, ratios and proportions, and statistics.

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1. INTRODUCTION

Mathematical communication skills determine whether students can translate situations into mathematical models, explain relationships among concepts, use symbols

accurately, and evaluate the reasoning presented by others. The National Council of Teachers of Mathematics (NCTM) identifies communication, together with problem solving, reasoning, connections, and representation, as fundamental processes in mathematics learning [1]. Mathematical communication is therefore not limited to students' ability to report correct answers but also involves explaining mathematical ideas, interpreting representations, constructing arguments, and responding to the reasoning of others. In practice, however, classroom communication may be reduced to reporting final answers. When teachers primarily explain procedures, pose closed questions, and evaluate responses as right or wrong, students have limited opportunities to develop mathematical explanations and arguments [2], [3]. This instructional pattern highlights a design problem: students need structured opportunities to communicate mathematical reasoning, while teachers need practical mechanisms for organizing such discourse.

Systems of linear equations in two variables provide a particularly suitable context for developing mathematical communication because a single situation can be represented through verbal descriptions, tables, ordered pairs, graphs, and equations. These multiple representations require students not only to manipulate procedures but also to explain relationships among representations and interpret the meaning of solutions. However, students may perform elimination or substitution without explaining the meaning of variables, justifying their choice of strategy, or interpreting the point of intersection. Previous studies on linear equations have reported difficulties in translating contextual information, connecting representations, and constructing mathematical justifications [4], [5]. Thus, the instructional challenge is not simply how to teach procedures for solving systems of equations, but how to design learning experiences in which procedural work and mathematical communication are developed together.

Productive mathematical discourse requires deliberate instructional orchestration. Teachers need to make students' thinking audible, request clarification, shift responsibility for reasoning to students, and connect different strategies to an intended mathematical goal [6]. Research on mathematical tasks further indicates that the nature and cognitive demand of tasks influence the opportunities students have to explain, justify, and connect mathematical ideas [7]. Talk moves provide operational discourse practices that can support such interactions, including revoicing, repeating, reasoning, adding on, agreeing or disagreeing, and allowing sufficient wait time [8]. These practices are closely related to accountable talk, in which students are expected to make their reasoning explicit and respond substantively to the ideas of others [9]. Studies of classroom discourse also emphasize that productive mathematical interaction is shaped by how language is used as a resource for collective problem solving [10] and by broader sociomathematical norms concerning what counts as an acceptable mathematical explanation, justification, or argument [11]. Accordingly, talk moves should not be treated merely as isolated questioning techniques; they need to be embedded within tasks, classroom norms, and sequences of interaction that progressively position students as participants in mathematical reasoning.

Previous studies have therefore established the importance of mathematical discourse, productive discussion, and communication, but they do not by themselves provide a sufficiently integrated instructional sequence specifically designed for systems of linear

equations in two variables. In particular, the existing literature provides limited guidance on how communication objectives can be aligned systematically with (a) the sequence of instructional phases, (b) anticipated student responses, (c) multi-representational mathematical tasks, and (d) complementary oral and written assessments. Research on mathematical discussions emphasizes the orchestration of productive talk [12], while research on classroom discourse provides principles for making reasoning visible and accountable [9]. However, these contributions do not necessarily specify a complete learning design in which talk moves are sequenced with mathematical representations, anticipated responses, teacher follow-up, and observable communication outcomes. This distinction is important because the present study addresses a design problem rather than assuming that the effectiveness of the resulting design has already been empirically established.

Design-based research (DBR) provides an appropriate methodological framework for addressing this design problem because it enables researchers to develop and refine educational interventions through iterative processes while generating knowledge about the principles underlying their design [13]. Educational design research emphasizes the relationship between systematic design, development, evaluation, and reflection in authentic educational settings [14], [15], while conjecture mapping can be used to articulate relationships among design features, mediating processes, and anticipated learning outcomes [16]. For the present study, these perspectives support the development of a learning design in which talk moves function as design mechanisms rather than as independent questioning techniques. The intended design can subsequently be subjected to empirical classroom validation, but the current study should distinguish the development and evaluation of the design from claims about its effectiveness.

The specific gap addressed by this study is therefore the absence of an integrated learning design that explicitly connects instructional sequencing, anticipated student responses, multi-representational tasks, talk moves, and oral and written mathematical communication assessment within the topic of systems of linear equations in two variables. The proposed design integrates six talk moves into four instructional phases: (1) modeling a contextual situation, (2) comparing mathematical representations, (3) examining the equivalence of solution strategies, and (4) defending interpretations of solutions. Each phase specifies a discourse objective, anticipated student responses, teacher follow-up questions, and observable evidence of mathematical communication. The design also incorporates principles of effective mathematical discussion [12], intentional classroom talk [17], and productive discussion orchestration [6], while recognizing that students' mathematical communication develops through participation in classroom discourse rather than through procedural practice alone [18].

The originality of this study lies in combining these established discourse principles into a coherent, phase-based learning design rather than proposing another set of isolated teacher questioning techniques. The contribution of the study is therefore threefold: a learning design for systems of linear equations in two variables, a set of explicit design principles linking talk moves with mathematical representations and anticipated responses, and an initial evaluation of the design's validity and practicality. Because the present analysis uses simulated data, the study does not claim empirical effectiveness or causal

improvement in students' mathematical communication. Instead, effectiveness should be treated as a subsequent empirical validation question to be examined with actual students in classroom implementation. For empirical evaluation, improvement in communication scores can subsequently be examined using appropriate pretest–posttest procedures and effect-size reporting [19].

Accordingly, the study is guided by the following research questions: (RQ1) What instructional design for systems of linear equations in two variables can validly and practically integrate mathematical communication objectives, multi-representational tasks, sequenced talk moves, anticipated responses, and oral and written assessments? (RQ2) How are the proposed talk moves and instructional phases designed to mediate classroom mathematical discourse? (RQ3) What changes in students' mathematical communication can be examined when the design is empirically implemented with actual learners? The first two questions concern the design and its mediating mechanisms, whereas the third explicitly represents a future empirical validation question rather than an effectiveness claim based on the current simulated data.

The study is expected to contribute theoretically by clarifying how talk moves can be translated into explicit design features within a mathematical learning sequence, methodologically by demonstrating a DBR-oriented approach to developing and refining such a design, and practically by providing teachers with a structured framework for supporting students' mathematical explanations, representations, and justifications. The design is intended to complement established standards for mathematical practice and classroom discourse [2], while its assessment component can draw on established approaches to evaluating the quality and reliability of educational ratings [20]. Together, these contributions position the study as a design-oriented investigation whose proposed instructional model requires subsequent empirical testing before conclusions about effectiveness can be made.

2. METHOD

2.1. Research Design, Setting, and Participants

This study employed design-based research (DBR) as a design-oriented research approach that combines the development of practical educational solutions with the generation of design knowledge [13], [15]. The study was structured around the educational design research framework proposed by Plomp and Nieveen, consisting of preliminary research, prototyping, and assessment [14]. Preliminary research was designed to examine the curriculum, students' difficulties with systems of linear equations in two variables, classroom communication patterns, and teachers' instructional needs. During prototyping, Prototype 1 was intended to undergo expert review, revision, and two iterative classroom cycles. Each cycle was planned to include implementation, observation, analysis of the emerging learning process, and revision of the design. Retrospective analysis was planned to compare the conjectured learning process with observed classroom events and to identify design features requiring modification. These analyses would subsequently be used to formulate and refine the design principles underlying the learning sequence.

The planned empirical setting was a Grade 8 junior high school mathematics classroom in Medan, Indonesia, with an intended class size of 32 students. The class was selected purposively based on the planned teacher participation and schedule compatibility. The intended expert-validation panel consisted of four experts: two mathematics education lecturers, one educational evaluation expert, and one experienced mathematics teacher. A model teacher and two observers were also specified in the implementation plan. However, because no field trial was conducted and no empirical participant data were available for this manuscript, these individuals are described as planned participants rather than actual research participants. Consequently, the numerical values presented in the manuscript, including validation, implementation, response, and student achievement data, are deterministic simulations developed to examine the internal consistency of the instruments, tables, and analytical procedures.

The planned sample of 32 students was intended to support classroom-level DBR cycles rather than statistical generalization to a wider population. If implemented empirically, student participation would require predefined inclusion criteria, including enrollment in the target Grade 8 class and participation in the relevant learning sequence. Student identities would be anonymized using codes such as S01–S32.

2.2. Learning Design and Development Procedure

The learning design comprised four lessons. The first lesson focused on modeling two quantities in a buying-and-selling context. The second connected tables, graphs, and equations to strengthen students' ability to move between mathematical representations. The third compared elimination, substitution, and informal strategies. The fourth required students to evaluate and defend interpretations of solutions. The sequence was designed so that each lesson progressively increased the demand for students to explain, compare, justify, and evaluate mathematical ideas.

Teachers were intended to select talk moves according to the anticipated student response and instructional objective rather than follow a list of questions mechanically. The presentation sequence adapted the practices of anticipating, monitoring, selecting, sequencing, and connecting [12], [17]. In the DBR process, the first prototype would be reviewed against the intended communication objectives and subsequently revised when expert feedback or classroom observations indicated misalignment between the task, anticipated response, talk move, and communication evidence. The second cycle would then be used to examine whether the revised design could be implemented more coherently. Thus, each DBR cycle would generate evidence for a specific design revision rather than function solely as a repetition of the preceding lesson.

The design was evaluated using three planned dimensions: content validity, practicality, and implementation quality. Content validity concerned the alignment of the learning objectives, mathematical content, discourse activities, assessment indicators, and instructional sequence. Practicality concerned the clarity, usability, feasibility, and suitability of the learning design for classroom implementation. Implementation quality concerned the extent to which the planned learning activities and talk moves could be enacted according to the intended design. Because no empirical field trial was conducted,

these dimensions are treated as evaluation criteria rather than as observed empirical outcomes in the present manuscript.

Table 1. Operationalization of talk moves in the SPLDV learning design

Talk move	Sample teacher utterance	Communication function
Revoicing	“So, are you saying that ...?”	Clarifies meaning and models mathematical language.
Repeating	“Who can repeat Rani’s idea?”	Encourages students to listen to and process a peer’s idea.
Reasoning	“Why does that method always work?”	Elicits reasons, evidence, and conceptual connections.
Adding on	“Who would like to add to that idea?”	Extends or completes an idea.
Agreeing/disagreeing	“Do you agree or disagree? Explain.”	Promotes mathematical evaluation of a claim.
Wait time	“Write down your reasoning first.”	Provides thinking time and broadens participation.

2.3. Instruments, Data Sources, and Data Collection

The planned data sources comprised expert-validation forms, implementation-observation forms, audio/video recordings and discussion transcripts, student response questionnaires, student artifacts, and a mathematical communication test. These sources were selected to capture both the characteristics of the learning design and the discourse processes that the design was intended to support. Because the study currently contains simulated data, no actual classroom recordings, student responses, or expert ratings are claimed to have been collected.

The mathematical communication test consisted of four constructed-response items with a maximum score of 100. The analytic rubric assessed four dimensions: (a) representation, (b) mathematical expression, (c) written explanation, and (d) argumentation, with each dimension assigned a maximum score of 25. Content validity was planned to be evaluated using Aiken’s *V* [20]. For an empirical implementation, the four planned experts would independently evaluate the relevance and alignment of each item and design component, and the resulting Aiken’s *V* coefficients would be interpreted according to a predefined validity criterion. The numerical Aiken’s *V* values presented in the current manuscript, if any, should therefore be understood as simulated values rather than expert judgments obtained from an actual validation process.

Implementation and response data were planned to be converted into percentages to facilitate descriptive evaluation of practicality and implementation quality. The planned reliability analysis for the mathematical communication scoring procedure would use the intraclass correlation coefficient (ICC) after rater training and independent scoring procedures had been completed. No observed ICC value is reported because no actual rater data were available. Accordingly, the previously stated ICC value of 0.89 is not treated as an empirical finding and is removed from the description of the present study.

2.4. Data Analysis

For the planned empirical implementation, the effectiveness analysis would include descriptive statistics, normalized gain, a Shapiro–Wilk test of the pretest–posttest difference scores, a paired-samples *t* test at the .05 significance level, a 95% confidence interval for the mean difference, and Cohen’s d_{z} . These procedures would be applied only to observed student data and would be used to describe changes in mathematical communication scores rather than to establish broad population-level generalization.

N-gain was calculated as:

$$g = \frac{\text{posttest} - \text{pretest}}{100 - \text{pretest}}$$

Following Hake [21]. Effect size would be reported together with the corresponding estimate and confidence interval where appropriate, with interpretation based on the study design and estimation precision rather than relying exclusively on conventional small, medium, or large categories [19]. Because the current manuscript uses deterministic simulated data, these analyses are presented as demonstrations of the planned analytical procedure and should not be interpreted as evidence of treatment effectiveness.

The frequencies of talk moves and discourse episodes would be analyzed descriptively to characterize the instructional processes accompanying changes in mathematical communication scores. The discourse analysis would focus on the occurrence and function of the six talk moves across the four instructional phases, allowing the researchers to examine whether the intended discourse mechanisms were enacted as specified in the design. In a subsequent empirical study, these observations would provide process evidence that could be examined alongside students’ quantitative communication scores.

2.5. Ethical Considerations

No human-participant field data were collected for the present manuscript. Therefore, the simulated dataset does not represent actual students, teachers, experts, or classroom observations. Before any future empirical implementation, the study protocol would obtain institutional approval through the relevant institutional mechanism and would follow applicable requirements for research involving minors. The planned procedure would include written informed consent from parents or legal guardians, student assent, voluntary participation, permission for audio/video recording, confidentiality and anonymization of participant data, secure storage of recordings and research files, and the right to withdraw without academic consequences. Audio/video files would be accessible only to authorized members of the research team and would be used solely for the stated research purposes.

This ethical distinction is essential because the current manuscript describes a DBR design and simulated analytical procedures rather than reporting completed human-participant research. Any claims concerning actual classroom implementation, participant responses, expert judgments, or changes in student outcomes should therefore be reserved for a subsequent empirical study conducted after the required ethical and institutional procedures have been completed.

3. RESULTS

3.1. Illustration of the Planned Analysis

Because no empirical field data were collected, this section does not report actual research findings. Instead, it presents a simulated analytical demonstration based on deterministic data to illustrate how the proposed learning design, validation procedures, implementation indicators, mathematical communication outcomes, and discourse data would be analyzed in a future empirical study. All tables and figures in this section are therefore explicitly identified as simulated, and their values should not be interpreted as evidence of actual classroom effectiveness.

3.1.1. Simulated Product Characteristics and Content Validity

The simulated final product was specified to consist of a teacher guidebook, four sequenced learning activities, multi-representational student worksheets, an anticipated-response map, talk-move cards, observation sheets, and oral and written communication rubrics. Within the simulated development scenario, revisions to Prototype 1 focused primarily on prompt clarity, the allocation of wait time, and the connection between informal strategies and algebraic forms. The simulated expert-feedback process included recommendations to replace the question “How did you do it?” with more diagnostic prompts, such as “Which information did you define as variables, and why?” and “How does the graph verify the ordered pair you obtained?”

Table 2. Simulated content-validity results

Validated component	Aiken's V	Decision
Objectives and indicators	0.94	Valid
Learning activities for linear equation systems	0.91	Valid
Talk-move guide	0.95	Valid
Student worksheets and representations	0.89	Valid
Communication assessment	0.92	Valid
Mean	0.922	Valid

In the simulated dataset, the V coefficients ranged from 0.89 to 0.95, with a mean of 0.922. The student worksheet and representation component had the lowest simulated coefficient ($V = 0.89$), whereas the talk-move guide had the highest simulated coefficient ($V = 0.95$). Within the simulated development scenario, these values were used to illustrate potential revision priorities: additional guidance could be incorporated to support translation from contexts into tables, tables into equations, and equations into graphs. Similarly, the simulated feedback on the talk-move guide illustrates how each move could be explicitly linked to a communication objective and possible follow-up responses.

These simulated values illustrate how content-validity evidence could inform design revision; they do not demonstrate that actual experts judged the product to be valid. Accordingly, the results should be interpreted as an analytical demonstration of the planned validation procedure rather than as empirical evidence of product validity or effectiveness.

3.1.2. Simulated Practicality and Implementation Analysis

Table 3. Simulated practicality and implementation results.

Indicator	Simulated result	Illustrative interpretation
Teacher practicality	91.3%	Very practical
Student practicality	87.6%	Very practical
Implementation meeting 1	78.6%	Good
Implementation meeting 2	85.7%	Very good
Implementation meeting 3	91.1%	Very good
Implementation meeting 4	94.6%	Very good

In the simulated dataset, the implementation percentage ranged from 78.6% in Meeting 1 to 94.6% in Meeting 4. This pattern is used only to illustrate how implementation data could be examined across successive DBR cycles. In the simulated development scenario, the lower first-meeting value was associated with a hypothetical tendency for the teacher to provide explanations immediately after revoicing, thereby reducing opportunities for students to continue their own reasoning.

The corresponding simulated revision introduced a “probe before explain” rule, whereby reasoning, repeating, or adding on would be used before direct clarification. A simulated 60–90-second individual writing period was also incorporated before whole-class discussion to provide additional preparation time for students. These revisions represent design conjectures derived from the simulated scenario and should not be interpreted as observations of an actual teacher or classroom.

The simulated practicality percentages similarly illustrate the planned procedure for summarizing teacher and student responses. They do not constitute empirical evidence that the learning design was practical or that participants actually rated it as such.

3.1.3. Simulated Mathematical Communication Analysis

Table 4. Simulated pretest–posttest analysis.

Measure	Pretest	Posttest	Change
Mean	49.25	74.22	24.97
Standard deviation	9.14	13.54	7.76
Minimum–maximum	32–62	51–96	—
Mean N-gain	—	—	0.517
Shapiro–Wilk (difference)	—	—	$W = 0.950; p = 0.148$
Paired-samples <i>t</i> test	—	—	$t(31) = 18.210; p < .001$
95% CI of mean difference	—	—	22.17–27.77
Cohen’s <i>d</i> _z	—	—	3.219

In the simulated dataset, the mean score increased by 24.97 points, from 49.25 to 74.22. The simulated mean N-gain was 0.517. The difference scores produced a simulated Shapiro–Wilk statistic of $W = 0.950$ ($p = .148$), while the simulated paired-samples *t* test produced $t(31) = 18.210$ ($p < .001$), with a 95% confidence interval of [22.17, 27.77]. The corresponding simulated Cohen’s d_{z} was 3.219.

These inferential statistics are presented solely to demonstrate the planned analytical procedure. The simulated *p* value does not establish a statistically significant intervention

effect, and the simulated Cohen's $d_{z} = 3.219$ should not be interpreted as evidence that the intervention has a very large effect. Effect-size reporting is intended to accompany estimates of change and their precision in a future empirical implementation [19].

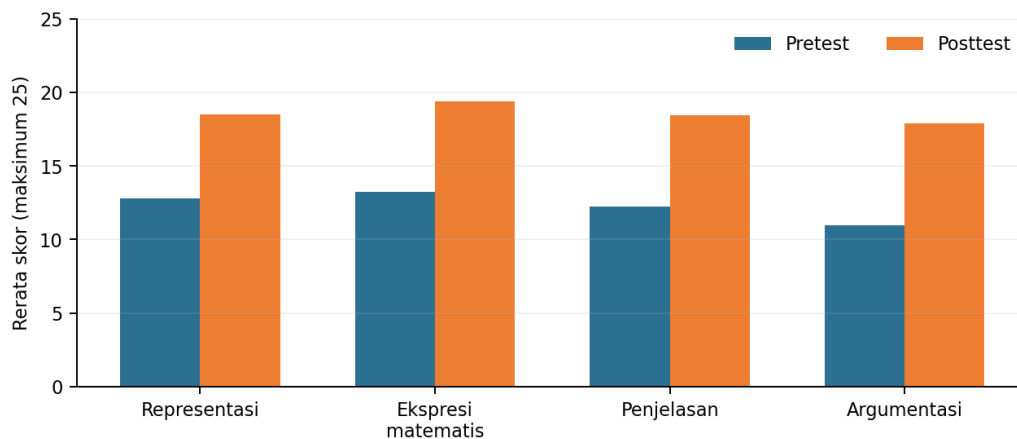


Figure 1. Simulated mean scores for each mathematical communication indicator

The simulated indicator-level pattern illustrates how changes across representation, mathematical expression, written explanation, and argumentation could be examined. In this simulated dataset, argumentation showed the largest numerical change, while mathematical expression had the highest simulated posttest mean. These patterns are not evidence of actual improvement. They instead illustrate the type of indicator-level comparison that could be conducted after real student data become available.

The simulated pattern also illustrates a potentially important design conjecture concerning oral and written communication. A student may be able to express an idea orally while still requiring additional support to produce a coherent written explanation. Therefore, the proposed design includes individual written synthesis following discussion. This feature represents a design hypothesis requiring empirical examination rather than a conclusion established by the simulated scores.

3.1.4. Simulated Classroom Discourse Analysis

Table 5. Simulated frequency of talk moves

Talk move	Cycle 1	Cycle 2	Change
Revoicing	14	18	4
Repeating	8	13	5
Reasoning	17	24	7
Adding on	7	15	8
Agree/disagree	6	14	8
Wait time	11	16	5

In the simulated dataset, the frequency of each talk move was higher in Cycle 2 than in Cycle 1. The largest numerical changes occurred for adding on and agreeing/disagreeing,

each increasing by eight simulated episodes. This pattern is presented only as an illustration of how discourse frequencies could be compared between DBR cycles.

The simulated pattern further illustrates a possible design conjecture: when students are explicitly invited to repeat, evaluate, and extend peers' ideas, classroom interaction may provide more opportunities for connections among utterances. This conjecture is consistent with the concept of accountable talk, in which classroom discourse is oriented toward the learning community, disciplinary knowledge, and standards of reasoning [9], [22]. However, simulated frequencies cannot demonstrate that such discourse development actually occurred.

Frequency alone would also be insufficient for evaluating discourse quality. In a future empirical implementation, talk-move episodes should therefore be examined together with their mathematical content, the nature of students' responses, the distribution of participation, and the extent to which the interaction advances the intended mathematical objective.

4. DISCUSSION

The findings presented above should be interpreted as a simulated analytical demonstration rather than as empirical evidence. Accordingly, the discussion focuses on the design conjectures that can be derived from the proposed learning architecture and on how these conjectures could be examined through future classroom implementation.

The design is hypothesized to operate through three interconnected mechanisms. First, multi-representational tasks are expected to generate differences among representations that provide substantive objects for mathematical discussion. Second, talk moves are intended to reduce the tendency toward immediate teacher evaluation by creating opportunities for students to clarify, extend, or challenge mathematical claims. Third, the sequencing of activities is intended to connect students' strategies with increasingly general interpretations of systems of linear equations in two variables. These conjectures are consistent with perspectives that conceptualize mathematical learning as participation in discourse and communication through mathematical representations and artifacts [18], [12].

The first conjecture concerns the role of multiple representations. A contextual situation can be represented verbally, numerically, graphically, and algebraically. The proposed design assumes that deliberately connecting these representations can create opportunities for students to explain why different forms describe the same mathematical relationship. This is a design assumption that requires empirical examination rather than a mechanism demonstrated by the simulated data [23], [24].

The second conjecture concerns the role of talk moves. Revoicing, repeating, reasoning, adding on, agreeing/disagreeing, and wait time are not intended to function as isolated sentence stems. Their proposed function depends on when and why the teacher uses them. For example, an agreeing/disagreeing prompt may support mathematical evaluation when students have a sufficiently explicit claim to examine, whereas the same prompt may generate superficial responses when students lack a representation or adequate thinking time. The design therefore positions talk moves as contingent instructional decisions linked to

anticipated student responses rather than as a fixed sequence of teacher questions [25], [26], [27].

The third conjecture concerns the relationship between oral and written mathematical communication. Oral discussion may provide opportunities for students to formulate, revise, and evaluate ideas, whereas written communication requires students to organize those ideas into an explicit and independently assessable form. For this reason, the proposed design concludes discussion episodes with individual written synthesis, including prompts for defining variables, organizing reasons, and interpreting solutions in context. The value of this design feature remains a proposition for future empirical testing rather than an established outcome of the simulated analysis [28].

From a design perspective, the contribution of the proposed model therefore extends beyond six talk-move sentence stems. The design integrates mathematical objectives, multi-representational tasks, classroom discourse norms, anticipated responses, talk moves, presentation sequences, and communication assessment. This integration is important because the same talk move may function differently depending on the mathematical task, students' current reasoning, the available representation, and the amount of thinking time provided [29].

The proposed DBR model can consequently be represented as a conjecture chain: design features → mediating discourse processes → expected communication outcomes. The design features comprise multi-representational tasks, anticipated-response maps, sequenced talk moves, wait-time structures, and oral–written assessment. The hypothesized mediating processes include clarification of mathematical meaning, comparison of representations, peer evaluation, extension of ideas, and individual synthesis. The expected outcomes are more explicit mathematical representations, expressions, explanations, and arguments. This conjecture map should be treated as a framework for future empirical investigation rather than as a set of relationships confirmed by the simulated data [30].

The main limitations of the present manuscript arise from its simulation-based status. The numerical results cannot establish actual product validity, practicality, implementation fidelity, classroom discourse development, or intervention effectiveness. In addition, the proposed classroom design involves a single intended class of 32 students and four lessons; these parameters would constrain transferability even if empirical data were subsequently collected. A future field study should document implementation fidelity, apply the planned inter-rater agreement procedures, and collect authentic classroom and student data. If the research objective is subsequently expanded to causal inference, a comparison group or an appropriate quasi-experimental design would be required.

Ethical procedures would also become essential once actual students and classroom recordings are involved, including institutional approval, parental consent, student assent, protection of participant identities, and secure storage of audio/video data. Thus, the present study should be understood primarily as a design-development and analytical-demonstration study, while empirical validation of the proposed learning design remains a subsequent research stage.

5. CONCLUSION

This study aimed to develop a talk-moves-based learning design for systems of linear equations in two variables that integrates mathematical communication with structured classroom discourse. The resulting design consists of a four-lesson instructional sequence incorporating multi-representational tasks, six talk moves, anticipated student responses, presentation orchestration, and oral and written mathematical communication assessment. The design also specifies an analytical protocol for examining content validity, practicality, implementation processes, mathematical communication, and classroom discourse in a subsequent empirical study.

The theoretical coherence of the design lies in the integration of mathematical representations, discourse processes, and communication assessment. Multi-representational tasks provide mathematical objects for explanation and comparison, while talk moves are positioned as contingent instructional practices that support clarification, reasoning, evaluation, and extension of students' ideas. The inclusion of individual written synthesis further connects oral classroom discourse with independently assessable mathematical communication. These relationships are formulated as design conjectures rather than established causal effects.

Because the present manuscript uses simulated data, the numerical results are intended only to illustrate the proposed analytical procedures and cannot be interpreted as evidence of product validity, practicality, implementation fidelity, improved mathematical communication, or instructional effectiveness. The main contribution of the study at this stage is therefore a testable instructional design and analytical protocol that can be subjected to empirical validation in authentic classroom settings.

Future research should implement the proposed design with actual students and teachers, document implementation fidelity, apply the planned inter-rater agreement procedures, and collect authentic classroom, discourse, and student outcome data. Subsequent studies may also examine whether the design principles can be transferred to other mathematical topics, including functions, ratios and proportions, and statistics, and may compare mathematical communication processes and outcomes across different instructional conditions.

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