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Exploring Computational Thinking in Geometric Proof Construction Across Gender Groups

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ABSTRACT

Mathematical proof requires students to coordinate concepts, procedures, reasoning, and justification, while computational thinking (CT) provides a lens for organizing such processes. How CT processes are manifested during geometric transformation proof construction, particularly in relation to variation in proof representation, remains insufficiently explored. This study examined CT processes in students' mathematical proof construction and described gender-related variation without assuming differences in mathematical ability. A qualitative descriptive design was conducted with 20 Grade XI students (10 male, 10 female) at a single senior high school. Data were collected through four open-ended geometric transformation tasks, reflection sheets, semi-structured interviews with six selected cases, and documentation. Data were analyzed through framework-based coding of decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization, alongside levels of proof construction. CT processes appeared unevenly across students. Evaluation and abstraction and generalization were the most challenging processes. Only 4 of 20 students reached the holistic-structural proof level. In the analyzed cases, some male cases showed more algebraic-structural representations, whereas some female cases showed more deductive-narrative representations. Conclusion: Procedural accuracy alone was insufficient for complete proof construction. The study contributes an analytical perspective linking CT processes with proof construction and highlights pedagogical attention to evaluation and generalization.

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1. INTRODUCTION

Mathematics requires not only the ability to obtain correct answers but also the ability to explain why a statement is true through logical, systematic, and defensible arguments.

Research on proof and proving indicates that proof construction is a central component of mathematics learning because it requires the coordination of concepts, representations, inferences, and justifications [1]. Understanding the structure of deductive proof and proof-production strategies is also essential to the development of students' proving abilities [2], [3]. In this process, obtaining a correct result is not sufficient; students must also validate the mathematical steps and relationships they use [4]. Research on learning trajectories in geometric proof construction shows that the development of proof understanding is associated with the ability to organize and connect components of an argument [5]. Difficulties in proving are also related to concept image, conceptualization processes, and the ability to connect facts with the proof objective [6], [7], [8]. Recent studies have further extended the discussion of proof to collaborative proof development and the use of technology to support formal proving activities [9], [10]. Accordingly, proving ability should be examined not only in terms of final outcomes but also in terms of the thinking processes students use to construct mathematical arguments. This process-oriented perspective is particularly important in geometric transformations, where students need to coordinate given information, transformation relationships, sequential procedures, and formal justification rather than merely obtain the transformed image or coordinates.

One relevant framework for tracing these processes is computational thinking. Systematic reviews show that computational thinking has received increasing attention in mathematics education, including primary mathematics, K–12 mathematics education, and the broader contexts of computing and mathematics education [11], [12], [13], [14]. In the present study, these literature-based perspectives are used as the theoretical basis for viewing computational thinking as a structured way of organizing information, identifying relationships, developing procedures, checking solutions, and moving from specific instances toward generalized representations. In this study, computational thinking is not understood merely as the use of computers or programming, but as a process of organizing problems and developing structured solutions. This distinction is important because computational thinking in this study refers to the organization of the reasoning process, whereas mathematical proof concerns the construction and justification of a mathematically valid argument. Thus, the two are analytically related but not treated as equivalent constructs. For analytical purposes, this process is operationalized through five components: decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization. These five components are treated as an operational analytical framework for tracing students' thinking during proof construction, rather than as a claim that they constitute the only possible classification of computational thinking. Decomposition helps students identify relevant facts and the proof objective; pattern recognition supports the identification of regularities; algorithmic thinking helps organize the steps of an argument; evaluation is used to examine the validity of procedures and conclusions; and abstraction and generalization enable students to move from specific cases to more general mathematical statements. Taken together, these processes provide a lens for examining how students move from problem interpretation and information organization toward procedural development, validation, and formal mathematical generalization.

A number of empirical studies have applied computational thinking in various mathematical contexts. Mubarokah et al. examined it in the solution of numeracy problems [15], Al Aziz et al. in mathematical problem solving among prospective teachers [16], and Hartawan et al. in mathematical problem solving among secondary school students [17]. Candraningtyas and Khusna linked computational thinking to mathematical critical thinking ability [18], while Afifah and Apriyono examined it in the context of social arithmetic [19]. Collectively, these studies indicate that computational thinking has been examined mainly as a process supporting problem solving, numeracy, critical thinking, and other mathematical activities. However, the specific processes through which computational thinking may be manifested during mathematical proof construction remain less clearly described.

In contrast, the literature on proving has focused more extensively on proof structure, validation, learning trajectories, concept image, and the process of constructing arguments [1]–[9]. The two strands of literature therefore provide complementary perspectives but have not been sufficiently integrated in empirical analyses of proof construction. This leaves a meaningful area for investigation at the intersection of the two fields, particularly regarding how computational thinking processes are manifested when students construct mathematical proofs involving geometric transformations.

This gap becomes particularly relevant when variations in thinking strategies across gender groups are considered. Harmini et al. examined computational thinking ability by gender in calculus learning [20], while Erawati and Purwati investigated mathematical proving ability in relation to gender and learning styles [21]. Nurwahyu et al. also identified variations in concept image by gender and mathematical ability [22]. Although these studies provide evidence that gender may be associated with variations in computational thinking, proving, or concept image, they do not specifically trace how such variations emerge across multiple thinking processes within a single proof-construction activity. However, these studies have not specifically explained how variations in male and female students' strategies emerge as they decompose a problem, recognize patterns, organize algorithmic steps, evaluate arguments, and engage in abstraction and generalization within a single proof-construction activity. Therefore, gender is not treated in this study as a deterministic factor that determines higher or lower mathematical ability, but as an analytical lens for identifying variations in representational strategies, information organization, the explicit articulation of reasoning, and argument structure. This approach allows gender-related tendencies to be described without interpreting group differences as evidence of inherent superiority or inferiority.

The importance of this investigation is further supported by preliminary observations in the research context. Some students tended to apply transformation formulas immediately without first identifying essential information, relationships among variables, or the statement to be proved. Some students were able to obtain transformation results procedurally but could not justify those results, whereas others could organize solution steps sequentially but experienced difficulty when converting numerical cases into symbolic forms and formal generalizations. Preliminary data also revealed variation in how students organized their solutions; some responses were concise and moved directly to symbolic forms, whereas others recorded information and explained solution steps more explicitly.

These conditions indicate that procedural success cannot be equated with successful proof construction. They also suggest that difficulties may occur at different points in the thinking process, from identifying relevant information and recognizing relationships to organizing procedures, evaluating validity, and formulating general statements. The more fundamental issue concerns how computational thinking operates throughout the transition from understanding a problem to developing a more complete mathematical argument. Examining this transition can provide a more detailed account of proof construction than an analysis based solely on the correctness of students' final answers.

Based on this research gap, the study aims to explore students' computational thinking processes in constructing mathematical proofs in the context of geometric transformations and to identify descriptive tendencies in these processes across male and female student groups. The study contributes to the literature by connecting a process-oriented computational-thinking perspective with the analysis of mathematical proof construction, while using gender as a descriptive analytical lens. The novelty of this study lies in three aspects. First, it integrates five computational thinking processes, namely decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization, with the structure of mathematical proof construction within a single analytical framework. Second, it shifts the focus from evaluating final outcomes to examining the underlying processes by tracing how each computational thinking process contributes to the development of mathematical argumentation from procedural solutions toward more structured proof construction. Third, the gender perspective is used to identify variations in cognitive and representational strategies rather than to determine which group demonstrates superior ability. Together, these features distinguish the present study from previous research that has primarily examined computational thinking in problem solving or mathematical proving without tracing the two perspectives simultaneously within geometric proof construction. Accordingly, this study addresses three research questions: (1) how do students' computational thinking processes emerge during mathematical proof construction in geometric transformations; (2) how are tendencies in these processes manifested across male and female student groups; and (3) what difficulties arise within each computational thinking process as students develop mathematical proofs? The findings are expected to contribute a process-based understanding of proof construction and provide an analytical basis for mathematics educators to identify where students encounter difficulties when developing mathematical arguments.

To clarify the analytical relationship proposed in this study, a conceptual diagram can be presented to illustrate the progression from the geometric transformation problem through decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization toward mathematical proof construction. The diagram should emphasize that the five computational-thinking processes function as an analytical lens for tracing students' reasoning during proof construction, rather than as a linear sequence that necessarily occurs in the same order for every student.

2. METHOD

2.1. Research Design and Context

This study employed a qualitative descriptive approach to explore students' computational thinking processes as they constructed mathematical proofs involving geometric transformations. A qualitative approach was selected because the study focused on how students identified and organized information, recognized patterns, developed solution steps, evaluated arguments, and engaged in abstraction and generalization, rather than on testing statistical differences between male and female student groups. This approach is consistent with the characteristics of qualitative research, which seeks an in-depth understanding of processes, experiences, and meanings [23], [24]. Accordingly, the unit of analysis was the reasoning process manifested in students' written responses, reflections, and explanations during interviews. The design therefore emphasized the description and interpretation of students' reasoning processes within their natural instructional context rather than the measurement of group differences.

The study was conducted at SMA Negeri 1 Panai Hulu, Labuhanbatu Regency, North Sumatra, during the second semester of the 2025/2026 academic year. Preliminary observations and initial information gathering were conducted from February to March 2026. This context was selected because the Grade XI students had already studied geometric transformations, and preliminary observations indicated variation in their ability to identify information, organize solution steps, provide justification, and construct mathematical generalizations. The research context was therefore considered appropriate for examining how computational-thinking processes were manifested when students were required to move from procedural transformation work toward mathematical justification and proof construction.

2.2. Participants and Subject Selection

The study involved 20 Grade XI students, comprising 10 male and 10 female students. The 20 students were selected from the Grade XI learning context in which the geometric-transformation material was taught and constituted the participants who completed the research activities. All participants completed a written assessment of computational thinking and mathematical proof construction. The number of participants was not intended to support statistical generalization, but rather to provide a range of responses that would allow students' thinking processes to be examined in depth.

After the written tests and reflection sheets had been analyzed, six students were purposively selected for in-depth interviews, comprising three male and three female students. Selection considered gender representation, variation in the attainment patterns of computational thinking indicators, the characteristics of proof construction evident in the responses, the completeness of information that could be obtained, and participants' ability to explain the reasoning and strategies underlying their solutions. The selection process was therefore intended to identify information-rich cases rather than a sample statistically representative of the population [23]. The six interview participants were thus treated as information-rich cases for qualitative interpretation, whereas the responses of all 20 participants were retained as part of the broader written-data corpus.

The mathematics teacher was involved as a supporting informant to provide contextual information about instruction and the characteristics of students' responses. Information from the teacher was used as supporting evidence in verifying interpretations, not as a basis for scoring students' abilities.

2.3. Data Sources and Research Instruments

In this qualitative study, the researcher served as the primary instrument for data collection, coding, interpretation, and verification. Supporting instruments included a written test, a post-test reflection sheet, a semi-structured interview guide, and documentation of students' responses. The instruments were designed to capture both the observable products of students' reasoning and the explanations underlying those products.

The written test consisted of four open-ended problems on geometric transformations covering translation, reflection, and composition of transformations, matrix-based transformations, and rotation. The tasks were designed to elicit computational thinking processes while also providing students with opportunities to construct mathematical justifications and proofs. The tasks differed in their mathematical demands so that students were required not only to perform transformation procedures but also to identify relevant information, establish relationships among mathematical objects, organize a logical sequence of reasoning, evaluate the validity of their work, and formulate conclusions beyond a single numerical case. The intended proof demand of each task was therefore connected to the five computational-thinking processes used in the analysis.

For example, a task involving a geometric transformation required students to identify the given coordinates and transformation information, determine the relevant transformation relationship, organize the transformation procedure, verify whether the resulting representation was consistent with the stated properties, and formulate or justify a conclusion in a general mathematical form. The task was consequently analyzed through decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization.

The five computational thinking processes used as the analytical framework were: (1) decomposition, defined as the ability to identify and organize known information, required information, and the elements to be proved; (2) pattern recognition, defined as the ability to identify regularities or relationships in coordinates, transformation properties, and mathematical representations; (3) algorithmic thinking, defined as the ability to organize solution steps and arguments logically and sequentially; (4) evaluation, defined as the ability to examine the accuracy of calculations, concepts, reasoning, and conclusions; and (5) abstraction and generalization, defined as the ability to focus on relevant mathematical structures and extend a specific case into a more general representation or conclusion. These definitions were applied consistently during coding to distinguish computational-thinking processes from the broader construct of mathematical proof. Computational thinking was used as the analytical lens for tracing how information and reasoning were organized, whereas proof construction was examined in terms of the coherence, connection, and mathematical justification of the resulting argument.

For the descriptive profile, students' performance on each indicator was categorized using a three-level rubric: 3 = capable, 2 = less capable, and 1 = not yet capable. The capable category was assigned when an indicator appeared completely, relevantly, and consistently; less capable was assigned when the indicator was present but remained partial or contained inconsistencies; and not yet capable was assigned when evidence of the indicator was absent or the response did not demonstrate the required process. The rubric was developed from the operational definitions of the five computational-thinking processes and was reviewed for alignment between each indicator, the task demands, and the observable evidence expected in students' responses. The review focused on whether each category could be distinguished from the others on the basis of students' written reasoning and explanations. These scores were used as an aid for categorization and description, not for inferential statistical testing.

Mathematical proof construction was analyzed separately by adapting proof-construction structures developed in mathematical proving research [2], [3], [5], [25], [7]. The three analytical levels were used to represent increasing degrees of organization and integration in students' proof construction. At the pre-structural level, students organized facts and basic steps; at the partial-structural level, students connected known facts with the statement to be proved; and at the holistic-structural level, students integrated premises, definitions, properties, or theorems to construct a logical and valid mathematical argument. Thus, the pre-structural level emphasized the presence of relevant mathematical elements without an adequate argumentative connection; the partial-structural level indicated that some relevant relationships had been established, but the argument remained incomplete; and the holistic-structural level indicated that the relevant mathematical elements were integrated into a coherent and justified argument.

The instruments were developed through a review of constructs and indicators, the preparation of items based on geometric transformation content, expert and mathematics-teacher review of content and construct alignment, examination of readability and content appropriateness, and revision prior to implementation. This process focused primarily on content validity and construct alignment because the instruments were intended to reveal thinking processes qualitatively rather than to function as large-scale psychometric tests. The review process also examined whether the tasks provided sufficient opportunities for students to demonstrate all five computational-thinking processes and different levels of proof construction.

The reflection sheet was administered immediately after the written test to obtain information about the strategies students used, the information they considered important, the patterns they recognized, how they checked their answers, and the difficulties they experienced. The semi-structured interview guide was developed around the five computational thinking processes and proof construction, with probing questions tailored to each student's responses. This individualized probing was intended to clarify reasoning that could not be fully inferred from the written responses alone.

2.4. Data Collection Procedure

Data were collected in several stages. First, the researchers coordinated with the school and mathematics teacher, established the research schedule, obtained research

permission, and prepared the instruments. The research procedures were explained to the participants before data collection, including the voluntary nature of participation, the use of the collected data for research purposes, and the protection of participants' identities. Second, all 20 students completed four open-ended problems and were asked to record the known information, solution process, mathematical reasoning, and conclusion.

Third, immediately after the test, students completed a reflection sheet to explain the thinking processes they had used while solving the problems. Fourth, written responses and reflections were examined preliminarily to identify variations in response profiles and to select six information-rich cases. The six selected participants then took part in semi-structured interviews to clarify the reasons for their strategy choices, relationships among concepts, pattern-recognition processes, evaluation of their answers, generalization, and sources of difficulty. Fifth, data from the written tests, reflections, interviews, documentation, and teacher information were compared to develop the final interpretation.

Under this procedure, the interviews served to deepen understanding of thinking processes that had first been identified through written responses and reflections, rather than functioning as a stand-alone source of data. The sequence was therefore designed to move from students' written mathematical work, to their own reflections, and finally to interview-based clarification of the reasoning underlying their responses.

2.5. Data Analysis

Data were analyzed iteratively from the beginning of data collection through conclusion drawing by adapting the interactive analysis model of Miles and Huberman [26], which includes data collection, data reduction, data display, and conclusion drawing and verification.

The first stage involved organizing written responses, reflection sheets, interview data, and documentation according to each participant's anonymous code. The second stage used deductive framework-based coding based on the five computational thinking processes. Data segments were coded as decomposition, pattern recognition, algorithmic thinking, evaluation, or abstraction and generalization according to the established operational definitions and rubric. Coding was conducted by the researcher using the predefined operational definitions, with repeated examination of the coded data to maintain consistency across participants and data sources. Interpretive decisions were checked against the original written responses, reflections, and interview explanations before being retained as findings.

The third stage analyzed proof construction by mapping students' responses onto the pre-structural, partial-structural, or holistic-structural levels based on the organization of facts, the connection between facts and the proof objective, and the use of concepts, definitions, properties, or theorems within the argument. The analysis kept computational-thinking coding and proof-construction analysis conceptually distinct so that the presence of a particular computational-thinking process was not automatically interpreted as evidence of a complete proof.

The fourth stage consisted of cross-case analysis. Response patterns were first compared among participants within the same group and then across male and female student groups. Comparisons focused on how students organized information, made their reasoning

explicit, used verbal and symbolic representations, evaluated their work, and engaged in abstraction and generalization. Counts were used as the primary descriptive representation of how many participants demonstrated particular categories, while percentages were used only to provide additional descriptive context. Neither measure was interpreted as evidence of statistically significant gender differences.

2.6. Trustworthiness and Research Ethics

The credibility of the interpretations was strengthened by comparing multiple sources and forms of evidence, including written responses, reflection sheets, student interviews, documentation, and supporting information from the teacher. Discrepancies between written responses and verbal explanations were revisited through probing and data checking before final interpretations were established. A pattern was retained as a finding when it was consistently supported by more than one form of evidence or was adequately explained through the interviews. The iterative comparison of evidence also helped reduce the possibility that a conclusion would be based solely on a single written response or a single interview statement.

Participants' identities were anonymized using subject codes, and the data were used solely for research purposes. Research permission was obtained from the school before data collection. Because the participants were Grade XI students and some may have been under the age of 18, informed parental or guardian consent and student assent should be explicitly reported if these procedures were implemented. Participation was voluntary, and students were informed that their participation would not affect their academic assessment. The grouping of male and female students was used solely as an analytical lens for exploring variations in thinking and representational strategies, not as a basis for establishing the superiority of one group's ability over the other. Any institutional ethical approval or formal ethics review should likewise be reported here by stating the approving institution and approval/reference number, if such approval was obtained.

3. RESULTS

3.1. Overall Profile of Students' Computational Thinking in Mathematical Proof Construction

This section presents the findings from the written responses, reflection sheets, interviews, and supporting documentation. The results are organized according to the five computational thinking processes, levels of mathematical proof construction, gender-related descriptive patterns, and the difficulties identified across the cases.

Analysis of the written responses, reflection sheets, and interview data showed that the five computational thinking processes did not emerge at the same level of attainment. Among the 20 students, the proportions classified as capable were 40% for decomposition, 35% for pattern recognition, 30% for algorithmic thinking, 25% for evaluation, and 20% for abstraction and generalization. Thus, the earlier processes of organizing information and recognizing regularities were identified more frequently than processes requiring students to examine the validity of arguments and construct generalizations.

Table 1. Students' computational thinking profiles in mathematical proof construction

Computational thinking process	Capable, n (%)	Less capable, n (%)	Not yet capable, n (%)	Dominant characteristics
Decomposition	8 (40%)	7 (35%)	5 (25%)	Information was identified, but the proof objective was not always stated explicitly.
Pattern recognition	7 (35%)	8 (40%)	5 (25%)	Coordinate patterns were recognized but were not consistently connected to transformation properties.
Algorithmic thinking	6 (30%)	9 (45%)	5 (25%)	Procedural sequences emerged, but logical transitions were sometimes incomplete.
Evaluation	5 (25%)	8 (40%)	7 (35%)	Verification focused more on calculations than on the validity of mathematical arguments.
Abstraction and generalization	4 (20%)	9 (45%)	7 (35%)	Specific cases could often be solved, but general mathematical statements were difficult to construct.

These findings show a decrease in the proportion of students categorized as capable, from 40% in decomposition to 20% in abstraction and generalization. This pattern does not imply that the five processes occurred in a linear sequence; rather, it indicates that fewer students demonstrated the corresponding indicators completely when the task required increasingly explicit validation, formal representation, and generalization.

Evidence from the written responses reinforced this pattern. In the translation task, SL-02 first identified the initial coordinates, translation vector, image coordinates, and proof objective before performing the operations. In contrast, SP3 proceeded directly to coordinate calculations without explicitly identifying what needed to be proved. This contrast indicates that successful decomposition was not determined solely by the ability to obtain a correct numerical result, but also by the ability to organize relevant information as a basis for the argument. SL-02's response displayed a more explicit decomposition structure, whereas SP3's response showed that procedural progress could still occur even when the initial structure of the proof had not been fully established.

In abstraction and generalization, only four students (20%) were able to move from a specific case to a general representation. In the rotation task, students classified as capable were able to develop the relevant mapping and connect it to the rotation matrix and distance invariance. In contrast, students in the lower categories tended to stop at numerical substitution or formula application without justifying its general validity.

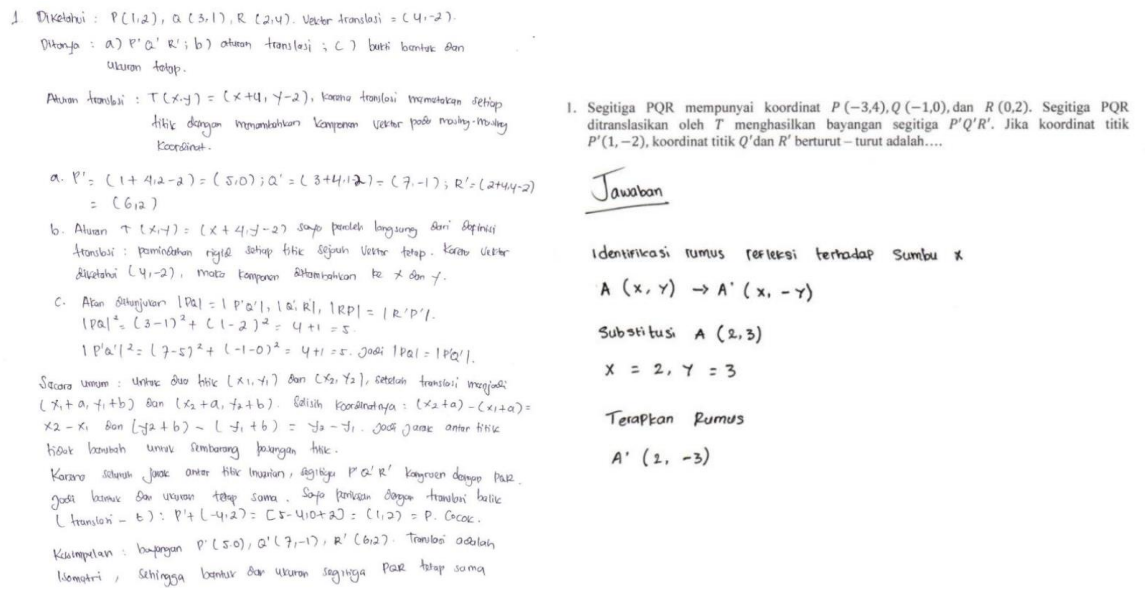


Figure 1. Contrasting written responses in decomposition: (a) SL-02 showing explicit decomposition and (b) SP3 showing incomplete decomposition.

3.2. Levels of Mathematical Proof Construction

Analysis of proof structure showed that 4 of the 20 students reached the holistic-structural level, eight were at the partial-structural level, and the remaining eight were at the pre-structural level. The distribution by gender is presented in Table 2.

Table 2. Distribution of mathematical proof-construction levels across gender groups

Proof-construction level	Male students (n=10)	Female students (n=10)	Total
Holistic-structural	2	2	4
Partial-structural	3	5	8
Pre-structural	5	3	8
Total	10	10	20

Descriptively, the number of students reaching the holistic-structural level was the same in both groups, with two students in each. Differences in the distribution were more evident at the partial-structural and pre-structural levels. These distributions are presented as descriptive characteristics of the participants rather than as evidence of a gender effect. These data were therefore not used to infer the superiority of either group, but rather as a basis for tracing how argument structures were constructed in the analyzed cases.

Cross-case analysis showed that differences across levels primarily concerned the completeness of the connection between procedure and justification. Pre-structural responses generally contained facts or calculation results but did not yet show a clear relationship to the proof objective. In partial-structural responses, students began to connect patterns, transformation rules, and solution steps with the statement to be proved, although the argument was not always complete. Holistic-structural responses, by contrast, were characterized by the integration of information, patterns, procedures, evaluation, and generalization into an argument that could be followed logically. Procedural accuracy alone was therefore insufficient to place a response at a higher proof level; completeness of

justification and the general validity of the argument were the principal distinguishing features. This relationship is understood as an analytical pattern emerging from the data and is not intended to imply a causal relationship among the processes.

3.3. Gender-Related Patterns in Computational Thinking and Proof Construction

Table 3. Computational thinking profiles across gender groups

CT process	Male: Capable	Male: Less capable	Male: Not yet capable	Female: Capable	Female: Less capable	Female: Not yet capable
Decomposition	4	3	3	4	4	2
Pattern recognition	3	4	3	4	4	2
Algorithmic thinking	2	5	3	4	4	2
Evaluation	1	4	5	4	4	2
Abstraction and generalization	1	4	5	3	5	2

Based on Table 3, the distribution of computational thinking profiles shows descriptive variation between the male and female student groups across the five processes analyzed. In decomposition, the number of students categorized as capable was relatively similar in the two groups, with four students in each. Differences became more apparent in pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization. Among female students, the numbers categorized as capable were 4, 4, 4, and 3, respectively, whereas among male students the corresponding numbers were 3, 2, 1, and 1. Conversely, the number of male students categorized as not yet capable increased to five in evaluation and in abstraction and generalization, while the number among female students remained relatively stable at two for each process. These descriptive distributions suggest that the observed variation between the groups was more visible in processes involving evaluation and formal generalization than in decomposition.

However, this distribution was not used to conclude that either gender group had generally higher ability. Given the qualitative design and limited number of participants, the frequencies in Table 3 functioned as descriptive mapping to identify cases and tendencies for subsequent analysis through written responses, reflection sheets, and interviews. The observed differences are therefore more appropriately understood as variations in how thinking processes and proof-construction strategies were manifested in the analyzed cases, rather than as evidence of universal gender-based differences in ability. The findings should consequently be interpreted as case-specific and exploratory rather than as evidence of stable gender-related differences in mathematical ability.

Analysis of selected cases showed that more substantive variation was evident not only in the category distributions but also in how students represented and articulated their proofs. In the case of SL-02, the argument tended to move more quickly toward symbolic representation through vectors, matrices, general variables, and invariance properties. The proof structure was relatively concise because several mathematical relationships were expressed symbolically. In contrast, SP-02 tended to develop the argument through a more

explicit sequence of definitions, transformation properties, solution steps, and conclusions. Both cases were capable of producing acceptable proof constructions, but they followed different representational pathways.

Based on these patterns, this study uses the term algebraic-structural proof to describe proofs constructed primarily through symbolic forms, matrices, vectors, and invariance relationships, and deductive-narrative proof to describe proofs that more explicitly emphasize the sequence of definitions, properties, reasons, and conclusions. These terms are used only as descriptive categories for the analyzed cases and should not be interpreted as established proof types or as characteristics inherently associated with either gender. Accordingly, the gender perspective in this study serves to identify variations in representational strategies and proof presentation rather than to establish a gender-based hierarchy of ability.

3.4. Main Difficulties Across Computational Thinking Processes

Triangulation of written responses, reflection sheets, and interviews showed that students' difficulties were not limited to obtaining correct transformation results, but were concentrated primarily in the transition from procedural solution to mathematical justification. These difficulties appeared across all computational thinking processes but were more pronounced in evaluation and in abstraction and generalization, where students had to examine the validity of arguments and extend results from specific cases into mathematical statements with broader validity.

Table 4. Observed difficulties across computational thinking processes and gender cases

CT process	Observed tendencies in male cases	Observed tendencies in female cases
Decomposition	Frequently moved directly to numbers, formulas, or vectors without explicitly identifying the proof objective.	More often recorded known and required information, but did not always identify which information was essential for the proof.
Pattern recognition	Patterns were often recognized quickly but were not always articulated as justification.	Patterns were traced more gradually but were not always connected to a general mathematical principle.
Algorithmic thinking	Some logical transitions were omitted because they were considered self-evident.	Procedures were generally more explicit but sometimes remained dependent on familiar examples.
Evaluation	Verification sometimes stopped after obtaining a plausible result	Calculations were rechecked more frequently, but conceptual validity was not always examined
Abstraction and generalization	Symbolic forms could be produced without sufficient explanation of their general meaning.	Verbal explanations were often more explicit, but translating them into formal generalizations remained challenging.

Both groups displayed the same fundamental difficulty: procedural success did not automatically develop into mathematical justification. The patterns in Table 4 should be interpreted cautiously because they describe the analyzed cases and may reflect individual differences in prior learning and mathematical communication rather than gender alone. In several male-student cases, difficulties appeared as jumps between steps, unarticulated

reasons, and limited conceptual evaluation. In several female-student cases, difficulties were more evident when an already organized verbal explanation had to be transformed into symbolic representation and formal generalization. The consistency of these patterns across written responses, reflection sheets, and interviews strengthened the interpretation that the central difficulty lay in the transition from carrying out procedures to constructing mathematically justifiable arguments.

At the evaluation stage, students in the less capable and not yet capable categories tended to check whether their results appeared procedurally correct, but they did not consistently assess whether those results were consistent with transformation properties or whether the proof arguments were valid. Checking was often limited to repeating calculations or examining visual agreement, while the logical relationships among premises, procedures, and conclusions received less attention. At the abstraction and generalization stage, some students were able to use formulas, symbols, or matrices but could not explain why those representations applied to an arbitrary point or to similar cases in general.

Interview data clarified the distinction between using mathematical representations and understanding their meaning. Some students could use matrices or symbolic forms correctly, yet their explanations remained procedure-oriented and did not show why the representation supported a generally valid conclusion. Conversely, some students could explain relationships among concepts verbally but were not yet able to express those relationships as formal generalizations. These findings indicate that correct use of symbols does not in itself constitute complete proof construction; students must also integrate representation, justification, evaluation, and generalization.

Overall, the empirical findings support a flexible pattern of computational thinking development rather than a fully linear sequence of stages. Evaluation and abstraction, and generalization emerged as the most challenging processes, as reflected in the proportions of students categorized as capable, which were only 25% and 20%, respectively. These findings indicate that holistic proof construction cannot be supported solely by the ability to obtain correct results or execute transformation procedures. The quality of an argument also depends on the ability to evaluate the validity of steps, connect representations with their underlying concepts, and extend relationships from specific cases to mathematical generalizations. In the analyzed cases, the integration of evaluation with abstraction and generalization was therefore the most salient characteristic as responses moved toward more complete proof construction.

3.5. Summary of Key Findings

Overall, the results indicate three main patterns. First, computational-thinking processes were manifested unevenly, with decomposition occurring more frequently at the capable level than evaluation and abstraction and generalization. Second, proof-construction levels differed primarily in the completeness of the relationship between mathematical procedures and their justification, with only four students reaching the holistic-structural level. Third, gender-related distributions showed descriptive variation, particularly in evaluation and abstraction and generalization, but the qualitative design and limited sample

do not support interpreting these patterns as general gender differences in mathematical ability.

4. DISCUSSION

4.1. Computational Thinking as a Process in Mathematical Proof Construction

The findings show that computational thinking processes did not develop uniformly as students constructed mathematical proofs. Decomposition and pattern recognition appeared relatively more frequently than evaluation and abstraction and generalization. This pattern indicates that some students were able to identify important information and recognize regularities in transformations but experienced difficulty when required to examine the validity of an argument and transform the solution of a specific case into a generally valid mathematical statement. These findings extend previous studies of computational thinking in mathematics education [11] [12][13][14] and empirical research in the contexts of numeracy, problem solving, critical thinking, and social arithmetic [15][16][17][18][19]. In contrast to previous studies that predominantly used problem-solving contexts, the present study shows how these processes are manifested as students move from procedural solutions toward the construction of proof arguments.

The finding is particularly relevant to the distinction between computational thinking and mathematical proof established in this study. Computational thinking provides a process-oriented lens for examining how students organize information, identify relationships, sequence procedures, evaluate results, and generalize representations, whereas mathematical proof requires these processes to be integrated into a logically justified mathematical argument. The results therefore suggest that demonstrating a computational-thinking process does not automatically indicate that a student has constructed a complete proof.

4.2. Proof Construction and the Transition from Procedure to Justification

Differences in proof-construction levels reinforce this interpretation. Pre-structural responses generally consisted of collections of facts or calculation results that had not yet been explicitly connected to the proof objective. At the partial-structural level, students began to connect facts, transformation rules, and calculation results, but the logical relationships among the components were not yet fully integrated. Holistic-structural responses, in contrast, demonstrated the ability to coordinate premises, definitions, transformation properties, evaluation, and generalization within a traceable argumentative structure. This pattern is consistent with research on proof structure and proof-production strategies [2], [3], learning trajectories in geometric proof construction [5], and studies of concept image and difficulties in proving [27][8]. Thus, proof quality cannot be judged solely by the correctness of the result, but must also be considered in terms of the connections among procedures, concepts, representations, and justifications.

The distinction among the three proof-construction levels also helps explain why a correct transformation result may coexist with an incomplete proof. A student may successfully execute a transformation algorithm while leaving the relationship between the procedure and the proposition to be proved implicit. The findings therefore reinforce the

need to examine the argumentative structure surrounding a result rather than treating procedural correctness as sufficient evidence of mathematical proof.

4.3. Evaluation and Abstraction-Generalization as Transition Points

The findings also show that evaluation and abstraction and generalization were the most challenging processes. Only 25% of students were categorized as capable in evaluation and 20% in abstraction and generalization. This result is important because validation is an integral part of proof construction [4], while the development of proof structure requires the ability to connect cases, representations, reasons, and statements with broader validity [3], [4], [5]. Obtaining a correct transformation result therefore cannot be equated with successfully constructing a proof. The findings indicate that evaluation and generalization are critical points bridging procedural solutions and more complete mathematical arguments.

The difficulty at the evaluation stage suggests that checking an answer and validating an argument are not necessarily equivalent activities. Repeating a calculation or confirming that an image appears visually plausible may establish procedural consistency without addressing whether the reasoning is mathematically valid. Similarly, the difficulty in abstraction and generalization suggests that using a symbolic representation is different from explaining the conditions under which that representation remains valid for a broader class of cases.

These findings provide a process-based explanation for the lower proportion of students classified as capable in the two processes. Evaluation requires students to move beyond producing a result toward questioning the validity of the reasoning, while abstraction and generalization require them to identify the mathematical structure that remains invariant across cases. The transition from procedure to proof consequently involves not only additional steps but also a change in the level at which students justify mathematical relationships.

4.4. Gender-Related Patterns: An Exploratory Interpretation

From a gender perspective, the study found no basis for concluding that either group demonstrated superior ability. The number of students reaching the holistic-structural level was the same in the male and female groups, with two students in each. More meaningful differences were observed in how arguments were represented and made explicit. These findings can be considered alongside Harmini et al.'s study of computational thinking by gender [20], Erawati and Purwati's study of mathematical proving by gender [21], and Nurwahyu et al.'s study of variations in concept image [22]. However, the present study extends these perspectives by suggesting that gender-related variation is more appropriately understood as variation in representational strategies and the presentation of arguments in the analyzed cases, rather than as evidence that one group possesses consistently greater mathematical capacity.

The gender-related findings should nevertheless be interpreted as exploratory and case-specific. The present study did not directly measure prior mathematical achievement, familiarity with symbolic notation, previous experience with proof, instructional exposure to geometric transformations, or individual differences in verbal and written mathematical

communication. Any of these factors could contribute to differences in how students presented or explained their proofs independently of gender. Because these factors were not systematically controlled or compared in the present study, the observed gender-related patterns cannot be attributed specifically to gender.

This qualification is particularly important because the same gender group contained students with different computational-thinking profiles and proof-construction levels. The findings therefore support the use of gender as an analytical lens for describing variation within the observed cases, but they do not support treating gender as an explanatory mechanism for the differences. Future studies would need designs that explicitly examine prior achievement, learning experience, instructional conditions, and mathematical communication alongside gender to investigate these relationships more systematically.

4.5. Representational Pathways in Proof Construction

The findings on students' difficulties reveal a gap between procedural solution and mathematical justification. Some students were able to use transformation formulas, matrices, or procedures correctly but could not explain why those procedures were valid. Conversely, some students could explain mathematical relationships verbally but were unable to express them as formal generalizations. This pattern is consistent with the importance of validation activities in proof construction [4] and findings on the development of understanding of geometric proof structure [5]. Difficulties in connecting procedures with reasons also align with studies of proving difficulties and conceptualization processes [28], [8]. The present findings add that this gap can be traced specifically through evaluation and abstraction-generalization processes within the computational thinking framework.

The two exploratory representational categories identified in the cases, namely algebraic-structural proof and deductive-narrative proof, further illustrate this variation. The former emphasizes concise symbolic relationships, whereas the latter makes definitions, properties, reasons, and conclusions more explicit. Neither pathway should be interpreted as inherently more effective or as characteristic of a particular gender. Rather, the categories indicate that students may make mathematical reasoning visible through different representational forms, and that each form still requires sufficient justification to constitute a complete proof.

4.6. The Non-Linear Relationship among Computational Thinking Processes

Analytically, the relationship between the five computational thinking processes and proof construction is better understood as an interconnected pattern that is not fully linear. Students did not necessarily proceed through decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization in the same sequence. Some students used symbolic representations before conducting adequate conceptual evaluation, whereas others could explain relationships verbally but had not yet developed formal generalizations. This non-linear interpretation is also relevant to research on the development of proving, which indicates that construction, presentation, and reflection can mutually reinforce one another as a proof develops [9]. Computational thinking in this study

is therefore understood as the dynamic coordination of processes rather than as a rigid sequence of cognitive stages.

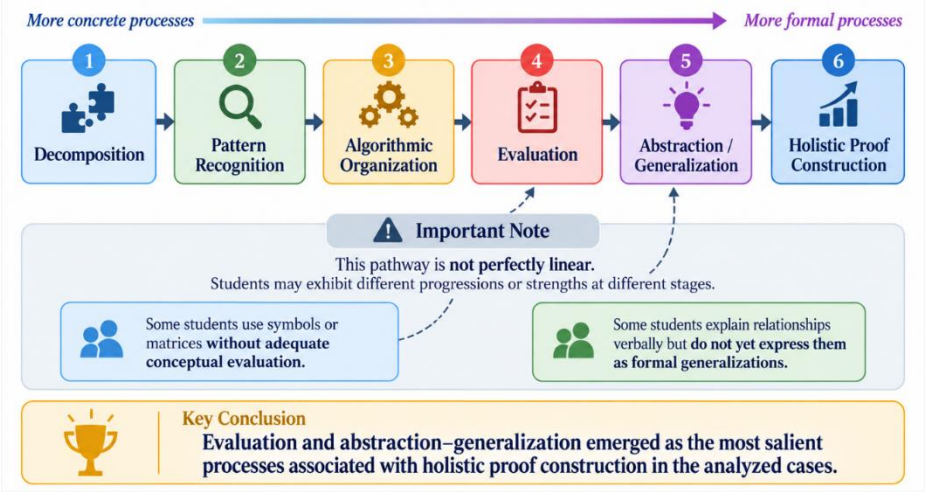


Figure 2. Analytical synthesis of computational thinking processes associated with mathematical proof construction

Figure 2 summarizes the analytical relationships that emerged from the cases examined and is not intended as a causal model. Evaluation and abstraction-generalization emerged as the most salient processes as responses progressed toward holistic proofs, but students could enter, revisit, or strengthen particular processes at different points during proving. Mathematical proof construction is therefore more appropriately viewed as flexible coordination among processes rather than as a completely fixed cognitive sequence.

To make this interpretation visually consistent, Figure 2 should use bidirectional or curved connections among the five computational-thinking processes rather than a single directional chain. Decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization may therefore be represented as interconnected processes surrounding mathematical proof construction, with arrows indicating that students can revisit earlier processes when evaluation reveals an incomplete argument or when generalization requires additional information and pattern recognition.

4.7. Instructional and Theoretical Implications

From an instructional perspective, these findings indicate that geometric transformation exercises should not stop at determining image coordinates or applying formulas. Instruction should provide opportunities for students to identify the proof objective, explain the reason for each step, conduct validation [4], develop proof structure progressively [3], [5], and reflect on the arguments they have constructed [9]. Questions such as “Why is this step valid?”, “Does this conclusion hold for all points?”, and “How else can this result be verified?” can be used to make evaluation and generalization explicit components of proving activities.

In higher education, the use of proof assistants such as Lean demonstrates the potential of formal proving technologies to support validation and proof representation [29]. In the context of the present school-level study, this example is more appropriately viewed

as a direction for future research rather than as a direct instructional implication of the current findings. Future studies could examine whether age-appropriate digital proof environments can provide additional opportunities for students to test, revise, and justify mathematical arguments.

The principal contribution of this study lies in integrating the five computational thinking processes with the structure of mathematical proof construction. Previous systematic reviews have documented the development of computational thinking in mathematics education and K–12 contexts [11], [12], [13], [14], while proof research has emphasized structure, validation, and the development of proof and proving activities [1]–[5]. The present study brings these two strands together by tracing how decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization are manifested throughout the construction of proofs involving geometric transformations. In addition, gender is used as an analytical lens for understanding variation in representational strategies without treating it as a basis for an ability hierarchy.

The theoretical contribution therefore lies not in proposing a new computational-thinking model, but in demonstrating how the five processes can be used as an analytical lens for examining proof construction. The findings particularly highlight evaluation and abstraction-generalization as processes that warrant explicit attention when analyzing the transition from procedural mathematical work to more complete argumentation.

4.8. Limitations and Future Research

Nevertheless, the findings should be interpreted in light of the study's limitations. Participants came from only one class in one school, the number of interview participants was limited, and the mathematical context was restricted to geometric transformations. The findings are therefore not intended for statistical generalization. The qualitative interpretation also involves researcher judgment in coding and categorizing students' responses, which may introduce interpretive subjectivity even when the researcher repeatedly checks the coded evidence against the original data. In addition, the study was cross-sectional and did not follow students over time; consequently, the findings cannot establish how computational-thinking processes or proof-construction levels develop longitudinally.

Future research could involve different mathematical topics, larger numbers of participants, and mixed-methods designs to examine whether the observed relationship among evaluation, abstraction-generalization, and the quality of proof construction also appears in broader populations. Future studies should also consider prior mathematical achievement, familiarity with symbolic notation, previous proof-learning experience, instructional practices, and mathematical communication ability when examining gender-related patterns. Intervention studies could also investigate the effectiveness of instruction that explicitly integrates evaluation and generalization into proving activities. Longitudinal research could further examine whether explicit instruction in validation and generalization changes the development of students' proof-construction processes over time.

5. CONCLUSION

This study explored how computational thinking processes are manifested in students' mathematical proof construction involving geometric transformations, with gender used as an analytical lens for examining variation in reasoning and proof representation. The findings show that decomposition, pattern recognition, algorithmic thinking, evaluation, and abstraction and generalization appeared at different levels, with evaluation and abstraction and generalization emerging as the most challenging processes. The findings indicate that procedural accuracy alone is insufficient for complete proof construction because obtaining a correct result does not necessarily involve explaining the validity of procedures, evaluating the strength of an argument, or establishing general validity. More complete proof construction involves the integration of information, patterns, procedures, evaluation, and generalization within a coherent mathematical argument.

The gender-related findings are best understood as variation in proof representation rather than variation in mathematical ability. Some male cases demonstrated a more algebraic-structural representation involving vectors, matrices, symbolic variables, and invariance, whereas some female cases more explicitly organized definitions, properties, procedural steps, and conclusions in a deductive-narrative form. These patterns were observed in selected cases and should not be generalized to male or female students as groups. Theoretically, the study contributes an empirically grounded use of computational thinking as an analytical lens for examining mathematical proof construction, particularly the transition from procedural execution to justification. Pedagogically, the findings highlight the need for learning activities that explicitly engage students in evaluating the validity of reasoning and developing abstraction and generalization beyond specific examples.

The findings should be interpreted in light of the study's limitations, including the single Grade XI learning context, focus on geometric transformations, limited interview cases, researcher-based qualitative interpretation, and cross-sectional design. Future research should employ broader quantitative or mixed-method designs across different mathematical topics and educational contexts to examine the consistency of these patterns. Future studies should also investigate whether the identified proof strategies remain consistent when controlling for prior mathematical achievement and proof experience, alongside factors such as familiarity with symbolic notation, instructional practices, and mathematical communication.

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