

Spatial-Algebraic Cognitive Barriers in Solving Ellipse and Hyperbola Problems: An Analysis Using Polya's Heuristics and Cognitive Load Theory

David Pratama¹, Aditya Prihandhika², Rafiq Zulkarnaen³, Novia Nurlatifah⁴, Putri Hoeriyah⁵, Seli Mauludani⁶

^{1,2,3,4,5,6}Universitas Singaperbangsa Karawang, Karawang, Indonesia

Article Info

Article history:

Received 2026-08-19

Revised 2026-09-19

Accepted 2026-09-30

Keywords:

Cognitive barriers

Cognitive load theory

Polya's heuristics

Problem-solving

Spatial algebraic representation

ABSTRACT

Ellipse and hyperbola problems require students to coordinate geometric, spatial, and algebraic representations, which may generate cognitive barriers. This qualitative preliminary study aims to characterise students' cognitive barriers when solving conic section tasks by integrating Polya's heuristics with Cognitive Load Theory (CLT) as an analytical lens, without directly measuring empirical cognitive load states. Participants included 25 undergraduate students across two groups (4A and 4B). Data were collected from students' written metacognitive reflections following a final examination alongside their problem-solving artefacts. Data were analysed using integrative coding across Polya's four stages and qualitatively interpreted through intrinsic, extraneous, and germane cognitive load dimensions. Among the recurring patterns identified in hyperbola tasks, students frequently struggled to coordinate the centre, vertex, and focus to determine curve orientation and key parameters a , b , and c . In ellipse tasks, barriers were commonly observed during general-to-standard equation transformations, specifically in grouping terms and completing the square. Procedural complexity, time constraints, and schema retrieval difficulties further interrupted problem-solving trajectories. These findings suggest that conic-section problem-solving difficulties reflect complex interactions between spatial-algebraic coordination and cognitive demands. Consequently, this study conceptualises the spatial-algebraic cognitive barrier as a preliminary analytical concept rather than an established psychological construct, offering pedagogical insights for analytic geometry instruction.

This is an open-access article under the [CC BY-SA](#) license.



Corresponding Author:

Aditya Prihandhika

Faculty of Teacher Training and Education, Universitas Singaperbangsa Karawang

Email: aditya.prihandhika@fkip.unsika.ac.id

1. INTRODUCTION

Conic sections, particularly ellipses and hyperbolas, represent a cognitively demanding domain in mathematics education because problem-solving requires coordinating geometric, visual, and algebraic representations rather than merely memorising formulas. In this topic, mathematical meaning is inherently distributed across multiple representations: learners must interpret key geometric attributes, such as the centre, vertices, foci, and axes, while simultaneously constructing or manipulating symbolic equations. Consequently, students often experience conceptual and procedural difficulties, such as struggling to interpret geometric properties and formulating equations of hyperbolas when focal or vertex information is provided [1]. Although multiple representations can support learning through complementary functions, their effectiveness depends heavily on learners' ability to coordinate and integrate information across these distinct representational systems.

In conic sections, this representational coordination manifests as a bidirectional demand between spatial reasoning and algebraic manipulation [2]. Solving hyperbola tasks requires translating spatial configurations into algebraic parameters (a , b , and c), whereas converting an ellipse equation into standard form demands sequential algebraic transformations (e.g., completing the square) to extract its underlying geometric properties. Recent research emphasises that spatial visualisation is an integral component of algebraic problem-solving in conic sections rather than a separate, auxiliary skill [3]. However, students persistently encounter obstacles at this interface, as evidenced by high rates of errors and misconceptions when determining conic equations from geometric parameters [4]. These failures demonstrate that difficulties in analytic geometry are not isolated calculation errors, but multifaceted obstacles spanning the conceptual, procedural, and technical dimensions of spatial-algebraic coordination [5].

To trace where these difficulties occur, Polya's four-stage heuristic framework, understanding the problem, devising a plan, carrying out the plan, and looking back, provides an established procedural model [6]. Empirical evidence shows that students' obstacle profiles vary systematically across each stage, ranging from reasoning deficiencies during problem comprehension and conceptual connection failures during planning, to procedural errors and weak evaluation in the final stage [7]. Nevertheless, Polya's framework is descriptive of process; it can map where errors occur, but cannot explain the cognitive demands that trigger these failures. For instance, two students who both stall at the execution stage may do so for fundamentally different reasons: an absence of algebraic schemas or working memory overload. Therefore, a cognitive perspective is necessary to unpack the underlying mechanisms behind these stage-specific failures.

Cognitive Load Theory (CLT) addresses these limitations by explaining how working memory constraints limit information processing during complex problem-solving [8]. In conic sections, intrinsic cognitive load is driven by high element interactivity, where students must simultaneously process and connect the centre, vertices, foci, orientation, and related variables [9]. When coordinating these interactive elements exhausts cognitive capacity, schema acquisition and automation are hindered [10]. The complexity of these tasks arises not merely from the quantity of elements, but from the requirement to integrate

geometric conceptual relationships with lengthy algebraic transformation procedures concurrently [11]. In the final stage of problem-solving, this heavy processing load also depletes cognitive resources allocated for verification and metacognitive monitoring [12], [13], explaining why students frequently fail to evaluate the validity of their solutions during the looking-backstage [14].

To date, the literature on conic sections [15], [16], Polya's stages [17], [18], CLT principles [19], [20], and spatial reasoning [21] has developed in silos. Integrating these perspectives does not merely combine frameworks; it provides theoretical visibility into phenomena that remain unexplained by individual theories: specifically, how friction at the spatial-algebraic interface amplifies element interactivity and triggers execution failures at specific Polya stages. To capture this specific phenomenon, this study proposes the analytical construct of the *spatial-algebraic cognitive barrier*. This construct is distinctly differentiated from a *misconception* (rooted in persistent incorrect concept structures) or a *schema retrieval failure* (a memory recall issue); it refers to a breakdown in the simultaneous coordination between visual-geometric interpretation and symbolic manipulation when the burden of element interactivity exceeds working memory capacity. Through the analysis of problem-solving artefacts and student metacognitive reflections on ellipses and hyperbolas, this study addresses the following research question: What are the characteristics of students' spatial-algebraic cognitive barriers when analysed through the integration of Polya's stages and Cognitive Load Theory?

2. METHOD

This study employed an exploratory qualitative design to investigate the characteristics of students' spatial-algebraic cognitive barriers when solving ellipse and hyperbola problems. A qualitative approach was chosen to interpret students' problem-solving processes and metacognitive reflections rather than to measure cognitive load quantitatively. The analysis was guided by the integration of Polya's problem-solving heuristics, understanding the problem, devising a plan, carrying out the plan, and looking back [22], with Cognitive Load Theory (CLT) serving as an interpretive lens for cognitive demands: intrinsic cognitive load (ICL), extraneous cognitive load (ECL), and germane cognitive load (GCL) [23]. Polya's framework served to trace *where* obstacles emerged, while CLT explained *why* specific stages imposed heavy demands on students' cognition [24]. Rather than functioning as a direct physiological or psychometric measurement tool, CLT was applied as a theoretical interpretive framework for analysing qualitative data [25].

Context, Participants, and Sampling

This study was conducted at Universitas Singaperbangsa Karawang and involved fourth-semester undergraduate students in the Mathematics Education Study Program who were enrolled in an Analytic Geometry course. Prior to this study, the participants had not received formal college-level instruction on conic sections. Sampling was conducted using a purposive sampling technique based on an error-focused criterion. Initially, 85 student answer sheets were collected from two intact classes: Class 4A and Class 4B. These 85 submissions were screened to identify students demonstrating conceptual, representational,

or procedural errors while solving ellipse and/or hyperbola tasks. Based on these inclusion criteria, 25 students were selected (11 from Class 4A and 14 from Class 4B). All selected participants completed every research instrument, including written problem-solving tasks and open-ended metacognitive reflection sheets, achieving a 100% data completion rate.

Mathematical Tasks and Data Sources

Data were collected through the triangulation of two sources: written problem-solving artefacts (observable mathematical traces) and metacognitive reflection sheets (students' self-reported perceptions of difficulty, strategies, and confidence) [26]. Specification of written problem-solving is shown in Table 1.

Table 1. Mathematical Tasks

Task	Mathematical Focus	Representational Demands & Typical Obstacles
Task 1: Hyperbola	Determining the standard equation given the centre, vertices, and foci.	Geometric/Spatial to Symbolic Algebraic Obstacle: Interpreting curve orientation and extracting parameters (a, b, c).
Task 2: Ellipse	Converting a general equation of an ellipse into standard form.	Symbolic Algebraic to Geometric Property Extraction Obstacle: Sequential symbolic manipulations, particularly completing the square.

Data Analysis Flow and Matrix

Data analysis was conducted through an integrated coding scheme that systematically linked empirical findings to Polya's stages and CLT dimensions [27], as shown in Table 2.

Table 2. Integrative Coding Matrix Based on Polya's Stages and Cognitive Load Theory

Polya Stage [26]	Observable Indicators	Difficulty	Initial Code Categories	Cognitive Load Interpretation (CLT)
1. Understanding the problem	Misinterpreting problem statements, given conditions, or spatial relationships between objects.		Curve orientation error, misplacement of center/foci.	ICL: Inherent complexity and high interactivity among geometric-spatial elements [27].
2. Devising a plan	Failing to retrieve formulas, strategies, or parameter relationship schemas (a, b, c).		Parameter-retrieval failure, formula gap.	ICL & GCL: Limited availability of schemas in long-term memory [28].
3. Carrying out the plan	Making symbolic manipulation errors or stalling mid-procedure during algebraic execution.		Failure in completing the square, substitution errors.	ECL: Processing load stemming from lengthy, concurrent procedural algebraic steps.
4. Looking back	Failing to verify solutions; demonstrating false confidence without evaluating mathematical validity.		Confidence without verification, ignoring inconsistencies.	GCL: Deficit in metacognitive resource allocation caused by cognitive overload [29].

The credibility of the findings was maintained through data source triangulation (cross-referencing self-reported reflections with observable error patterns on answer sheets) and by delineating empirical evidence from theoretical interpretations [28]. All participant identities were anonymised using pseudonyms and identification codes. The research procedure is illustrated in Figure 1:

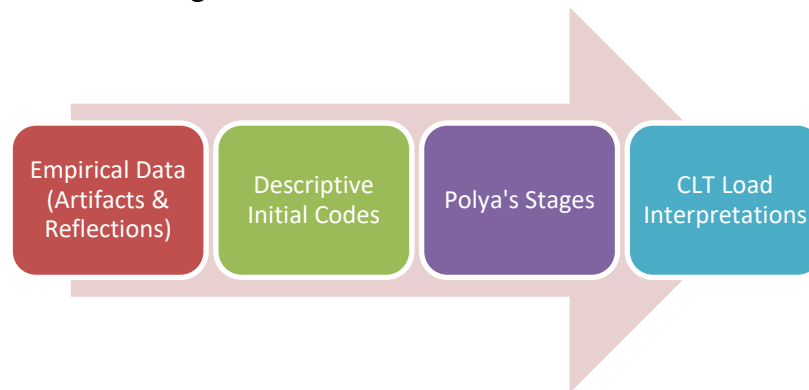


Figure 1. Data analysis workflow integrating empirical data, descriptive coding, Polya's problem-solving stages, and Cognitive Load Theory (CLT) interpretations

3. RESULTS AND DISCUSSION

3.1. Results

Analysis of the written metacognitive reflections and problem-solving artefacts from the 25 participants revealed four distinct characteristics of spatial-algebraic cognitive barriers. These barriers were not limited to simple computational errors; rather, they emerged as students coordinated geometric information, mathematical schemas, algebraic procedures, and solution evaluations [29]. Spatial interpretation and schema-retrieval difficulties were concentrated in *understanding the problem* (P1) and *devising a plan* (P2) stages of Polya's framework, whereas procedural interruptions occurred in the *carrying out the plan* stage (P3). Looking *backstage* (P4) revealed variations in students' capacity to evaluate solution completeness and validity [30]. These patterns are summarised in Table 3.

Table 3. Distribution of Participants Across Cognitive Barrier Characteristics, Polya's Stages, and CLT Lenses

Characteristics of barrier	Representative manifestation	Polya stage	CLT Lens	Frequency	Percentage
Spatial interpretation difficulty	Difficulty identifying hyperbola orientation and interpreting centre, vertex, and focus	P1	ICL	9	36%
Schema retrieval and parameter construction	Difficulty recalling formulas and relating a , b , and c to geometric information	P2	ICL	14	56%
Procedural-algebraic interruption	Difficulty grouping terms and completing the square in the ellipse equation	P3	ECL	16	64%
Solution evaluation variation	Recognition of incomplete solutions versus confidence despite uncertainty	P4	GCL-related interpretation	13	52%

Spatial Interpretation Difficulties in Task 1 (P1 – ICL)

A total of 9 of 25 participants (36%) exhibited barriers at the P1 stage during the hyperbola task. While participants recognised terms such as *centre*, *vertex*, and *focus*, they struggled to coordinate these spatial attributes when the centre was located away from the origin $(h,k) \neq (0,0)$. The student subsequently reported becoming confused during the algebraic conversion in Table 4.

Table 4. Representative Excerpts: Spatial Interpretation

Participant	Polya stage	Initial code	Reflection excerpt
4A-P02	P1	Center misinterpretation	I wasn't careful enough when reading it; it turned out the centre wasn't at (0,0).
4B-P03	P1	Orientation uncertainty	"I forgot if this is vertical or horizontal."
4B-P09	P1	Spatial-symbolic confusion	"I forgot (h,k) is for peak or centre."

A more complex manifestation of the spatial interpretation barrier was observed in Participant 4B-P09. The student attempted to determine the orientation of the hyperbola by considering both horizontal and vertical forms of its standard equation. However, uncertainty subsequently emerged in interpreting the roles of the centre (h,k) and the parameters a and b , resulting in an incomplete construction of the equation. The student also produced a coordinate sketch, but the spatial representation was not fully consistent with the algebraic form being considered. This response illustrates how difficulty in coordinating geometric information with symbolic representation can interrupt the problem-solving process [30]. Figure 2 presents the student's written response as a representative artefact of this spatial-algebraic barrier.

a) Tentukan persamaan standar hiperbola H

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-1)^2}{a^2} - \frac{(y-4)^2}{b^2} = 1 \quad \text{or} \quad (x-1)^2 - (y-4)^2 = 1$$

Figure 2. Representative written response illustrating a spatial-algebraic barrier in interpreting the relationship among the centre, vertex, focus, and hyperbola orientation (Participant 4B-P09)

Schema Retrieval and Parameter Construction in Task 1 (P2 – ICL & GCL)

Over half of the cohort (14 of 25 participants, 56%) encountered obstacles at the P2 stage. Although these students identified the general algebraic structure of a hyperbola, they could not retrieve the schemas required to convert geometric distances into parameters a , b , and c . This error subsequently affected the calculation of b and the final standard equation in Table 5.

Table 5. Schema Retrieval and Parameter Construction

Participant	Polya stage	Initial code	Reflection excerpt
4A-P05	P2	Parameter retrieval difficulty	"I've forgotten how to determine the values of a and b ."
4B-P10	P2	Formula retrieval difficulty	"I need some time to recall the method or formula I learned."
4A-P11	P2	Focus-parameter confusion	"I thought the focus itself was 9, without calculating the distance to the centre."

The case of 4A-P11 demonstrates an error in representational conversion, wherein the ordinate of the focus was directly equated to parameter c without accounting for the distance displacement from the centre (h, k) . In contrast, several students (e.g., 4B-P01) systematically activated the parameter schema, deriving a from the centre–vertex distance, c from the centre–focus distance, and subsequently computing b , which highlights that these schematic impediments are inherently tied to the stability of participants' long-term memory retrieval.

Procedural-Algebraic Interruptions in Task 2 (P3 – ECL)

In the ellipse task, 16 of 25 participants (64%) stalled during algebraic execution at the P3 stage. Although participants recognised completing the square as the necessary method, extraneous cognitive load (ECL) associated with multi-step symbolic manipulations disrupted execution, as shown in Table 6.

Table 6. Representative Reflection Excerpts: Procedural-Algebraic Interruption

Participant	Polya stage	Initial code	Reflection excerpt
4B-P01	P3	Completing-the-square barrier	"I struggled with completing the square and eventually stopped because time ran out."
4B-P02	P3	Algebraic grouping difficulty	"I was confused when grouping like variable terms together."
4A-P04	P3	Procedural interruption	"Problem 4 was left unfinished due to time constraints; I only reached the completing-the-square step."

The response of 4A-P04 further illustrates the role of procedural continuity. The participant reported that the ellipse problem was not completed because time had run out and only the square-completion stage had been reached. The data therefore indicates a distinction between strategy recognition and strategy execution. Students could know that completing the square was required but still fail to operate the procedure. This pattern was coded primarily as P3-ECL in the initial analysis.

A different form of cognitive barrier emerged in the ellipse problem, particularly during the execution of the algebraic transformation. Participant 4B-P01 recognised that completing the square was necessary to transform the general equation into standard form and attempted to apply the procedure. However, the student had trouble maintaining the sequence of algebraic operations and was unable to complete the transformation successfully. The written work therefore provides evidence of a procedural interruption

occurring after the appropriate strategy had already been identified. Figure 3 presents the student's written response as a representative artefact of this procedural-algebraic barrier.

a. Persamaan elips
 Untuk ke bentuk umum, gunakan kuadrat sempurna $3x^2 + 4y^2 - 30x - 8y - 5 = 0$
 $3x^2 + 4y^2 - 30x - 8y - 5 = 0$
 $3x^2 - 30x + 4y^2 - 8y = 5$
 $3(x^2 - 10x) + 4(y^2 - 2y) = 5$
 $3[(x - 5x) - 25] + 4[(y - 1) + 1] = 5$
 $3(x - 5x) + 4(y - 1) = 5$

Figure 3. Representative written response illustrating a procedural-algebraic barrier during the completion-of-the-square process for an ellipse equation (Participant 4B-P01)

As illustrated in Figure 3, the student initiated the expected algebraic transformation but was unable to complete the sequence required to obtain the standard form. The fourth characteristic concerned students' evaluation of their own solutions. The reflections revealed considerable variation in the extent to which students recognised errors, incomplete procedures, or unresolved uncertainty. Some participants demonstrated explicit awareness that their solutions were incomplete.

Variations in Solution Evaluation (P4 – GCL Deficit)

A total of 13 out of 25 students (52%) exhibited anomalies at the P4 stage. Metacognitive reflections revealed two contrasting evaluation patterns: explicit incompleteness awareness and unwarranted confidence (Table 5). Participant 4B-P09, for example, stated that the answer was not correct because the solution had not been completed. Similarly, participant 4B-P01 reported very low confidence because the standard form of the ellipse had not been obtained. Other participants were more confident in their solutions. Participant 4B-P01 in the hyperbola response described the sequence of calculating a , b and c , followed by substitution into the standard equation, and reported being "Sure, but a little doubtful". Another participant, 4B-P08, described drawing the positions of the centre, vertex, and focus before calculating a , b and c , and reported being confident in the procedure despite some difficulty determining b and the positive-negative signs in Table 7.

Table 7. Representative Reflection Excerpts: Solution Evaluation

Participant	Polya stage	Initial code	Reflection excerpt
4B-P01	P4	Partial confidence	"I am confident, yet slightly doubtful."
4B-P09	P4	Recognition of incomplete solution	"I don't feel I answered correctly because my answer wasn't fully complete."
4B-P08	P4	Confidence after procedural completion	"While writing the answer, I was quite confident in the steps I took."

These responses show that students' confidence was not uniform and did not necessarily depend only on whether the mathematical procedure was completed. Some students explicitly monitored the incompleteness of their work, whereas others expressed

confidence after constructing a coherent sequence of procedures. Comparison of the two mathematical tasks revealed a clear difference in the dominant form of students' difficulties. The hyperbola problem primarily exposed difficulties in coordinating geometric information with algebraic parameters, whereas the ellipse problem more frequently exposed difficulties in maintaining and executing a sequence of algebraic transformations.

3.2. Discussion

Spatial-Algebraic Barriers as a Coordination Problem

The findings indicate that students' difficulties in solving ellipse and hyperbola problems cannot be adequately characterised as either purely spatial or purely algebraic difficulties. Instead, the central issue appears to lie in the coordination between spatial meaning and algebraic representation. In the hyperbola task, students were required to interpret the relative positions of the centre, vertex, and focus before constructing the parameters a , b , and c and subsequently forming the standard equation. In the ellipse task, students were required to transform a general algebraic expression into standard form while maintaining the mathematical meaning of the transformation. Thus, successful problem-solving required students to move continuously between representations rather than operate within a single representational system.

This finding extends previous research on students' difficulties with conic sections. Sudihartini and Purniati reported misconceptions among university students involving the determination of conic equations, the interpretation of hyperbola properties, and relationships involving the focus and other geometric elements [30]. The present findings provide a more process-oriented account of such difficulties by showing where and how the representational coordination breaks down. For example, participant 4A-P11 did not simply make an incorrect calculation of c ; the student treated the focus coordinate itself as c , indicating that a geometric distance relationship had not been successfully transformed into its corresponding algebraic parameter. From this perspective, the term spatial-algebraic cognitive barrier captures a process that is broader than a conventional misconception. A misconception describes an incorrect conception of a mathematical relationship, whereas the present findings suggest that some students possessed parts of the relevant knowledge but failed to coordinate those elements during problem solving. Participant 4A-P05, for instance, recognised that the problem concerned the standard equation of a hyperbola but was uncertain about how to determine a and b . The difficulty therefore emerged at the interface between recognising the mathematical object and operationalising its relationships.

Polya Reveals Where the Barrier Emerges

The findings also demonstrate the usefulness of Polya's heuristic framework for locating the emergence of these barriers. The spatial interpretation difficulties were concentrated in the understand the problem stage, while schema retrieval and parameter construction appeared primarily during devise a plan. Procedural interruptions occurred predominantly during carry out the plan, and differences in students' awareness of correctness appeared during look back. This pattern is consistent with broader research showing that students' difficulties in mathematical problem solving can occur at different

stages, with conceptual misunderstandings and weak connections appearing during understanding and planning, procedural errors during execution, and inadequate evaluation during the final stage [31].

However, the present findings suggest that Polya's stages should not be interpreted as isolated or strictly sequential categories. A difficulty that begins during understanding may propagate into later stages. For example, when participant 4B-P09 became uncertain about whether (h, k) represented the centre or another geometric element, the uncertainty subsequently affected the selection of the hyperbola equation and the determination of a and b . Thus, the stages provide a useful diagnostic trajectory, but the cognitive barrier itself can extend across multiple stages. This interpretation is consistent with contemporary views of problem solving in mathematics, in which heuristics provide ways of organising and monitoring mathematical activity rather than merely prescribing a fixed sequence of procedures [32]. Recent systematic research also indicates that Polya-based analysis can distinguish difficulties in understanding, planning, execution, and evaluation, but that these difficulties are associated with different cognitive and conceptual factors [33]. The contribution of the present study is therefore not simply to confirm that students make errors at different Polya stages, but to connect those stages specifically to the spatial-algebraic coordination required by conic-section problems.

Cognitive Load Is Sensitive to the Nature of the Mathematical Demand

The findings can also be interpreted through Cognitive Load Theory. The hyperbola task required students to simultaneously coordinate several interacting elements: the centre, vertex, focus, orientation, distances, parameters, and equation structure. According to CLT, intrinsic cognitive load is strongly associated with the number and interactivity of elements that must be processed in relation to one another [34]. Consequently, a student who has not sufficiently automated the relationships among these elements may experience substantial cognitive demands even when each individual component appears mathematically familiar.

The responses of participants 4A-P02 and 4B-P09 illustrate this condition. Participant 4A-P02 knew the standard hyperbola formula but failed to attend to the non-origin centre, while participant 4B-P09 simultaneously struggled with orientation, the meaning of (h, k) , and the roles of a and b . The problem, therefore, was not necessarily the absence of individual pieces of knowledge. Rather, the students had difficulty coordinating multiple interacting elements within a single solution process. This provides a plausible CLT-based interpretation of the spatial and schema-related barriers. The ellipse problem presents a different cognitive configuration. Students often recognised the intended strategy, such as completing the square, but became unable to sustain the sequence of algebraic transformations. Participant 4B-P01 explicitly understood that completing the square was required but reported becoming stuck because the expected standard form did not emerge despite repeated calculations. This pattern suggests that the cognitive demand was associated less with identifying the strategy and more with maintaining and coordinating intermediate algebraic states during execution.

Here, however, the term extraneous cognitive load should be interpreted cautiously. The present study did not measure cognitive load directly, and procedural difficulty cannot

automatically be equated with ECL. Rather, the findings are consistent with an interpretation in which cumbersome or insufficiently automated symbolic procedures consume working-memory resources that could otherwise be used for mathematical reasoning. This distinction is important because CLT emphasises that intrinsic and extraneous demands are theoretically distinct, and element interactivity can play a role in both [35].

From Strategy Recognition to Strategy Execution

One of the more important findings is the distinction between knowing what to do and being able to carry out what is known. Several students could identify the appropriate mathematical strategy but could not execute it successfully. This was particularly evident in the ellipse problem, where students frequently mentioned completing the square as the intended method but stopped during the algebraic transformation. This distinction is theoretically important because a student may possess a relevant schema without having sufficiently automated the procedures required to operationalise it. Participant 4B-P02, for example, reported forgetting the steps and becoming confused when grouping terms, while participant 4B-P01 understood the intended transformation but could not obtain the expected standard equation. Thus, the barrier is better understood as a schema-to-procedure transition problem rather than simply a lack of conceptual knowledge. This distinction also helps explain why students who appear to have similar knowledge could perform differently. Participant 4B-P01 in the hyperbola task was able to construct a coherent sequence—determining a , b , and c followed by substitution into the standard equation, whereas other participants struggled to establish these relationships. The difference suggests that prior knowledge becomes effective only when its constituent relationships are sufficiently connected and accessible during problem solving.

The Role of Metacognitive Evaluation

The findings concerning the look backstage provide another important insight. Students differed substantially in their ability to evaluate the quality of their own solutions. Some students explicitly recognised that their solutions were incomplete or incorrect, whereas others expressed confidence despite unresolved uncertainties. This variation is important because successful problem solving does not end when a mathematical procedure produces an expression. The solver must also determine whether the resulting representation is mathematically coherent. Participant 4B-P09 recognised that the solution was incomplete, while participant 4B-P08 expressed confidence after constructing a coherent sequence of geometric and algebraic procedures. Such differences suggest that metacognitive evaluation functions as an important mechanism for detecting representational inconsistencies. This finding is consistent with research identifying inadequate evaluation, errors in interpreting results, and lack of thoroughness as persistent difficulties during the looking back phase of mathematical problem solving [36]. At the same time, the present study adds a representational dimension to this issue. Students were not merely checking whether a numerical answer was correct; they needed to evaluate whether the geometric interpretation and algebraic form remained consistent with one another. Consequently, looking back on the conic-section problem, solving should involve more than rechecking arithmetic. It should

include questions such as: Does the orientation represented by the equation agree with the spatial configuration? Does the value of a correspond to the centre-to-vertex distance? Does c represent the centre-to-focus distance? Does the final standard equation preserve the geometric properties given in the problem? Such questions can potentially transform metacognitive checking from a superficial verification step into a mechanism for reconstructing mathematical meaning.

Why Hyperbola and Ellipse Problems Produce Different Barriers

An important contribution of the study is the observation that the two conic-section tasks exposed different dominant cognitive vulnerabilities. The hyperbola task predominantly required students to coordinate geometric information with algebraic parameters, whereas the ellipse task required them to maintain a sequence of symbolic transformations. This difference can be explained by the direction of representational transformation required by each task. In the hyperbola problem, students moved from spatial/geometric information to parameters and an algebraic equation. In the ellipse problem, students moved primarily from general algebraic representation to symbolic transformation and standard representation.

The distinction is pedagogically important because it suggests that treating “conic sections” as a single content category may conceal substantially different cognitive demands. Previous research has already documented misconceptions across several conic-section concepts, including difficulties involving hyperbola equations and focus-related relationships [37]. The present findings extend that observation by suggesting that the direction of representation transformation may influence the type of barrier that becomes visible. Thus, instructional support should not be designed solely around memorising the standard equations of ellipses and hyperbolas. Students need opportunities to coordinate geometric configurations, parameters, symbolic equations, and transformations. Instruction should make explicit the relationships between the centre, vertex, focus a , b , and c , rather than presenting these elements as disconnected formula components.

Theoretical Contribution: Toward a Spatial-Algebraic Cognitive Barrier

The central theoretical implication of the findings is the possibility of conceptualising students’ difficulties as spatial-algebraic cognitive barriers. This concept refers to difficulties that emerge when students must coordinate spatial/geometric relationships with symbolic-algebraic representations during mathematical problem solving. The concept is narrower than general “cognitive difficulty” because it specifically concerns coordination across representational systems. It is also broader than a conventional algebraic error because the source of the difficulty may occur before symbolic manipulation begins. In the hyperbola task, for example, an incorrect value of c may originate from a failure to interpret the focus as a point whose distance from the centre must be calculated. In the ellipse task, by contrast, the barrier may emerge after the correct strategy has already been identified, when the student must maintain a sequence of symbolic transformations. The combination of Polya and CLT provides useful analytical architecture for this concept. Polya identifies the process location of the barrier, while CLT helps interpret the cognitive demands associated with

coordinating interacting elements. The resulting framework can therefore be represented as a mathematical representation, problem-solving process, and cognitive demand. This is potentially the strongest theoretical contribution of the study. Rather than treating spatial reasoning, algebraic manipulation, and problem-solving stages as separate dimensions, the findings suggest that students' difficulties may be better understood through their interaction.

Pedagogical Implications

The findings have several implications for mathematics instruction. First, instruction on conic sections should explicitly support transitions between geometric and algebraic representations. For hyperbola problems, students should be encouraged to construct or interpret a spatial representation before selecting the standard equation. For example, determining the orientation from the relative position of the centre, vertex, and focus should precede formula selection. Second, procedural transformations such as completing the square should be taught not merely as symbolic algorithms but as transformations whose mathematical purpose is made explicit. Students who know that completing the square is required but cannot execute it may benefit from instruction that decomposes the procedure into meaningful intermediate states while gradually increasing procedural complexity. Such an approach is consistent with the broader CLT principle that instructional design should manage unnecessary processing demands so that working-memory resources can be directed toward essential mathematical relationships [38].

Third, the *look-back* stage should be strengthened as a representational verification process. Students should be encouraged to compare the final equation with the original geometric information rather than merely checking whether calculations appear complete. This aligns with evidence that inadequate evaluation remains a persistent challenge in mathematical problem solving [39]. Finally, the findings suggest potential value in visual and dynamic representations. However, technology should not simply be added to reduce procedural effort. Its role should be to help students connect spatial relationships with algebraic parameters and thereby support schema construction. The use of technology in mathematical problem solving is most productive when it supports the construction of mathematical reasoning rather than merely automating calculations.

Limitations and Directions for Further Research

Several limitations should be considered when interpreting these findings. First, the study involved 25 students from two class groups and therefore should be understood as a preliminary qualitative investigation, rather than a basis for generalising the prevalence of cognitive barriers. Second, the study relied primarily on written metacognitive reflections and written problem-solving artefacts. These sources provide valuable evidence concerning students' reported experiences and observable procedures, but they do not provide direct access to moment-to-moment cognitive processing. Third, CLT was used as an interpretive framework rather than as a direct measurement system. Consequently, the classification of difficulties as ICL, ECL, or GCL-related should be understood as theoretically informed interpretations rather than measurements of actual cognitive load. Future research could

strengthen this line of inquiry by combining qualitative analysis with validated cognitive-load measures, process tracing, think-aloud protocols, eye-tracking, or experimental comparisons of different representational supports.

Future studies could also investigate whether the identified spatial-algebraic barriers can be reduced through targeted interventions that integrate dynamic visualisation, explicit schema development, worked examples, and metacognitive prompts. Such studies would allow the preliminary categories identified here to be tested beyond the descriptive context of the present dataset. Overall, the findings suggest that students' difficulties with ellipse and hyperbola problems are best understood not as isolated failures of formula recall or algebraic computation, but as breakdowns in the coordination of spatial meaning, mathematical schemas, symbolic procedures, and metacognitive evaluation. Polya helps reveal the trajectory of these breakdowns across the problem-solving process, while CLT provides a complementary explanation of why coordinating multiple interacting elements may become cognitively demanding. The most consequential insight is therefore that spatial and algebraic reasoning should not be treated as separate instructional targets in conic-section learning. Their coordination constitutes a central part of successful problem solving. The concept of a spatial-algebraic cognitive barrier provides a preliminary framework for describing this interface and offers a basis for designing subsequent research and instructional interventions.

4. CONCLUSION

This study investigated the characteristics of students' spatial-algebraic cognitive barriers when solving ellipse and hyperbola problems through the integration of Polya's problem-solving heuristics and Cognitive Load Theory. The findings indicate that students' difficulties were not limited to isolated conceptual or computational errors but emerged from the coordination of spatial interpretation, mathematical schema activation, algebraic procedures, and solution evaluation.

Four interconnected characteristics of spatial-algebraic cognitive barriers were identified. The first was spatial interpretation difficulty, particularly in determining the orientation of a hyperbola and interpreting the relationships among the centre, vertex, and focus. The second was schema retrieval and parameter-construction difficulty, in which students recognised the relevant mathematical topic or equation but struggled to translate geometric information into parameters a , b , and c . The third was procedural-algebraic interruption, particularly in transforming the general equation of an ellipse into standard form through completing the square. The fourth was variation in solution evaluation, reflected in differences between students who recognised that their solutions were incomplete and those who expressed confidence despite remaining uncertainty.

The integration of Polya's heuristics and Cognitive Load Theory provided complementary insights into these barriers. Polya helped trace the barriers across the problem-solving process, while CLT provided a theoretical basis for interpreting the cognitive demands associated with coordinating interacting mathematical elements. Importantly, the CLT categories in this study represent qualitative interpretations rather than direct measurements of students' cognitive load. The findings therefore suggest that the

central difficulty in conic-section problem solving lies not exclusively in spatial reasoning or algebraic manipulation, but at the interface between spatial meaning and algebraic representation.

Theoretically, the study proposes a spatial-algebraic cognitive barrier as a useful analytical concept for describing difficulties that arise when students coordinate geometric-spatial information with symbolic-algebraic representations during problem solving. This concept provides a more integrated perspective than treating spatial reasoning, algebraic manipulation, and problem-solving stages as independent dimensions. It also suggests that the direction of representational transformation matters: hyperbola problems primarily exposed difficulties in moving from geometric information toward algebraic parameters, whereas ellipse problems more strongly exposed difficulties in maintaining sequential algebraic transformations. Pedagogically, the findings suggest that instruction on conic sections should move beyond formula memorisation and explicitly support the coordination of geometric representations, algebraic parameters, symbolic procedures, and metacognitive evaluation. Instructional scaffolding should help students establish the relationships among the centre, vertex, focus, and parameters in hyperbola problems, while providing structured support for algebraic transformations such as completing the square in ellipse problems. Future research should examine whether targeted representational and metacognitive scaffolds can reduce these barriers and should combine qualitative evidence with direct or validated measures of cognitive load to examine further the cognitive mechanisms underlying students' spatial-algebraic difficulties.

ACKNOWLEDGEMENTS

The authors would like to express their sincere gratitude to the students who participated in this study and contributed their written reflections and problem-solving artefacts. The authors also gratefully acknowledge the support of Universitas Singaperbangsa Karawang, particularly the Institute for Research and Community Service (LPPM), which supported and funded the implementation of this research.

REFERENCES

- [1] E. Sudihartinih and T. Purniati, "Students' mistakes and misconceptions on the subject of conics," *International Journal of Education*, vol. 12, no. 2, pp. 92–100, 2020, doi: 10.17509/ije.v12i2.19130.
- [2] M. Dintarini, M. T. Budiarto, and Y. Fuad, "Spatial thinking in conic sections: A study of undergraduate mathematics students by sex and spatial category," *Jurnal Pendidikan Matematika*, vol. 19, no. 2, pp. 275–298, 2025.
- [3] A. I. Sugandi, D. Sofyan, L. Linda, and D. R. Sari, "Identifikasi kesalahan mahasiswa dalam menyelesaikan soal-soal pada mata kuliah geometri analitik," *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, vol. 5, no. 4, 2022, doi: 10.22460/jpmi.v5i4.11151.
- [4] S. Ainsworth, "DeFT: A conceptual framework for considering learning with multiple representations," *Learning and Instruction*, vol. 16, no. 3, pp. 183–198, 2006, doi: 10.1016/j.learninstruc.2006.03.001.
- [5] R. Duval, "A cognitive analysis of problems of comprehension in the learning of mathematics," *Educational Studies in Mathematics*, vol. 61, pp. 103–131, 2006, doi: 10.1007/s10649-006-0400-z.
- [6] A. Arcavi, "The role of visual representations in the learning of mathematics," *Educational Studies in Mathematics*, vol. 52, no. 3, pp. 215–241, 2003, doi: 10.1023/A:1024312321077.
- [7] M. Hegarty and M. Kozhevnikov, "Types of visual-spatial representations and mathematical problem solving," *Journal of Educational Psychology*, vol. 91, no. 4, pp. 684–689, 1999, doi: 10.1037/0022-0663.91.4.684.
- [8] M. T. Battista, "The development of geometric and spatial thinking," in F. K. Lester Jr., Ed., *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: Information Age

- Publishing, 2007, pp. 843–908.
- [9] D. H. Clements and M. T. Battista, “Geometry and spatial reasoning,” in D. A. Grouws, Ed., *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan, 1992, pp. 420–464.
 - [10] A. J. Bishop, “Review of research on visualisation in mathematics education,” *Focus on Learning Problems in Mathematics*, vol. 11, no. 1–2, pp. 7–16, 1989.
 - [11] G. A. Goldin, “Representation in mathematical learning and problem solving,” in L. D. English, Ed., *Handbook of International Research in Mathematics Education*. Mahwah, NJ: Lawrence Erlbaum Associates, 2002, pp. 197–218.
 - [12] C. Janvier, Ed., *Problems of Representation in the Teaching and Learning of Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1987.
 - [13] J. J. Kaput, “Representation systems and mathematics,” in C. Janvier, Ed., *Problems of Representation in the Teaching and Learning of Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1987, pp. 19–26.
 - [14] R. Duval, *Sémiosis et pensée humaine: Registres sémiotiques et apprentissages intellectuels*. Bern, Switzerland: Peter Lang, 1995.
 - [15] G. Pólya, *How to Solve It: A New Aspect of Mathematical Method*, 2nd ed. Princeton, NJ: Princeton University Press, 1957.
 - [16] A. A. Wahab, Y. S. Kusuma, D. Juandi, T. Turmudi, B. Buhaerah, and S. Syaiful, “Understanding students’ struggles in solving mathematical problems: A systematic literature review using Polya’s framework,” *Jurnal Pendidikan Progresif*, vol. 14, no. 3, 2024, doi: 10.23960/jpp.v14.i3.2024118.
 - [17] A. H. Schoenfeld, *Mathematical Problem Solving*. Orlando, FL: Academic Press, 1985.
 - [18] A. H. Schoenfeld, “What’s all the fuss about metacognition?” in *Cognitive Science and Mathematics Education*, A. H. Schoenfeld, Ed. Hillsdale, NJ: Lawrence Erlbaum Associates, 1987, pp. 189–215.
 - [19] A. H. Schoenfeld, “Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics,” in D. A. Grouws, Ed., *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan, 1992, pp. 334–370.
 - [20] J. Mason, L. Burton, and K. Stacey, *Thinking Mathematically*, 2nd ed. Harlow, UK: Pearson, 2010.
 - [21] National Research Council, *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001, doi: 10.17226/9822.
 - [22] L. R. Izzati and A. Mahmudi, “The influence of metacognition in mathematical problem solving,” *Journal of Physics: Conference Series*, vol. 1097, no. 1, Art. no. 012107, 2018, doi: 10.1088/1742-6596/1097/1/012107.
 - [23] J. Sweller, “Cognitive load theory, learning difficulty, and instructional design,” *Learning and Instruction*, vol. 4, no. 4, pp. 295–312, 1994, doi: 10.1016/0959-4752(94)90003-5.
 - [24] J. Sweller, J. J. G. van Merriënboer, and F. G. W. C. Paas, “Cognitive architecture and instructional design,” *Educational Psychology Review*, vol. 10, no. 3, pp. 251–296, 1998, doi: 10.1023/A:1022193728205.
 - [25] J. Sweller, J. J. G. van Merriënboer, and F. Paas, “Cognitive architecture and instructional design: 20 years later,” *Educational Psychology Review*, vol. 31, pp. 261–292, 2019, doi: 10.1007/s10648-019-09465-5.
 - [26] J. Sweller, “Element interactivity and intrinsic, extraneous, and germane cognitive load,” *Educational Psychology Review*, vol. 22, pp. 123–138, 2010, doi: 10.1007/s10648-010-9128-5.
 - [27] S. Kalyuga, “Cognitive load theory: How many types of load does it really need?” *Educational Psychology Review*, vol. 23, no. 1, pp. 1–19, 2011, doi: 10.1007/s10648-010-9150-7.
 - [28] F. Paas, J. E. Tuovinen, H. Tabbers, and P. W. M. van Gerven, “Cognitive load measurement as a means to advance cognitive load theory,” *Educational Psychologist*, vol. 38, no. 1, pp. 63–71, 2003, doi: 10.1207/S15326985EP3801_8.
 - [29] P. Chandler and J. Sweller, “Cognitive load theory and the format of instruction,” *Cognition and Instruction*, vol. 8, no. 4, pp. 293–332, 1991, doi: 10.1207/S1532690XCI0804_2.
 - [30] J. Sweller and P. Chandler, “Evidence for cognitive load theory,” *Cognition and Instruction*, vol. 8, no. 4, pp. 351–362, 1991, doi: 10.1207/S1532690XCI0804_5.
 - [31] F. Paas, A. Renkl, and J. Sweller, “Cognitive load theory and instructional design: Recent developments,” *Educational Psychologist*, vol. 38, no. 1, pp. 1–4, 2003, doi: 10.1207/S15326985EP3801_1.
 - [32] S. Kalyuga, P. Ayres, P. Chandler, and J. Sweller, “The expertise reversal effect,” *Educational Psychologist*, vol. 38, no. 1, pp. 23–31, 2003, doi: 10.1207/S15326985EP3801_4.
 - [33] S. Kalyuga, P. Chandler, J. Tuovinen, and J. Sweller, “When problem solving is superior to studying worked examples,” *Journal of Educational Psychology*, vol. 93, no. 3, pp. 579–588, 2001, doi: 10.1037/0022-0663.93.3.579.
 - [34] J. Sweller and G. A. Cooper, “The use of worked examples as a substitute for problem solving in learning

- algebra,” *Cognition and Instruction*, vol. 2, no. 1, pp. 59–89, 1985, doi: 10.1207/S1532690XCI0201_3.
- [35] J. Hiebert and P. Lefevre, “Conceptual and procedural knowledge in mathematics: An introductory analysis,” in J. Hiebert, Ed., *Conceptual and Procedural Knowledge: The Case of Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1986, pp. 1–27.
- [36] T. R. Azzahra and S. Mariani, “Mathematical problem solving skills reviewed from students’ metacognition performance in online-based PME learning model,” *Unnes Journal of Mathematics Education*, vol. 11, no. 1, pp. 48–57, 2022, doi: 10.15294/ujme.v11i1.54664.
- [37] J. R. Star, “Reconceptualising procedural knowledge,” *Journal for Research in Mathematics Education*, vol. 36, no. 5, pp. 404–411, 2005, doi: 10.2307/30034943.
- [38] J. Sweller, R. Low, and J. Sweller, “Some cognitive factors relevant to mathematics instruction,” *Mathematics Education Research Journal*, vol. 4, pp. 83–94, 1992, doi: 10.1007/BF03217233.
- [39] D. A. Sholihah, “Strategi pembelajaran matematika berdasarkan Cognitive Load Theory untuk meminimalkan extraneous cognitive load,” *EQUALS: Jurnal Ilmiah Pendidikan Matematika*, vol. 5, no. 1, pp. 13–23, 2022, doi: 10.46918/equals.v5i1.1197.
-