

Development of APOS Model Student Worksheet Assisted by GeoGebra on Line and Angle Material Representation in Mathematics in Junior High School

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ABSTRACT

Preliminary observations and interviews with teachers identified low levels of students' mathematical representation skills in junior high school mathematics learning, particularly in representing mathematical concepts in visual, symbolic, and verbal forms. The limited use of technology-based teaching materials also highlights the need for learning resources that integrate an appropriate instructional model with interactive technology. This study aimed to develop an APOS (Action, Process, Object, and Schema) model Student Worksheet assisted by GeoGebra for the topic of Lines and Angles and to examine its validity, practicality, and effectiveness. This Research and Development study employed the ADDIE model, comprising Analysis, Design, Development, Implementation, and Evaluation stages. The research subjects were 30 seventh-grade students at SMP Negeri 28 Bengkulu Selatan, with 10 selected through purposive sampling for a small-group practicality trial. Data were collected using expert validation sheets, student response questionnaires, and pretest and posttest instruments. The results showed an Aiken's V coefficient of 0.86, indicating a high level of validity; a practicality score of 83.7%, categorized as practical; and an N-Gain score of 0.60, indicating a moderate improvement in students' mathematical representation skills. The developed Student Worksheet integrates APOS-based concept construction with dynamic GeoGebra visualization, thereby providing a technology-based learning resource for the topic of Lines and Angles.

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1. INTRODUCTION

Mathematics education plays an important role in developing students' logical, critical, analytical, and creative thinking skills [1], [2]. In contemporary mathematics learning, students are expected not only to master mathematical procedures but also to understand and communicate mathematical ideas through appropriate representations [3].

Mathematical representation skills enable students to express, interpret, and communicate mathematical ideas through visual, symbolic, tabular, diagrammatic, and verbal forms. They are closely related to conceptual understanding, reasoning, problem-solving, and mathematical communication [4], [5]. However, students have not yet fully mastered these skills in mathematics learning. Students often struggle to translate ideas across representations, interpret diagrams, use mathematical symbols, and explain mathematical relationships [6], [7]. These difficulties are particularly relevant to geometry, where students must coordinate visual, symbolic, and verbal information to understand abstract relationships.

Lines and Angles is a fundamental geometry topic that requires students to understand relationships among points, lines, angles, parallel lines, and intersecting lines.[8], [9]. Students may have difficulty identifying geometric objects, determining relationships among lines, and representing angular relationships [10]. Preliminary observations and interviews with teachers in this study indicated that students had difficulty representing mathematical concepts visually, symbolically, and verbally. In addition, technology-based teaching materials have not yet been optimized to support students' understanding of abstract mathematical concepts. Student Worksheets can provide structured learning activities that guide students in constructing and applying mathematical concepts [11]. However, worksheets dominated by explanations and routine exercises may not sufficiently support progressive concept construction or connections among mathematical representations [12], [13]. Therefore, a learning framework is needed that systematically guides students' conceptual development.

The APOS (Action, Process, Object, and Schema) model provides a framework for the progressive construction of mathematical concepts [14]. At the Action stage, students perform mathematical procedures explicitly through internalization; these actions develop into a Process that can be mentally performed and understood [15]. At the Object stage, students view the process as a complete mathematical object, whereas at the Schema stage, students coordinate related actions, processes, and objects into a coherent cognitive structure.

These stages are relevant to mathematical representation because they enable students to construct, interpret, and connect representations progressively, rather than merely producing a representation as a final answer. To support this conceptual construction, GeoGebra provides a dynamic environment for constructing, visualizing, manipulating, and exploring mathematical objects [16], [17]. Its dynamic features can help students observe geometric relationships and connect visual and symbolic representations [18]. However, APOS and GeoGebra are not merely separate components of instruction. Rather, APOS supports progressive conceptual construction, while GeoGebra externalizes this construction through dynamic visual and symbolic representations [19]. Thus, APOS provides the cognitive structure for students' conceptual development, whereas GeoGebra provides a dynamic environment in which students can visualize and manipulate mathematical objects and relationships. Integrating both into a structured Student Worksheet can give students opportunities to perform actions, explore relationships, construct concepts, and connect different forms of mathematical representation.

Previous studies have examined APOS in mathematics concept learning, GeoGebra in geometry learning, and Student Worksheets developed using various instructional models. Although these studies demonstrate the potential of each component, relatively few studies have specifically integrated all four APOS stages (Action, Process, Object, and Schema) with GeoGebra in a structured Student Worksheet designed to develop students' mathematical representation skills on the topic of Lines and Angles at the junior high school level. Accordingly, the integration of APOS-based conceptual construction, dynamic GeoGebra visualization, and mathematical representation in a structured worksheet remains underexplored. This study is novel in developing an APOS-based Student Worksheet assisted by GeoGebra that integrates the four APOS stages with dynamic GeoGebra activities to support students' mathematical representation skills in the topic of Lines and Angles. Based on this research gap and novelty, this study aimed to develop an APOS-based Student Worksheet assisted by GeoGebra for teaching Lines and Angles and to evaluate its validity, practicality, and effectiveness in supporting the mathematical representation skills of seventh-grade students.

2. METHOD

This study was a Research and Development study aimed at producing an APOS model Student Worksheet assisted by GeoGebra for the topic of Lines and Angles for Grade VII junior high school. The study was conducted at SMP Negeri 28 Bengkulu Selatan. The subjects consisted of 30 seventh-grade students from one class who participated in the implementation of the developed Student Worksheet. All 30 students were involved in the effectiveness analysis through pretest and posttest assessments. For the practicality trial, 10 students were selected from the 30 participants using purposive sampling. Thus, the 10 students represented the small-group practicality trial, while all 30 students participated in the effectiveness analysis. This study used the ADDIE model, which consists of five stages: Analysis, Design, Development, Implementation, and Evaluation. The ADDIE model was selected because it provides systematic development steps and is appropriate for developing technology-based instructional materials. According to Branch (2019), the ADDIE model supports the structured development of instructional products so the resulting products can meet learning needs [20].

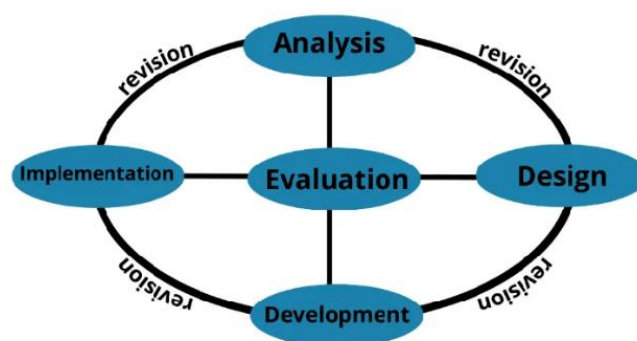


Figure 1. ADDIE Model Diagram

The Analysis stage focused on students' needs and the existing learning situation, as well as identifying the product that needed to be developed. This study included an analysis of students' needs, curriculum analysis, and analysis of student characteristics to identify the need for learning media aligned with the curriculum and students' needs [21]. Next, in the Design stage, the researchers prepared the product design by developing an APOS model Student Worksheet assisted by GeoGebra to improve students' mathematical representation skills in the topic of Lines and Angles. The researchers also designed instruments, including pretest and posttest instruments.

Subsequently, the Development stage involved transferring the prepared materials from Microsoft Word into a template previously designed using Canva. The Student Worksheet consisted of three parts: introduction, main content, and closing. The product developed at this stage was then evaluated through validation by a construct expert, a subject-matter expert, a language expert, and a media expert. Next, the Implementation stage involved applying the developed product in the classroom learning process. First, students were given a pretest before using the product. Students then had the opportunity to use the Student Worksheet during the learning process. Subsequently, students provided feedback on using the worksheet. Finally, students were given a posttest [22]. The final stage was Evaluation. Each preceding stage of the ADDIE model was evaluated to obtain more accurate data and satisfactory results [23].

In this study, the Student Worksheet expert validation questionnaire used five response options adjusted to the content of each statement. Each response option was assigned a different score. Table 1 presents the scoring for each response option [24].

Table 1. Content Validation Scoring Criteria

Criterion	Score
Very Good	5
Good	4
Fairly Good	3
Poor	2
Very Poor	1

Overall, the assessment scores provided by the expert validators, namely the construct, subject-matter, language, and media experts, were then converted into validation criteria, as shown in Table 2.

Table 2. Expert Validation Criteria

Percentage	Criterion
$80\% < V \leq 100\%$	Very Valid
$60\% < V \leq 80\%$	Valid
$40\% < V \leq 60\%$	Fairly Valid
$20\% < V \leq 40\%$	Less Valid
$0\% < V \leq 20\%$	Not Valid

In this development study, Aiken's V content validity test was used to measure the feasibility of the product being developed. Aiken's V assesses content validity with a minimum of three validators [25]. The formula used was in equation (1) [26].

$$V = \frac{\sum S}{[n(c-1)]} \text{ with } S = R - L_o \quad (1)$$

where:

- V = Aiken's validity index
- S = Score assigned by each rater minus the lowest Score in the category
- R = Score assigned by the rater
- L_o = Lowest rating score
- c = Highest rating score
- n = Number of validators (raters)

The scoring criteria were as follows:

Table 3. Content Validity Criteria

Percentage	Criterion
$0,80 < V \leq 1,00$	Very High Validity
$0,60 < V \leq 0,80$	High Validity
$0,40 < V \leq 0,60$	Moderate Validity
$0,20 < V \leq 0,40$	Low Validity
$0,00 < V \leq 0,20$	Very Low Validity

Next, the practicality test was administered to students, who were required to answer or respond to statements concerning the use of the developed product. Each response option was adjusted to the content of the statement [27]. Each response was assigned a different score. The total Score obtained from each student was then converted into an assessment percentage to determine the level of practicality. Table 4 presents the conversion of scores into practicality criteria.

Table 4. Practicality Test Criteria

Nilai	Criteria
$RT \geq 85\%$	Very Practical
$70\% \leq RT < 85\%$	Practical
$50\% \leq RT < 70\%$	Not Very Practical
$RT < 50\%$	Not Practical

Finally, the effectiveness test was conducted. In this study, the effectiveness of the developed product was measured using the Normalized Gain formula introduced by Hake. The N-Gain value was calculated based on the average gain (g) obtained from the pretest and posttest scores. The interpretation categories for the Normalized Gain Index (g) are presented in Table 5 [28]

Table 5. Interpretation of the Normalized Gain Index

N-Gain	Category	Treatment Criterion Using the Student Worksheet
$N - \text{Gain} > 0,7$	High	Effective
$0,3 \leq N - \text{Gain} \leq 0,7$	Moderate	Fairly Effective
$N - \text{Gain} < 0,3$	Low	Not Effective

3. RESULTS AND DISCUSSION

3.1. Results

This Research and Development study produced an APOS model Student Worksheet, assisted by GeoGebra, to improve students' mathematical representation skills in the topic of Lines and Angles. The first stage was the Analysis stage. Before developing the product, a needs analysis, curriculum analysis, and analysis of student characteristics were conducted through interviews with mathematics teachers and questionnaires administered to students at SMP Negeri 28 Bengkulu Selatan. The results showed that seventh-grade students had varying levels of mathematical ability. Students had difficulty understanding the learning material, and the learning process remained teacher-dominated. Students also tended to depend on teacher explanations, showed limited initiative in asking questions, and were not sufficiently independent in completing learning tasks. Further analysis of the available learning materials showed that the existing materials had not optimally integrated mathematical concepts with technology. These findings indicated the need for learning materials that could facilitate students' independent learning and support the construction and representation of mathematical concepts. Therefore, an APOS-based Student Worksheet assisted by GeoGebra was developed for the topic of Lines and Angles.

The Design stage involved preparing the framework of the APOS model Student Worksheet assisted by GeoGebra in accordance with the Basic Competencies (KD) and learning objectives based on the applicable curriculum. The Student Worksheet structure included an opening section consisting of the cover, title, group identity, learning outcomes, learning objectives, instructions, and time allocation, followed by content presenting Lines and Angles through Action, Process, Object, and Schema (APOS) activities assisted by GeoGebra to encourage students to construct concepts independently and improve their mathematical representation skills. Evaluation at this stage was conducted through guidance from the academic supervisors, who provided feedback concerning improvements to the visual design and the accuracy of the material as a basis for refining the product before entering the development stage.



Figure 2. Appearance of the Learning Media Used by the Teacher



Figure 3. Appearance of the Student Worksheet Design Developed by the Researchers

In the Development stage, the Student Worksheet was developed based on the four stages of the APOS model: Action, Process, Object, and Schema. The Student Worksheet consisted of an introductory section, main learning activities, and a closing section. The introductory section included the cover, learning objectives, learning outcomes, instructions, and time allocation. The main section contained learning activities on Lines and Angles organized according to the APOS stages. GeoGebra was integrated into the learning activities to support students in dynamically exploring and visualizing mathematical concepts. The developed product was then submitted to four experts for validation, covering construction, subject matter, language, and media aspects. The assessment results from all validators are presented below.

Table 5. Results of Validation by the Four Experts

Validator	Validation Score	Category
Construct Expert	0.90	Very Valid
Subject-Matter Expert	0.83	Very Valid
Language Expert	0.86	Very Valid
Media Expert	0.86	Very Valid
Average	0.86	Very Valid

At the Implementation stage, after the Student Worksheet was declared feasible by the validators, a product trial was conducted to determine the practicality of the developed teaching material. The practicality test involved 10 seventh-grade students at SMP Negeri 28 Bengkulu Selatan. Students were given a response questionnaire as an assessment instrument to obtain data regarding their responses to and evaluations of the APOS model Student Worksheet assisted by GeoGebra for the topic of Lines and Angles. The results of the practicality analysis are presented below.

Table 6. Results of the Student Worksheet Practicality Test

Respondent	Number of Respondents	Obtained Score	Maximum Score	Percentage (%)	Category
Students	10	83.7	100	83.7%	Practical

In addition to the practicality test, an effectiveness test was also conducted at the Implementation stage based on students' pretest and posttest results. The effectiveness test aimed to determine whether the APOS model Student Worksheet assisted by GeoGebra for the topic of Lines and Angles was suitable for continued application in the learning process or still required further refinement. If the effectiveness test results fell into the good category, the developed product could be used as an alternative solution to improve students' mathematical representation skills. The calculated pretest and posttest scores for seventh-grade students at SMP Negeri 28 Bengkulu Selatan are presented below.

Based on Table 7 below, the N-Gain Score was 0.60, categorized as "moderate," with a percentage of 60.3%, which falls into the "effective" category. The effectiveness test results indicate that the APOS model Student Worksheet assisted by GeoGebra for the topic of Lines and Angles was effective in improving students' mathematical representation skills and suitable for application in learning activities. During the Evaluation stage, continuous review and improvement were carried out at each development stage to minimize errors and

ensure that the quality of the APOS model Student Worksheet assisted by GeoGebra for improving students' mathematical representation skills in the topic of Lines and Angles was maintained throughout the development process.

Table 7. Results of the N-Gain Effectiveness Test

No	Student Code	N-Gain Effectiveness of the Contextual Learning Material				
		Score		Skor Ideal (100) - Pretest	N-Gain Score	N-Gain Score (%)
		Pretest	Posttest			
1.	PD-1	53	79	47	0,55	55,3%
2.	PD-2	56	80	44	0,55	54,5%
3.	PD-3	58	82	42	0,57	57,1%
4.	PD-4	60	84	40	0,60	60,0%
5.	PD-5	52	79	48	0,56	56,3%
6.	PD-6	52	80	48	0,58	58,3%
7.	PD-7	57	81	43	0,56	55,8%
8.	PD-8	54	83	46	0,63	63,0%
9.	PD-9	51	79	49	0,57	57,1%
10.	PD-10	59	87	41	0,68	68,3%
11.	PD-11	62	87	38	0,66	65,8%
12.	PD-12	61	88	39	0,69	69,2%
13.	PD-13	53	82	47	0,62	61,7%
14.	PD-14	50	79	50	0,58	58,0%
15.	PD-15	60	83	40	0,58	57,5%
16.	PD-16	58	86	42	0,67	66,7%
17.	PD-17	56	82	44	0,59	59,1%
18.	PD-18	57	83	43	0,60	60,5%
19.	PD-19	59	80	41	0,51	51,2%
20.	PD-20	63	85	37	0,59	59,5%
21.	PD-21	60	90	40	0,75	75,0%
22.	PD-22	64	88	36	0,67	66,7%
23.	PD-23	62	85	38	0,61	60,5%
24.	PD-24	51	80	49	0,59	59,2%
25.	PD-25	58	83	42	0,60	59,5%
26.	PD-26	53	80	47	0,57	57,4%
27.	PD-27	59	81	41	0,54	53,7%
28.	PD-28	62	82	38	0,53	52,6%
29.	PD-29	60	85	40	0,63	62,5%
30.	PD-30	59	86	41	0,66	65,9%
		Average Criterion			0,60	60,3%
					Moderate	Effective

Overall, the developed APOS-based Student Worksheet assisted by GeoGebra met the three predetermined product-quality criteria. The Student Worksheet obtained an average Aiken's V value of 0.86, indicating very high validity; a practicality score of 83.7%, indicating that the product was practical; and an average N-Gain value of 0.60, indicating a moderate improvement in students' mathematical representation skills. Thus, the developed Student Worksheet met the criteria for validity, practicality, and effectiveness.

3.2. Discussion

The results showed that the APOS-based Student Worksheet assisted by GeoGebra was very valid, practical, and effective for learning Lines and Angles. The developed Student Worksheet obtained an average Aiken's V coefficient of 0.86, a practicality score of 83.7%,

and an average N-Gain of 0.60. These findings indicate that the Student Worksheet was feasible in terms of content and presentation and showed potential to improve students' mathematical representation skills. The improvement in students' mathematical representation skills can be understood from the learning activities embedded in the Student Worksheet. The APOS stages were not implemented as separate activities; rather, they were organized as a progressive learning sequence. At the Action stage, students performed explicit activities involving lines and angles using GeoGebra. These actions then developed into a Process as students explored relationships and patterns. These processes were subsequently conceptualized as Objects, while the Schema stage required students to connect related concepts and apply them to mathematical problems. This sequence gave students opportunities to move from performing mathematical procedures to constructing and connecting mathematical concepts.

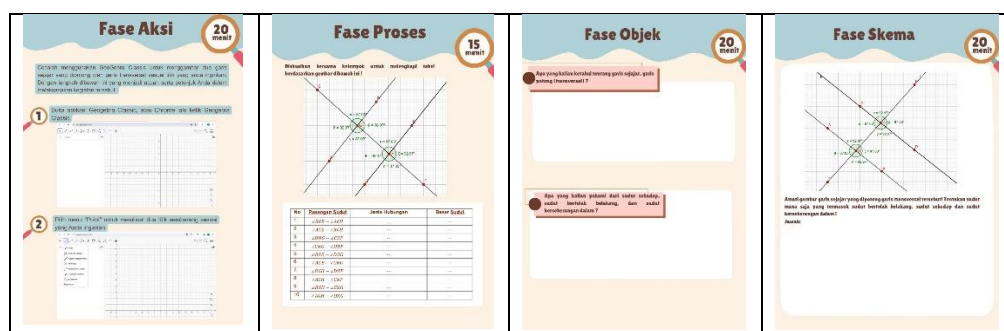


Figure 4. Appearance of the Student Worksheet Based on the APOS Model to Improve Junior High School Students' Mathematical Representation Skills

Therefore, the learning activities positioned students as active participants in constructing mathematical knowledge rather than merely receiving information from the teacher. These activities align with the preliminary analysis, which showed that students tended to rely on teacher explanations and demonstrated limited independence in completing learning tasks. The APOS-based structure provided a systematic sequence through which students could gradually develop their understanding and representations of Lines and Angles. The main pedagogical contribution of this study is integrating conceptual construction through APOS with dynamic visualization through GeoGebra in a structured worksheet. APOS provides a cognitive framework that organizes how students progressively construct mathematical concepts, whereas GeoGebra provides a dynamic environment in which these concepts can be explored, manipulated, and visually represented.

This relationship is particularly relevant to geometry learning. Through GeoGebra, students can construct and modify geometric objects and observe changes dynamically. These activities provide external representations that can support students as they progress through the APOS stages. In this respect, APOS structures the conceptual construction process, while GeoGebra externalizes and supports that construction through dynamic visual and symbolic representations. The integration of these components is also reflected in the structure of the developed Student Worksheet. The worksheet organizes student activities according to the APOS stages while providing opportunities to explore lines and angles using GeoGebra. The product-development results showed that the learning activities were

explicitly organized around Action, Process, Object, and Schema, with GeoGebra integrated into the mathematics learning tasks. Therefore, GeoGebra was not used merely as an additional technological tool. Instead, it supported students' conceptual exploration within the APOS learning sequence. This integration distinguishes it from studies that use GeoGebra primarily as a visualization or teaching medium.

The findings of this study support previous research on the use of APOS and GeoGebra in mathematics learning, while also extending the existing literature by integrating both within a representation-oriented worksheet. Previous studies have shown that APOS can support the development and analysis of students' mathematical understanding, including the construction of mathematical representations. Similarly, research on GeoGebra has shown that dynamic geometry environments can support students' exploration and representation of geometric concepts. According to Lestari (2023), applying Realistic Mathematics Education through the APOS model, assisted by GeoGebra, improved students' mathematics learning outcomes [29]. In addition, Palupi (2026) developed learning materials based on Problem-Based Learning assisted by GeoGebra and reported that the resulting materials met the criteria of validity, practicality, and effectiveness [30]. These studies support the potential of APOS and GeoGebra as technology-supported components of mathematics learning.

However, these studies did not directly investigate the specific integration of APOS stages, GeoGebra, and mathematical representation in a structured worksheet for Lines and Angles. Therefore, this study extends previous research by combining these three elements into a single learning product. Its contribution lies not only in the simultaneous use of APOS and GeoGebra but also in their functional complementarity: APOS provides a cognitive progression for conceptual construction, while GeoGebra provides dynamic representations that support students throughout that progression. At the same time, these findings should not be interpreted as evidence that the combined approach is superior to APOS or GeoGebra used independently. Because the study did not include a comparison group, the specific contribution of each component and their interaction remain uncertain. Therefore, further experimental research is needed to determine the added value of the integrated approach.

Although the overall average N-Gain was 0.60, individual N-Gain values ranged from 0.51 to 0.75, indicating variation in students' responses to the intervention. Some students showed greater improvement, while others showed more moderate gains. This variation may be related to differences in students' initial mathematical abilities and their readiness to engage in the learning activities. The preliminary analysis identified differences in students' mathematical abilities and showed that some students were highly dependent on teacher explanations and demonstrated limited independence in completing learning tasks. These conditions may have influenced how effectively individual students engaged with APOS-based activities and GeoGebra-supported exploration.

However, these factors should be considered possible interpretations rather than established causal explanations, because the study did not statistically examine the relationships among students' initial abilities, digital skills, and individual N-Gain scores. Therefore, the observed variation highlights an area that requires further investigation. The overall average N-Gain of 0.60 indicates a moderate level of improvement rather than a high

level. One possible explanation is that students needed to adapt to a learning environment that required active conceptual construction and the use of dynamic mathematical technology. The preliminary analysis indicated that students were accustomed to teacher-centered learning and tended to depend on teacher explanations. Therefore, adapting to APOS-based activities may require additional time and guidance.

Variations in students' mathematical proficiency may also have contributed to the moderate overall improvement. However, this interpretation should be treated cautiously because the study did not directly measure students' familiarity with GeoGebra, digital independence, or the effect of the implementation duration. Therefore, future studies should investigate these factors rather than treating them as confirmed explanations for the moderate N-Gain improvement.

Theoretically, this study contributes to mathematics education by connecting progressive conceptual construction and dynamic mathematical representation within a single instructional framework. APOS provides a theoretical basis for organizing students' development from Action to Process, Object, and Schema, while GeoGebra provides a dynamic environment for representing and exploring mathematical objects. Therefore, the findings suggest that combining a cognitive learning framework with dynamic visualization technology can provide a meaningful basis for designing representation-oriented mathematics learning materials.

Practically, the findings suggest that teachers can use structured worksheets to guide students through progressive conceptual activities while incorporating GeoGebra to facilitate dynamic visualization. GeoGebra can be positioned not only as a tool for displaying geometric figures but also as an environment in which students construct, manipulate, observe, and represent mathematical objects. For learning-material developers, the findings emphasize the importance of aligning technology with the instructional model. In this study, the Student Worksheet served as a structure connecting the APOS stages with GeoGebra activities, rather than treating technology as a separate instructional component.

Several limitations should be considered when interpreting these findings. First, the study was conducted at a single school, SMP Negeri 28 Bengkulu Selatan, which limits the generalizability of the findings to other educational contexts. Second, the effectiveness analysis involved 30 seventh-grade students, whereas the practicality evaluation involved only 10 students. Third, the study did not include a comparison or control group; therefore, the observed improvement cannot be attributed exclusively to the combined use of APOS and GeoGebra. Fourth, the implementation was conducted over a limited period, which may not fully reflect the long-term impact of the developed Student Worksheet. Finally, the practicality assessment relied on students' self-reported responses, which may not fully reflect their actual learning behavior or independent use of the Student Worksheet. These limitations indicate that the findings should be interpreted as evidence of the feasibility and potential effectiveness of the developed product rather than as definitive evidence of its superiority over other instructional approaches.

Future research should examine APOS-based Student Worksheets supported by GeoGebra using larger and more diverse samples from multiple schools. Experimental or quasi-experimental designs involving comparison groups are also recommended to

determine whether the integrated approach provides additional benefits compared with APOS-based learning or GeoGebra-supported learning used separately.

4. CONCLUSION

This study aimed to develop an APOS-based Student Worksheet assisted by GeoGebra on Lines and Angles to support students' mathematical representation skills. The developed Student Worksheet met the predetermined criteria for validity, practicality, and effectiveness, obtaining an average Aiken's V coefficient of 0.86, a practicality score of 83.7%, and an average N-Gain of 0.60. These findings indicate that the developed Student Worksheet is feasible for use in mathematics learning and has the potential to support the development of students' mathematical representation skills. The main contribution of this study is integrating APOS and GeoGebra into a structured worksheet. APOS provides a progressive framework for students to construct mathematical concepts through the stages of Action, Process, Object, and Schema. At the same time, GeoGebra supports this conceptual construction through dynamic visualization and manipulation of geometric objects. This integration connects students' conceptual construction with mathematical representation in learning Lines and Angles.

However, several limitations should be considered when interpreting the findings. The study was conducted at a single school with a relatively small sample, did not include a comparison group, and was implemented over a limited period. In addition, the practicality assessment relied primarily on students' self-reported responses. Therefore, the findings should not be generalized to broader educational contexts or interpreted as conclusive evidence that the integrated APOS–GeoGebra approach is superior to other instructional approaches. Future research should examine the effectiveness of the APOS–GeoGebra Student Worksheet using a quasi-experimental design with comparison groups and multiple schools and larger, more diverse samples. Further studies should also compare the APOS–GeoGebra Student Worksheet with conventional worksheets, examine its effects on individual mathematical representation indicators, and investigate long-term retention of mathematical concepts. Such studies would provide stronger evidence regarding the specific contribution and sustainability of integrating APOS with GeoGebra in mathematics learning.

REFERENCES

- [1] V. N. Candra, B. R. Arvy, and A. Nasrullah, "Pengaruh Pbl Berbantuan Desmos Terhadap Kemampuan Komunikasi Matematis Dan Self-Confidence Siswa," *J. Innov. Creat.*, vol. 6, no. 1, pp. 724–733, 2026.
- [2] N. Zuhaida, E. Pujiastuti, Zaenuri, and Walid, "Trends in Students' Mathematical Problem-Solving Skills: A Systematic Review and Bibliometric Analysis of GeoGebra-Assisted Problem-Based Learning Research (2021 – 2025)," *Media Pendidik. Mat.*, vol. 13, no. 2, pp. 1118–1133, 2025.
- [3] A. A. Amelia, L. Rianti, N. Jazilah, and S. Nabila, "Pemahaman Logika Matematika dalam Pembelajaran Matematika," *J. Creat. Innov. Res.*, vol. 3, no. 1, 2026.
- [4] D. A. Cahyaningsih and D. Wijayanti, "Deskripsi Aspek Kontekstual pada Soal Buku Teks Matematika SMP Kurikulum Merdeka Kelas VII," *JagoMIPA J. Pendidik. Mat. dan IPA*, vol. 6, pp. 41–54, 2026, doi: <https://doi.org/10.53299/jagomipa.v6i1.3611>.
- [5] N. Izzati and R. S. Adhany, "The Influence of Mathematical Representation Ability on Students' Mathematical Communication Skills," *J. Teach. Math. Soc. Res. Opin.*, vol. 5, no. 3, pp. 2099–2109, 2026, doi: <https://doi.org/10.58421/misro.v5i3.1540>.
- [6] T. J. Rati, Nursupiamin, and Yulia, "Perbandingan Kemampuan Representasi Matematis Ditinjau dari Self-Concept Peserta Didik," *J. Cendekia J. Pendidik. Mat.*, vol. 09, no. 61, pp. 1250–1263, 2025, doi:

- <https://doi.org/10.31004/cendekia.v9i3.4355>.
- [7] H. M. Siregar, Nurjanah, Turmudi, Darhim, and E. Nurlaelah, "Student Strategies And Errors In Solving Mathematical Representation Problems On A Quadratic Function Material," *AKSIOMA J. Progr. Stud. Pendidik. Mat.*, vol. 15, no. 1, pp. 236–251, 2026, doi: <https://doi.org/10.24127/ajpm.v15i1.14891>.
 - [8] P. O. Yesika, W. Ramadianti, and R. Jumri, "Pengembangan LKPD Berkonteks Budaya Dol dengan Pendekatan STEAM Untuk Memfasilitasi Literasi Numerasi," *J. e-DuMath*, vol. 12, no. 1, pp. 248–262, 2026, doi: <https://doi.org/10.26638/>.
 - [9] A. N. Harahap, Nurdalilah, and S. S. Lubis, "Development of Web-Based Learning Media Using Google Sites for Lines and Angles," *JDIME J. Dev. Innov. Math. Educ.*, vol. 2, no. 1, pp. 26–36, 2024, doi: <http://dx.doi.org/10.32939/jdime.v2i1.4116>.
 - [10] I. Septiani, Nursupiamin, and Yulia, "Fenomenologi pembelajaran matematika smp pada materi garis dan sudut," *Leibniz J. Mat.*, vol. 5, no. 2, pp. 182–194, 2025, doi: <https://doi.org/10.59632/leibniz.v5i02.574>.
 - [11] I. G. S. A. I. Jayaningrata, I. G. P. Suharta, and I. G. N. Y. Hartawan, "Pengembangan E-LKPD Berbasis Konsep Pembelajaran Mendalam Untuk Meningkatkan Kemampuan Pemecahan Masalah Matematis Trigonometri," *J. Edukasi Mat. dan Sains*, vol. 15, no. 2, pp. 449–460, 2026, doi: <https://doi.org/10.59672/emasains.v15i2.6529>.
 - [12] S. Salsabila, S. Nurhaliza, R. Qadri, and A. W. Ara, "Analisis Ketidakkonsistenan Guru dalam Penggunaan Lembar Kerja Peserta Didik (LKPD) pada Pembelajaran di MIN 6 Aceh Utara," *J. Educ. Innov.*, vol. 2, no. 3, pp. 147–157, 2026.
 - [13] I. B. Asfyra, W. Rumite, H. Arianto, N. R. Kalla, and U. Negeri, "Pendampingan Penyusunan LKPD Guru MGMP Matematika Tingkat SMP Kabupaten Maros," *COMSEP J. Pengabd. Kpd. Masy.*, vol. 7, no. 2, pp. 443–452, 2026, doi: [10.54951/comsep.v7i2.1315](https://doi.org/10.54951/comsep.v7i2.1315).
 - [14] Hanifah, "Model APOS sebagai Solusi Pembelajaran Abad 21." Deepublish, pp. 1–192, 2026.
 - [15] V. Omarisa, E. E. Muchlis, and A. Susanta, "Development of a mathematics teaching module based on the APOS model integrated with GeoGebra," vol. 6, no. 2, pp. 660–675, 2025, doi: <https://doi.org/10.59672/ijed.v6i2.5217>.
 - [16] D. P. Sari, A. Efriani, and H. Nizar, "Peran GeoGebra dalam Menggeser Pembelajaran Geometri dari Hafalan Prosedural Menuju Pemahaman Konseptual," *GeoMath J. Pendidik. Mat.*, vol. 7, no. 1, pp. 1–16, 2026, doi: <https://doi.org/10.55171/geomath.v7i1.1692>.
 - [17] W. Astuti, D. Nurjamil, and H. Patmawati, "Pengaruh penggunaan GeoGebra dengan model Discovery Learning terhadap kemampuan pemahaman konsep matematis peserta didik SMP pada materi transformasi geometri," *Primatika J. Pendidik. Matematika*, vol. 15, no. 1, pp. 125–140, 2026, doi: <https://doi.org/10.30872/primatika.v15i1.6526>.
 - [18] T. Siregar, Rusdinal, and Yerizon, "Pendekatan Kualitatif Deskriptif: Eksplorasi Perintah Geogebra dalam Pemahaman Persamaan , Pertidaksamaan , dan Fungsi," *J. Teach. Learn. Res.*, vol. 8, no. 1, pp. 180–212, 2026, doi: <https://doi.org/10.24256/jtlr.v8i1.8068>.
 - [19] A. Hamid, "Pemanfaatan GeoGebra Dalam Pembelajaran Geometri Analitik Untuk Meningkatkan Visualisasi Dan Pemahaman Konseptual Mahasiswa," *Prox. J. Penelit. Mat. dan Pendidik. Mat.*, vol. 9, no. 2, pp. 489–496, 2026, doi: <https://doi.org/10.30605/proximal.v9i2.8794>.
 - [20] R. M. Branch, *Instructional design: The ADDIE approach. In Digital Learning: The Key Concepts*. 2019. doi: <https://doi.org/10.4324/9780429425240-105>.
 - [21] L. N. Azizah and T. D. Prihatin, "Analisis Kebutuhan Pengembangan Media Pembelajaran Matematika Adaptif Berbasis Artificial Intelligence pada Kurikulum Merdeka di SMP," *J. Inov. Pendidik. Sains dan Terap.*, vol. 05, no. 01, pp. 1–8, 2026.
 - [22] M. Antasari, Hanifah, A. Susanta, and I. Andriani, "Penerapan Pendekatan Realistic Mathematics Education Melalui Model APOS Berbantuan Geogebra Untuk Meningkatkan Hasil Belajar Matematika," *AKSIOMA J. Progr. Stud. Pendidik. Mat.*, vol. 12, no. 2, pp. 2468–2478, 2023, doi: <https://doi.org/10.24127/ajpm.v12i2.6928>.
 - [23] E. S. Priandini and R. Faulina, "Analisis Persepsi Guru Matematika Tingkat Smp -Sma Sederajat Di Kecamatan Socah Terhadap Penggunaan Teknologi Dan Informasi (TIK) Untuk Pembelajaran Matematika," *SIGMA J. Pendidik. Mat.*, vol. 16, no. 1, pp. 121–129, 2024, doi: <https://doi.org/10.26618/sigma.v16i1.14544>.
 - [24] Sugiyono, *metode penelitian pendidikan (kuantitatif, kualitatif, kombinasi, R&D dan penelitian pendidikan)*. 2019.
 - [25] S. D. E. Yuliyanti and N. H. Waryanto, "LKPD Berbantuan GeoGebra Book Materi Segiempat Dan Segitiga Dengan Pendekatan Kontekstual Berorientasi Pada Kemampuan Pemecahan Masalah," *J. Pedagog. Mat.*, vol. 9, no. 3, pp. 224–236, 2023, doi: <https://doi.org/10.21831/jpm.v9i3.18566>.
 - [26] L. R. Aiken, "Three coefficients for analyzing the reliability and validity of ratings," *Educ. Psychol.*

- Meas.*, vol. 45, no. 1, pp. 131–142, 1985, doi: 10.1177/0013164485451012.
- [27] L. Damayanti, Nurhanurawati, W. T. Pratitis, F. Rohman, and I. Kurniawan, “Pengembangan Lkpd Berbasis PBL: Studi Validitas Dan Praktikalitas Untuk Kemampuan Berpikir Kritis Dalam Pembelajaran Matematika SD,” *Pendas J. Ilm. Pendidik. Dasar*, vol. 10, no. 3, pp. 535–547, 2025, doi: <https://doi.org/10.23969/jp.v10i3.29390>.
- [28] Amri, M. Arsyad, and P. Palloan, “Penerapan Media Buku Saku Digital Untuk Meningkatkan Literasi Sains Peserta Didik SMA Negeri 10 Makassar,” *J. Sains dan Pendidik. Fis.*, vol. 22, no. 1, pp. 337–348, 2026, doi: <https://doi.org/10.35580/h8s7c478> Abstrak.
- [29] D. Lestari, Zetriuslita, A. Dahlia, and E. Istikomah, “Pengembangan Perangkat Pembelajaran dengan Model Problem Based Learning dengan Berbantuan Software Geogebra pada Materi Persamaan Garis Lurus Kelas VIII SMP,” *Juring (Journal Res. Math. Learn.*, vol. 6, no. 1, pp. 73–88, 2023, doi: <http://dx.doi.org/10.24014/juring.v6i1.19600>.
- [30] T. C. R. Palupi, I. Nursit, and S. Zuhri, “Penggunaan Software Geogebra Geometry dalam Meningkatkan Kemampuan Matematis Siswa pada Materi Transformasi Geometri di SMPN 13 Malang,” *J. Pustaka Lokapedia*, vol. 1, no. 1, pp. 53–59, 2026.
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