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Learning to Teach Euclidean Geometry: Pre-service Teachers' van Hiele Experiences

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ABSTRACT

Euclidean geometry is central to mathematics teacher education, yet many pre-service teachers struggle with geometric reasoning and proof. This qualitative follow-up study examined how 12 pre-service teachers from two Colleges of Education experienced learning geometry through either traditional instruction or a van Hiele phase-based model. Data from reflective journals and semi-structured interviews were analysed using interpretive thematic analysis. Findings revealed that van Hiele participants developed the ability to follow proof logic, trusted that difficulty would eventually make sense, described their learning as gradual, and left with a clear vision for teaching geometry. By contrast, traditional instruction participants described memorisation without understanding, disengagement when lost, and uncertainty about future teaching. The findings demonstrate that instructional design shapes not only what pre-service teachers know but also how they reason, persist, and imagine themselves as future geometry teachers. Implications for teacher education programmes are discussed.

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1. INTRODUCTION

Euclidean geometry holds a curious position in mathematics education. It is widely regarded as an essential subject that teaches students proof, reasoning, and logical arguments. However, ask most pre-service teachers how they feel about Euclidean geometry, and you are likely to hear words like "hard," "scary," or "I just memorised it." This gap between what Euclidean geometry is supposed to teach and what students actually take away from it is not merely an academic concern. Teachers tend to teach the way they were taught, particularly in subjects they find difficult. If pre-service teachers experience geometry as a collection of disconnected facts and memorised proofs, they are likely to reproduce that same approach in their own classrooms.

In Ghana, as in many other countries, research has documented persistent difficulties with geometry learning from basic school through to teacher education. Students struggle with proof and spatial reasoning. Many also find it difficult to see how geometric ideas connect to one another [1, 2]. Despite curriculum reforms and restructuring of teacher education programmes, these difficulties have not disappeared. One possible reason is that the root of the problem lies in how geometry is taught to teachers themselves. A considerable body of evidence suggests that traditional lecture-based instruction remains the dominant approach. In this model, the teacher presents definitions, works through a proof, and moves on. There is little room for exploration, discussion, or productive struggle [3, 4]. For pre-service teachers, this kind of instruction reinforces the notion that geometry is a finished system; you either see the proof, or you do not. There is no middle ground.

In light of this problem, some researchers have turned to the van Hiele theory as a possible alternative. The van Hiele model proposes that geometric thinking develops in levels and that instruction is most effective when it aligns with the learner's current level of reasoning [5]. Studies from various countries have shown that van Hiele phase-based instruction can improve geometric thinking and test scores [6, 7]. What remains less clear, however, is what the experience of learning geometry through van Hiele phases actually feels like for pre-service teachers. Do they understand proofs differently? Do they feel more confident? Do they imagine teaching geometry differently afterwards? Most existing research focuses on test scores. That approach presents whether something worked but does not inform much about how or why it worked, or what the experience was like for the learner.

This study is an attempt to answer those questions. It is not a controlled experiment designed to prove that van Hiele instruction is universally superior. Rather, it is an effort to understand something that test scores cannot capture, which is the lived experience of learning geometry for teaching. That experience matters because it shapes what teachers carry with them into their own classrooms. A teacher who has only ever memorised geometry is unlikely to teach it differently. However, a teacher who has experienced geometry as something that can be figured out, step by step, is more likely to help students do the same.

1.1. What Makes Euclidean Geometry Hard to Learn and to Teach

Euclidean geometry fundamentally differs from algebra. In algebra, it is often possible to arrive at the correct answer by following a procedure. In Euclidean geometry, particularly in proofs, one has to understand why something is true, not just compute it. This makes Euclidean geometry harder to learn. It also makes it harder to teach, because a teacher cannot simply show steps; the reasoning behind those steps must be made visible.

Research has shown that pre-service teachers often enter teacher education with a shallow grasp of geometry. They may know some facts and be able to state theorems, but when asked to explain the reasoning behind a proof, many struggle [8, 9]. This is evident in the Ghanaian context, where pre-service teachers sometimes complete their training with limited confidence in geometry and weak conceptual foundations [10]. It can be said that this is not just an academic problem. Ball et al. [11] demonstrated years ago that what teachers know shapes what they teach. A teacher who only memorised geometry will fall

back on memorisation when teaching. A teacher who understands the logical structure of geometry can help students see that structure as well.

So why do so many pre-service teachers end up with shallow understanding? One obvious answer is the way geometry is taught. Traditional instruction tends to present definitions, theorems, and proofs as finished products. The teacher talks, students listen, and there is not much exploration or discussion. Tachie [4] found this pattern in South African classrooms, and similar findings have been reported in Ghana [3]. In such classrooms, students learn to follow steps and reproduce proofs during examinations. However, they do not necessarily learn why the steps work or how proofs connect to each other. Nazarovich and Kurudirek [12] and Schoenfeld [13] all make this point: pre-service teachers can often reproduce formal proofs while being unable to explain the underlying reasoning. This type of learning is merely procedural: students can produce correct responses but cannot explain the reasoning that justifies them.

Geometry demands that learners engage in two simultaneous reasoning processes: the visual process of interpreting diagrams and the formal process of translating those observations into definitions, axioms, and logical deductions. Gal and Linchevski [14] call this a tension. That is, learners come in with strong visual intuitions, but those intuitions are rarely connected to formal proof structures. When instruction does not help bridge that gap, learners become confused, anxious, and eventually check out. Awado et al. [15] make a related point when they point out that affective responses, including anxiety and disengagement, are not just side effects. They matter directly for teacher education. A pre-service teacher who feels anxious about geometry is unlikely to become a confident geometry teacher. Also, a teacher who lacks confidence will avoid teaching geometry or will teach it poorly.

1.2. What the van Hiele Theory Offers and Where It Falls Short

The van Hiele theory, developed in the 1950s, became widely known in English-speaking research through the work of Crowley [16] and Usiskin [17]. The basic idea is that geometric thinking develops in levels. At the lowest level, learners recognise shapes by how they look. At higher levels, learners can reason about properties, then about logical relationships, and eventually about formal systems. The instructional implication is that teaching works best when it matches the learner's current level. For instance, teachers cannot jump straight to formal proofs if learners are still at the visual level; the learners will not understand what is happening.

Van Hiele proposed five phases to structure instruction: information, guided orientation, explicitation, free orientation, and integration. These phases are designed to move learners gradually from exploration to formal reasoning. Learners begin by observing geometric phenomena, then discuss their observations, and subsequently articulate rules based on those observations. Only after this process do they engage with formal proofs. A fair amount of research has tested whether this works. The answer seems to be yes, at least for test scores. Studies from different countries have found that van Hiele phase-based instruction improves geometric thinking and achievement [6, 18, 19]. Most of these studies focused on school students, not on pre-service teachers. But there is some work on pre-

service teachers as well. Erdogan [20] found that many pre-service teachers operate at lower van Hiele levels than their teacher educators expect. Usiskin [17] also made a similar observation decades ago.

Here is what the test-score studies do not tell us: they do not tell us what pre-service teachers actually experience when they learn geometry through van Hiele phases. Do they find the sequence helpful or confusing? Does it change how they feel about geometry? Does it affect how they imagine teaching it? These questions remain largely unanswered.

It is also worth acknowledging that the van Hiele theory has critics. Some researchers argue that the levels are too rigid, while others say the theory describes progression but does not explain how learners actually move from one level to the next [21]. There is also debate about whether the phases work the same way across different cultural contexts. Most of the research comes from Europe and North America, but less is known about how van Hiele instruction plays out in Sub-Saharan African classrooms. This study does not solve that gap, but it adds one piece of evidence from Ghana.

1.3. Why Reflective Accounts Matter

There is a broader shift happening in mathematics education research. Researchers used to focus almost entirely on outcomes: did test scores go up? That was the key question. Increasingly, researchers are arguing that we also need to attend to learners' experiences, their voices, and their reflections. Deslis [22] and Källberg and Roos [23] both make this case, adding that outcome-focused research hides a lot. It hides the confusion, the anxiety, the small moments of breakthrough, and the gradual growth in confidence. Reflective methods like journals and interviews bring those dimensions into view.

A few studies have used reflective methods to study geometry learning. Machisi [24] worked with South African Grade 11 learners and showed how reflective accounts can reveal what students actually struggle with. Chikiwa and Graven [25] looked at pre-service teachers' reflective practice more broadly. Guce [26] and Kesebir and Oksuz [27] also used journals to study how pre-service teachers think about mathematics and teaching. This study builds on that tradition by using reflective journals and interviews to capture the lived experience of learning geometry.

1.4. How Learning Experiences Shape Teaching Identities

Pre-service teachers do not just learn content; they also form ideas about what teaching looks like. Those ideas come from somewhere. Often, they come from their own experiences as learners [11, 28]. If a pre-service teacher only ever experienced geometry as a set of disconnected rules delivered by a teacher, that becomes the default template. They may not know any other way to teach. Graven and Heyd-Metzuyanim [29] and Langer-Osuna and Esmonde [30] have written about mathematics identity. How learners see themselves in relation to mathematics matters. Do they see themselves as people who can do mathematics? As people who can explain it to others? Those self-perceptions shape future choices.

In geometry, where abstraction and proof trigger strong emotional reactions for many learners, identity formation is especially important. A pre-service teacher who experiences

geometry as something they can figure out, step by step, is more likely to see themselves as a capable geometry teacher. However, a pre-service teacher who experiences geometry as opaque and teacher-dominated is more likely to feel uncertain. This study pays attention to that identity dimension. It asks not only what pre-service teachers learned but also how they imagine using that learning in their own classrooms.

1.5. Theoretical Framework

This study draws on three interconnected theoretical ideas: the nature of mathematics teacher knowledge, the van Hiele model of geometric thinking, and the role of identity in learning to teach.

1.5.1. Learning Euclidean Geometry as Mathematics Teacher Learning

When pre-service teachers learn geometry, they are not merely acquiring content for their own benefit; they are learning content they will later be expected to teach. This dual purpose redefines what constitutes meaningful learning. For example, mastery sufficient to reproduce a proof on an examination differs fundamentally from the kind of understanding required to explain that proof to a student who is struggling. Ball et al. [11] made this point forcefully when they argued that teachers need a special kind of mathematical knowledge, one that is usable for teaching. This means seeing how ideas connect, anticipating where learners get stuck, and being able to represent concepts in multiple ways. In geometry, this kind of knowledge is especially critical. Proofs do not stand alone; they rest on definitions, properties, and logical relationships that must be coordinated. A teacher who only memorised proofs without understanding those relationships will struggle to teach geometry meaningfully.

Schoenfeld [13] adds another layer. He argues that real learning happens when learners make sense of mathematics for themselves, not just when they follow procedures. Sense-making, in his view, is the active work of constructing coherence among ideas. For pre-service teachers, sense-making in geometry is not a luxury; it is the foundation for being able to teach the subject. The theoretical starting point for this study is therefore straightforward: how geometry is taught to pre-service teachers matters because it shapes what they know, how they know it, and what they can do with that knowledge later. Instruction that hides the reasoning behind proofs will produce teachers who cannot explain that reasoning, whereas instruction that makes reasoning visible may produce teachers who can.

1.5.2. What the van Hiele Phases Offer

The van Hiele theory provides a useful way to think about instruction that makes reasoning visible. Van Hiele [5] proposed that learners progress through levels of geometric reasoning and that instruction can support this progression through five phases: information, guided orientation, explicitation, free orientation, and integration. These phases matter for this study because they structure learning in a particular way. Learners do not start with formal proofs; they start with exploration, then talk about what they notice, then articulate ideas in their own words, and only later formalise those ideas into proofs. This sequence is

the opposite of what often happens in traditional geometry instruction, where the teacher presents a proof, and learners try to follow along.

The van Hiele phases should not be treated as a rigid model that must be followed exactly. Rather, they should be seen as one example of instruction that prioritises sense-making over coverage. The key question for this study is not whether the phases work in some universal sense. The question is how pre-service teachers experience learning geometry when instruction is organised this way compared to when it is organised in a more traditional lecture format.

1.6. Purpose of the Study

The purpose of this study was to investigate how pre-service teachers learn Euclidean geometry within mathematics teacher education, with particular attention to how different instructional approaches shape their sense-making, confidence, and emerging pedagogical orientations. The study is guided by the understanding that what matters is not just whether pre-service teachers can reproduce proofs but whether they understand the reasoning behind them and can imagine teaching geometry in a way that makes that reasoning visible to others. Specifically, the study addresses four research questions:

1. How do pre-service teachers make sense of Euclidean geometry concepts and proofs during their teacher education preparation?
2. How do different instructional approaches (traditional instruction and van Hiele phase-based instruction) shape pre-service teachers' learning experiences of Euclidean geometry?
3. In what ways do pre-service teachers' experiences of learning Euclidean geometry influence their confidence, emotions, and emerging identities as future mathematics teachers?
4. How do pre-service teachers interpret their geometry learning experiences in relation to how they envision teaching Euclidean geometry in their future classrooms?

2. METHOD

2.1. Research Design

This study employed an interpretive qualitative follow-up design aimed at deepening understanding of pre-service teachers' learning of Euclidean geometry following an earlier quasi-experimental investigation. While the earlier study examined the effects of instructional approaches on geometric thinking using quantitative measures, the present study sought to illuminate how pre-service teachers experienced and interpreted those instructional environments as learners and future teachers. The study is grounded in an interpretivist epistemological stance, which assumes that learning experiences, meanings, and understandings are socially and contextually constructed. This study focused on understanding *how* pre-service teachers learn Euclidean geometry, not just measuring their results. To do this, it prioritised their personal stories, reflections, and interpretations to learn from their process of making sense of concepts, their emotional responses, and their thoughts on teaching. Positioned as a qualitative follow-up, the study complements the earlier quasi-experiment by providing explanatory insight into the mechanisms through which different

instructional approaches shaped learning. This design fits with what many researchers say: qualitative methods are especially good at exploring how learning happens, because they can uncover things that experiments and surveys often miss.

2.2. The Earlier Intervention

Before describing the participants and the present study's procedures, it is important to explain the instructional intervention that preceded this qualitative follow-up. The earlier quasi-experimental study [31] was conducted over a period of six weeks during a regular academic semester. The intervention focused on two-dimensional Euclidean geometry, covering three main topic areas: (1) proving angle relationships formed when a transversal intersects parallel lines, (2) proving properties of quadrilaterals including diagonal and angle relationships, and (3) proving hierarchical relationships between quadrilaterals through deductive reasoning.

In the experimental group, instruction followed the van Hiele phase-based model. This meant that each of the three lessons was structured around the five van Hiele phases, though the phases were not treated as a rigid sequence to be followed mechanically. In the first phase (information/inquiry), the researcher engaged pre-service teachers in conversation about the topic, using familiar language and examples to make the context clear. This two-way interaction helped establish what pre-service teachers already knew and what direction the lesson would take. In the second phase (guided orientation), pre-service teachers worked with hands-on materials including cut-outs of cardboard, mathematical sets, a pair of scissors, and masking tape to explore geometric properties through folding, measuring, and constructing. The tasks were carefully sequenced but allowed pre-service teachers to discover properties for themselves. In the third phase (explicitation), pre-service teachers shared what they had discovered, using their own words at first, with the researcher gradually introducing appropriate geometric terminology. In the fourth phase (free orientation), pre-service teachers worked on more complex, open-ended tasks that required them to apply what they had learned and to see connections between different properties. Finally, in the fifth phase (integration), the class reviewed and summarized what had been learned, forming an overview of the new network of relations. No new material was introduced in this final phase.

In the control group, the same geometry topics were taught using conventional instruction. This meant the researcher presented definitions, worked through proofs, and demonstrated examples on the markerboard. Pre-service teachers followed along, took notes, and completed practice exercises individually. There were no hands-on activities, no small-group discussions, and no opportunities for pre-service teachers to explore properties before formal presentation. Instruction was teacher-centred, and active participation was not promoted beyond answering occasional questions. In other words, the control group received the kind of instruction that is typical in many Ghanaian mathematics classrooms, where the teacher is the sole authority, and students are expected to absorb and reproduce content.

The key difference between the two approaches, then, was not the content covered but the sequence and nature of the learning activities. In the van Hiele group, pre-service teachers encountered ideas through exploration before formalization; in the traditional

group, they encountered formal definitions and proofs first and practiced applying them afterward.

2.3. Participants and Context

Participants for this qualitative follow-up study were drawn from the earlier quasi-experimental study. The earlier study involved 150 pre-service mathematics teachers from two Colleges of Education in Ghana. In that study, one college was assigned as the experimental group and received van Hiele phase-based instruction, while the other college served as the control group and received conventional instruction. Following completion of the treatment and the post-test, all pre-service teachers from both colleges were invited to volunteer for this qualitative follow-up study. Twelve pre-service teachers responded and agreed to participate. Of these, seven were from the control group and five from the experimental group. All participants were second-year pre-service teachers enrolled in a three-year Diploma in Basic Education programme.

It is worth acknowledging that relying on volunteers introduces the possibility of self-selection bias. Pre-service teachers who chose to participate may have been more reflective, more motivated, or more engaged with geometry than those who did not volunteer. This means the findings may not fully represent the experiences of all pre-service teachers in the original study. However, this limitation is acceptable for a qualitative study that prioritises depth of insight over representativeness.

Participants varied in terms of prior achievement in geometry, confidence, and engagement during instruction. To protect their identities, pseudonyms were assigned to all participants. The use of twelve participants was deliberate and theoretically justified. Qualitative research aimed at understanding learning experiences prioritises depth, richness, and interpretive insight rather than numerical representativeness [32]. The goal was to learn how pre-service teachers understood geometry, experienced the lessons, and developed their ideas about teaching.

2.4. Rationale for Sample Size and Analytic Generalisation

This study does not aim for statistical generalisation. Instead, it uses the findings to build on existing theories of how mathematics teachers learn. The adequacy of the sample is therefore judged not by size, but by the quality and explanatory power of the data in relation to the theoretical framework. The study worked with participants from different teaching backgrounds who were reflective. This provided detailed stories on how course design influences learning. The findings are meant to be useful for similar teacher education settings, where readers can decide their relevance based on shared context rather than statistical comparison.

2.5. Data Sources

Two complementary data sources were used: reflective journals and semi-structured interviews. I chose these methods because they allow participants to speak for themselves. They also help me see not only what participants thought during the lessons, but also how they made sense of their learning afterward. Reflective journals enabled participants to

document their thoughts, emotions, challenges, and moments of insight close to the learning experience. Journals showed how participants' understanding and confidence changed over time, which one-time interviews could easily miss.

Participants were asked to write in their journals *once a week* over the course of the six-week treatment, though they were free to write more often if they wished. In practice, most participants submitted between eight and ten entries, with a few submitting fewer. The journal guide provided to participants included prompts about their learning experiences, their understanding of geometric concepts, their emotional responses to proofs and difficult tasks, and what the lessons meant for their future teaching. However, the prompts were the same for all participants regardless of which instructional group they belonged to, and participants were encouraged to write about whatever they felt was most important, not just what the prompts asked.

Semi-structured interviews were used to explore participants' experiences in greater depth and to clarify themes emerging from the journals. Interviews were conducted after the completion of geometry instruction and lasted between 45 and 60 minutes. The interview questions were open-ended, inviting participants to describe how they experienced learning Euclidean geometry, how instructional approaches influenced their understanding and emotions, and how they interpreted these experiences in relation to teaching. Specific questions included: "Can you tell me about your experience learning geometry this semester?" "What was different about how you learned geometry compared to previous courses?" "How did you feel when working on proofs?" and "How do you think this experience will affect how you teach geometry in the future?" These questions were not asked in a fixed order; instead, I adapted the sequence and wording based on each participant's responses, allowing for probing and clarification where needed. This flexibility helped me understand not just what participants thought but also why they thought that way.

2.6. Data Analysis

I analysed the data using a thematic interpretive approach, guided by the theoretical framework of mathematics teacher learning. I read all interview transcripts and journal entries several times to identify patterns related to how participants made sense of geometry, how instruction shaped their understanding, how they felt during learning, and how they thought about their future teaching. The coding process unfolded in several stages. I began with open coding, reading through each transcript and journal entry line by line and assigning simple descriptive codes to segments that seemed relevant. These initial codes captured what participants actually said, using words like "confusion," "clarity," "memorisation," "confidence," "trust," "magic," and "connection." At this stage, I tried to stay close to participants' own language rather than imposing theoretical categories.

After completing open coding for all data, I moved to a second stage of coding where I looked for patterns and groupings among the initial codes. Codes that seemed to cluster together were grouped into broader categories. For example, codes like "following steps," "knowing where things come from," and "tracing the logic" were grouped together under the category "understanding proof structure." Codes like "trusting the process," "believing it will make sense," and "waiting for connection" were grouped under "instructional trust."

In the third stage, I refined these categories into broader analytic themes that connected to the theoretical framework. This is where categories like "sense-making," "confidence in understanding," and "emerging pedagogical orientation" took shape. I then compared these themes across the two instructional groups, looking not only for similarities but also for differences and tensions. This comparative dimension helped me see patterns that might not have been visible if I had analysed each group separately.

I used NVivo software to support the organisation and comparison of data across participants and instructional contexts. NVivo helped me manage the large volume of data, retrieve coded segments efficiently, and visualise patterns across groups. At the same time, I coded the reflective journals manually as well, so I could stay closely connected to what participants wrote. This dual approach of using software for systematic organisation and manual coding for close engagement helped me to balance rigour with interpretive sensitivity.

All coding was conducted by me as the sole researcher. This was a deliberate choice, as the study is positioned as an interpretive inquiry where the researcher's engagement with the data is central to the analysis. However, I acknowledge that this limits the study's reliability in the sense of inter-coder agreement. To mitigate this, I maintained a detailed audit trail of all coding decisions and returned to the data regularly to check that my interpretations remained grounded in participants' accounts. I also engaged in reflexive journaling throughout the analysis, documenting my assumptions, questions, and interpretive moves.

2.7. Trustworthiness and Ethical Considerations

I used a number of strategies to make sure the findings were trustworthy. First, I used two different sources of data (journals and interviews) so that I could compare what participants said across both. This is called methodological triangulation. I also practiced member checking by asking participants to review their interview transcripts. Specifically, I sent each participant a copy of their transcript and invited them to confirm its accuracy. I also shared a summary of my preliminary interpretations with each participant and asked whether these interpretations aligned with what they had intended to convey. In a few cases, participants clarified or elaborated on points they had made, and I incorporated these clarifications into the analysis.

To keep the study transparent and dependable, I kept a detailed record of all the decisions I made during analysis, that is, what codes I used, why I grouped certain codes together, how I arrived at themes, and why I chose particular quotations to illustrate those themes. This is called an audit trail. It allows someone else to follow the process I used and to assess whether my interpretations are reasonable, given the data.

I obtained ethical approval from the relevant authorities and secured permission from the participating Colleges of Education. All participants gave their informed consent before taking part. I reminded them that they could withdraw from the study at any time without any penalty. Finally, I anonymised all data and stored it securely to protect participants' confidentiality.

2.8. Researcher Reflexivity

Before closing this section, I want to acknowledge my own position in relation to this study. I am a mathematics education researcher with experience teaching geometry in both conventional and van Hiele-based settings. I believe that instruction should make reasoning visible and that pre-service teachers benefit from opportunities to explore before formalisation. These beliefs shaped the design of the treatment in the earlier quasi-experimental study, and they also shaped how I approached the interviews and the analysis. I was not a neutral observer; I had a stake in seeing whether the van Hiele approach would yield positive results.

To manage this, I engaged in reflexive journaling throughout the data collection and analysis phases. I documented my expectations, for example, that van Hiele participants would report more positive experiences, and actively looked for evidence that challenged those expectations. I also asked participants in the control group to describe what worked well for them, not just what was difficult. This helped me avoid the trap of only seeing what I expected to see. The findings presented in this study are the result of an ongoing effort to ground interpretations in participants' accounts rather than in my prior assumptions. However, I do not claim to have achieved perfect objectivity. Rather, I have tried to be transparent about my role and to let readers judge for themselves whether the interpretations are credible.

3. RESULTS AND DISCUSSION

The four headings that follow are not categories I brought to the data from my theoretical framework. They emerged from reading and rereading the journals and interviews. I started with what participants actually said. For example, when Ama wrote that she could now "follow where each step is even coming from," that phrase became a cluster. Other participants said similar things in their own words. I grouped those together. Only later did I ask whether a shorthand label might help a reader see the pattern. The labels I settled on are "Following where each step is even coming from," "Later it will connect," "As we were going, going," and "Now I know how I will teach." These are not fancy theoretical terms. They are the participants' own voices compressed into headings. Where a label risks oversimplifying what participants said, I note that in the interpretation. The aim throughout is to let the participants' words do the work and to treat the headings as conveniences, not as the finding itself.

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Table 1. Summary of Themes, Participant Groups, Data Sources, and Representative Quotations

Theme	Participant Group	Data Source(s)	Representative Quotation
"Following where each step is even coming from"	van Hiele group	Journals, Interviews	"Before, the proofs were like... I don't know, like magic. You just see the answer. But now I can follow where each step is even coming from." (Ama, journal)
"Later it will connect"	van Hiele group	Journals, Interviews	"Some of the parts were hard, yeah. But I just told myself, later it will connect. And true true, it connected." (Ama, journal)
"As we were going, going"	van Hiele group	Journals, Interviews	"In the beginning the activities were confusing me, honestly. But as we were going, going, the thing started to make sense small." (Esi, interview)
"Now I know how I will teach"	van Hiele group	Journals, Interviews	"Now I know how I will teach geometry. I've experienced a method that makes sense." (Nana, interview)
Fragile grasp of proofs	Traditional group	Journals, Interviews	"I could follow the steps, but I didn't really know why we were doing them. I just tried to memorise." (Kwesi, journal)
Disengagement when lost	Traditional group	Journals, Interviews	"Once I got lost at the beginning, the rest of the lesson didn't make sense to me." (Kojo, interview)
No model for teaching	Traditional group	Journals, Interviews	"I might teach the same way I was taught because I don't know another way." (Kojo, interview)

3.1. Results

Theme 1: "Following where each step is even coming from"

Participants across both instructional contexts described Euclidean geometry as cognitively demanding. However, their narratives revealed fundamentally different ways of engaging with geometric concepts and proofs. The phrase that captured this difference came from Ama, a participant in the van Hiele group. She wrote in her journal:

"*Before, the proofs were like... I don't know, like magic. You just see the answer. However, now I can follow where each step is even coming from*."

Other participants in the van Hiele group described similar experiences. Akosua reflected in her journal:

"*This time around I think I understood why the theorem is working before we even proved it –which never used to happen. The activities, they made it clearer for me somehow*."

Yaw similarly explained during his interview:

"*The phases, eh, they kind of prepared my mind first. So by the time we get to the proof, I'm not seeing it for the first time. My mind has already passed through it small small*."

What these accounts share is a shift from seeing proofs as magic or as steps to memorise toward seeing them as something they could follow, anticipate, and partly reconstruct on their own. I call this pattern "following where each step is even coming from" because that is what Ama said. It is not the same as full mastery. Participants still found geometry hard. However, they stopped treating proofs as external authority. They started to see themselves as people who could trace the logic.

Two other van Hiele participants, Adjei and Efua, also contributed to this theme. Adjei noted in his interview:

"*Before, I would just copy proofs from the board. Now, when I see a proof, I can predict the next line sometimes. That never happened to me before*."

Efua wrote in her journal after the third lesson:

"*I didn't understand everything in the lesson today. But I understood the structure. I knew what we were trying to do even if I couldn't do all of it myself*."

These accounts suggest that for van Hiele participants, understanding was not all-or-nothing. Even when they could not fully reconstruct a proof, they had a sense of its direction and purpose. This was something none of the traditional group participants described.

In contrast, participants taught through traditional instructional approaches described a different relationship with proofs. Dede noted in her interview:

"*Sometimes the teacher just started with the proof, and I didn't understand where the ideas came from*."

Kwesi similarly wrote in his journal:

"*I could follow the steps, but I didn't really know why we were doing them. I just tried to memorise*."

Kojo added during his interview:

"*If I miss one step, the whole proof is lost for me*."

These participants could reproduce proofs when asked. However, their language reveals a fragile grasp. One missed step and everything falls apart. They did not describe following where steps come from. They described managing steps.

Abena, another traditional group participant, captured this in her journal entry:

"*I can write the proof but I don't understand what it means. When the teacher explains, it makes sense. When I close my book, it disappears*."

This sense of knowledge that evaporates when not immediately prompted was something that appeared repeatedly in the traditional group's data but was almost entirely absent from the van Hiele participants' accounts.

Theme 2: "Later it will connect"

Participants' responses to difficulty in Euclidean geometry were shaped not only by the complexity of content but also by whether they believed the lessons would eventually make sense. The phrase that best captures this belief came from Ama:

"*Some of the parts were hard, yeah. But I just told myself, later it will connect. And true true, it connected*."

Other participants in the van Hiele group said similar things. Esi reflected in her interview:

"*In the beginning the activities were confusing me, honestly. But as we were going, going, the thing started to make sense small*."

Yaw added in his journal:

"*Even when I didn't understand immediately, I knew we would come back to it*."

Kofi, another van Hiele participant, wrote in his journal:

"*The first time we did the folding activity, I didn't see why we were doing it. But by the third week, I started to see the connection. The activities were building on each other*."

What these accounts show is a kind of trust. Participants did not understand everything right away. But they believed the instruction was designed in a way that confusion would be temporary. This belief allowed them to stay engaged when things got hard. Difficulty became productive rather than discouraging.

It is important to be careful here. Trust of this kind is not an unqualified good. A learner who assumes everything will eventually cohere might stop asking questions and just wait to be carried along. One or two of the van Hiele journals hint at exactly this. The difficulty was tolerated rather than genuinely worked through. For example, one van Hiele participant, whose pseudonym is Ato, wrote in his journal:

"*I just followed what we were doing. Sometimes I didn't understand, but I knew the teacher would explain later*."

This sounds positive on the surface. However, reading it closely, I wondered whether Ato was actively working through his confusion or simply waiting for the teacher to resolve it. His journal entries throughout the study were brief and lacked the kind of detailed reflection that other van Hiele participants offered. I think this is worth noting because it suggests that the van Hiele phases alone do not guarantee deep engagement. Some students, even in a well-designed instructional sequence, may still slip into passive compliance.

Nor did the contrast fall cleanly along group lines. Kwesi, who was taught traditionally, could at times articulate the logic of a proof more sharply than some participants in the experimental group. In his interview, he said:

"*I know that when two parallel lines are cut by a transversal, the alternate interior angles are equal. I understand why—it's because the corresponding angles are equal first. But when the proof has more than three or four steps, I lose track*."

This was a more sophisticated explanation than what some van Hiele participants offered. I had not expected this. It complicates the story. It suggests that a few students in the traditional group already had some grasp of proof structure, while a few in the van Hiele group were not as engaged as I had hoped. These are not anomalies to explain away. They are reminders that trust is one strand in a messier process.

In contrast, participants who learned through traditional methods often felt lost when they faced difficulty. Kojo stated in his interview:

"*Once I got lost at the beginning, the rest of the lesson didn't make sense to me*."

Abena noted in her journal:

"*When the proof became tough, ah! I just stopped following*."

Dede added in her interview:

"*Sometimes I didn't even know what the proof was trying to show*."

Here, difficulty did not lead to persistence but rather to disengagement. These participants did not have the same belief that things would eventually connect. Without that belief, confusion became a dead end rather than a temporary state.

Theme 3: "As we were going, going"

Participants' reflections reveal that learning Euclidean geometry unfolded gradually. It was not a single moment of understanding but a slow accumulation. The phrase that captures this came from Esi, who said in her interview:

"*Geometry used to scare me, especially proofs*."

Later in the same interview, she reflected:

"*Now I see proofs as something that explains, not something to fear*."

Yaw similarly noted in his journal:

"*At first I was confused, but as the phases continued, my confidence increased*."

Adwoa, another van Hiele participant, wrote in her journal after the fourth lesson:

"*I don't think I could teach this yet. But I understand more than I did three weeks ago. Each lesson adds something*."

These trajectories were not smooth. Participants still had hard days. But over time, the direction was toward greater confidence and clearer understanding. The van Hiele phases seemed to provide a rhythm that allowed this gradual change to happen. Participants were not rushed to prove before they had explored. They were given time.

In contrast, participants in traditional instructional contexts showed limited evidence of this kind of progression. Kwesi wrote late in the semester:

"*Geometry still feels like memorising many things*."

Dede similarly reflected in her interview:

"*I don't think my understanding really changed*."

Kojo, in his final journal entry, wrote:

"*I know more formulas now. But I don't think I understand geometry better*."

For these participants, time passed, but learning did not accumulate. Geometry remained something to manage rather than understand.

Theme 4: "Now I know how I will teach"

As pre-service teachers, participants frequently connected their learning experiences to their future classrooms. The phrase that best captures this came from Nana in her interview:

"*Now I know how I will teach geometry. I've experienced a method that makes sense*."

Ama similarly reflected in her journal:

"*The way we learned showed me that geometry doesn't have to be scary*."

Yaw added in his interview:

"*Before this course, I didn't know how I would teach geometry. I only knew how I was taught. Now I have an example I can follow*."

What these reflections show is that learning experiences function as templates. Participants who experienced geometry as a structured, sense-making process could imagine themselves teaching that way. They had a model to work from.

Participants from traditional instructional contexts expressed a very different relationship to future teaching. Abena stated in her interview:

"I want to teach better, but I'm not sure how. Geometry still feels complicated to me."

Kojo added in his final interview:

"*As for me, hmmm... I might teach the same way I was taught because I don't know another way*."

Kwesi, in his interview, echoed this sentiment:

"*I know I don't want to teach the way I was taught. But I don't know what else to do. So maybe I will just teach the same way*."

Here, the absence of a positive model left participants stuck. They wanted to teach differently but could not picture what that would look like. Their own learning had not shown them another path.

3.2. Discussion

This study set out to understand how pre-service teachers experience learning Euclidean geometry under different instructional conditions. The findings reveal patterns that are not simply about whether one instructional approach produced better outcomes than another. They are about the *nature* of the learning that occurred, that is, how participants made sense of proofs, how they responded to difficulty, and how they imagined themselves as future teachers. In this discussion, I interpret these patterns in relation to existing literature and consider their implications for mathematics teacher education, particularly in contexts like Ghana.

What the Findings Reveal About Learning Geometry for Teaching

The clearest difference between the two groups was not about test scores. I did not measure test scores in this study. The difference was about whether participants felt they understood where proofs came from. Van Hiele participants like Ama described proofs as something they could now follow, anticipate, and partly reconstruct. Traditional participants like Kojo described managing steps without grasping the underlying logic. This distinction echoes what Ball et al. [11] argued about mathematical knowledge for teaching. Knowing a proof well enough to reproduce it is not the same as knowing it well enough to explain it to someone else. The van Hiele participants' accounts suggest a shift toward the kind of usable knowledge that Ball et al. [11] described. They did not just memorise proofs; they learned the logical path. That is what they will need to explain to their future students.

There is something else here that connects to Schoenfeld's [13] work on sense-making. Van Hiele participants described their learning as gradual and cumulative. Esi said, "as we were going, going, the thing started to make sense small." Yaw said his confidence increased as the phases continued. These are not just descriptions of time passing. They point to a process of constructing coherence among ideas. This is what Schoenfeld means by sense-making: the active work of building connections so that mathematics feels coherent

rather than fragmented. The van Hiele phases seemed to provide a rhythm that allowed this sense-making to happen. Participants were not rushed to proof before they had explored. They had time to notice, discuss, and articulate ideas before formalization. Traditional participants did not describe this kind of progression. Time passed, but learning did not accumulate. Geometry remained something to manage rather than understand.

This also connects to the van Hiele theory itself. Van Hiele [5] proposed that learners progress through levels of geometric thinking and that instruction can support this progression through five phases. The findings of this study align with that claim. Van Hiele participants described moving from seeing proofs as "magic" to being able to follow the "logic". That is a shift from a lower level of reasoning to a higher one. However, the study also reveals something the van Hiele theory does not fully address, which is the role of trust in getting through difficulty. Participants who believed the lessons would eventually make sense were more willing to stay engaged when geometry became hard. This belief was not automatic. It came from experiencing the van Hiele phases as coherent, with each phase building on the previous one. This suggests that the van Hiele phases do more than structure content; they also structure learners' expectations about what learning will feel like. When learners can see where instruction is heading, they are more likely to persist through confusion. When they cannot see the path, confusion becomes a dead end.

Negative Cases and What They Tell Us

I did not go into this analysis expecting to find as much messiness as I did. A few participants in the traditional group could articulate the logic of proofs more sharply than some participants in the van Hiele group. Kwesi was one of them. He struggled overall, but in his interview, he said something sharp about why a particular angle relationship held. That did not fit the pattern. I also expected van Hiele participants to report uniformly positive trajectories. They did not. Ato, for instance, described tolerating difficulty rather than working through it. His journal entries were brief and showed little evidence of active struggle or reflection. He waited to be carried along by the phases rather than engaging deeply.

These negative cases matter. They remind us that instructional design is not destiny. The van Hiele phases alone do not guarantee deep engagement. Some students, even in a well-designed instructional sequence, may still slip into passive compliance. Moreover, some students, even in a traditional lecture format, may already have some grasp of proof structure. This complicates the story. It suggests that individual differences, including prior knowledge, motivation, and disposition toward reflection, also shape learning outcomes. These differences should not be ignored in the rush to declare one instructional approach superior to another.

This is one reason the study is based on two different colleges, not two groups within the same college. That decision was practical. It was not feasible to split one cohort into two instructional groups without contamination. However, it also introduces complexity. The two colleges had different institutional cultures, different schedules, and different student populations. The differences in learning experiences between the two groups cannot be

attributed solely to the instructional approach. Some of what participants reported may reflect broader differences between the two institutions.

That said, the patterns in the data are consistent enough to be meaningful. Van Hiele participants across the board described a different relationship with geometry than traditional participants. The contrast was not absolute; Kwesi's case shows that, but it was systematic. This suggests that the instructional approach made a difference, even if it was not the only factor at play.

Why Reflective Accounts Matter

This study used reflective journals and interviews to capture participants' experiences. I chose these methods because they allow participants to speak for themselves. They also help to reveal things that test scores miss; the confusion, the anxiety, the small moments of breakthrough, the gradual growth in confidence. This is consistent with what Deslis [22] and Källberg and Roos [23] have argued about the importance of attending to learners' experiences. Outcome-focused research hides a lot. Reflective methods bring those hidden dimensions into view.

The journals were particularly useful in showing how understanding developed over time. They showed that learning geometry was not a single moment of insight but a slow accumulation. The van Hiele phases provided a rhythm that allowed this gradual change to happen. Participants were not rushed. They had time to explore, to get confused, and to recover from confusion. This is something that outcome studies cannot capture. A pre-test and post-test can show whether scores went up, but they cannot show whether understanding deepened gradually or whether students just got better at test-taking.

Implications for Mathematics Teacher Education

The findings of this study have practical implications for how geometry is taught to pre-service teachers, particularly in contexts like Ghana where traditional lecture-based instruction remains dominant.

First, the study suggests that pre-service teachers benefit from instructional sequences that allow them to explore before formalization. The van Hiele phases, namely, information, guided orientation, explicitation, free orientation, and integration, provided a structure that allowed participants to encounter ideas multiple times before being asked to produce formal proofs. This seems to have helped them develop a sense of where proofs come from, rather than just memorizing steps. Teacher educators might consider adopting similar sequences in their own geometry instruction.

Second, the study highlights the importance of helping pre-service teachers develop trust in the instructional process. Van Hiele participants were more willing to stay engaged through difficulty because they believed the lessons would eventually make sense. This belief was not automatic; it came from experiencing the phases as coherent. Teacher educators can foster this trust by making the structure of instruction visible and by signaling where lessons are heading.

Third, the study underscores the value of reflective practice. Participants who kept journals described their learning in ways that showed growth over time. Reflective methods

seem to support sense-making by giving learners space to articulate what they are experiencing. Teacher education programmes might consider incorporating structured reflection as a regular part of geometry instruction, not just as an add-on but as a core component of learning.

Fourth, the study suggests that teacher educators need to be attentive to individual differences. Not all van Hiele participants showed deep engagement, and not all traditional participants showed shallow understanding. Some students may need additional support to engage actively with the material, even in a well-designed instructional sequence. Others may already have a grasp of proof structure that can be built upon. Differentiated instruction, within the constraints of large classes, is worth considering.

Finally, the study has implications for how we think about teacher identity. Van Hiele participants described leaving the course with a clearer sense of how they would teach geometry. Traditional participants described wanting to teach better but not knowing how. This suggests that learning experiences function as templates. When pre-service teachers experience coherent, sense-making instruction, they leave with a model they can try to reproduce. When they do not, they are left with nothing but the default template they already had, which is often the one they experienced as students. Teacher educators who want to change how geometry is taught in schools need to change how it is taught to teachers.

How This Study Extends the Literature

This study adds to the existing literature in three ways. First, it extends the van Hiele theory by showing how the phases work not just as a sequence of activities but as a structure that shapes learners' expectations and trust. The van Hiele theory has been used primarily to describe levels of geometric thinking and to design instruction that supports progression through those levels. This study suggests that the phases also have an affective dimension. They influence how learners respond to difficulty and whether they persist when confused. This is not something the van Hiele theory addresses explicitly, but it matters for how instruction is implemented in practice.

Second, the study extends Ball et al.'s [11] concept of mathematical knowledge for teaching by providing concrete examples of what that knowledge looks like in geometry. Ama's description of being able to follow where each step comes from is a specific instance of the kind of usable knowledge that Ball et al. [11] described. It is not just knowing a proof; it is knowing the logical path in a way that can be explained to someone else. This is what makes knowledge usable for teaching.

Third, the study adds to the literature on mathematics identity by showing how learning experiences shape how pre-service teachers see themselves as future teachers. Graven and Heyd-Metzuyanim [29] and Langer-Osuna and Esmonde [30] have written about how learners' self-perceptions in relation to mathematics matter for their future choices. This study provides evidence from a specific context, that is, geometry teacher education, showing that positive learning experiences can strengthen pre-service teachers' confidence and their sense of themselves as capable teachers.

Practical Implications for Geometry Methods Courses

For mathematics teacher educators designing geometry methods courses, the findings suggest several concrete strategies. First, consider structuring geometry instruction around the van Hiele phases. This does not mean following the phases rigidly. However, it means giving pre-service teachers time to explore before formal proof. Let them fold, measure, discuss, and notice. Let them articulate ideas in their own words before introducing formal terminology. This sequence seems to support sense-making in ways that traditional lecture-based instruction does not.

Second, embed reflective practice into the course. Ask pre-service teachers to keep journals and to write about what they are experiencing. The journals in this study were not just data collection tools; they were also learning tools. Participants who wrote regularly described their understanding and confidence growing over time. Reflective writing seems to support the sense-making process.

Third, make the structure of instruction visible. Let pre-service teachers know where the course is heading and why certain activities are included. This can help them develop the kind of trust that allows them to stay engaged through difficulty. When learners understand why they are doing a particular activity, they are more likely to take it seriously.

Fourth, anticipate individual differences. Not all pre-service teachers will engage deeply with the material, even in a well-designed course. Some may need additional support to move beyond passive compliance. Others may already have a grasp of proof structure that can be built upon. Differentiated approaches, within the constraints of large classes, are worth considering.

Limitations in Context

It is important to acknowledge that the findings of this study are situated within a specific context. The study was conducted with a small number of pre-service teachers from two Colleges of Education in Ghana. The findings may not transfer directly to other contexts. However, the aim of the study was analytic generalisation, not statistical. The findings are meant to be useful for readers who can judge their relevance based on shared context.

Another limitation is that the study relied on self-report. I did not observe participants teaching. I do not know whether Nana, who said she now knows how to teach geometry, will actually teach that way in a real classroom. That is a question for longitudinal research. I also did not analyse participants' written geometric work. It is possible that some participants who described understanding still made errors on proofs. These limitations do not invalidate the findings, but they should temper any strong claims.

Finally, the reliance on two different colleges introduces the possibility that factors other than the instructional approach, including institutional culture, instructor personality, and student demographics, shaped participants' experiences. I have tried to account for this by emphasising patterns that were consistent across participants and by acknowledging negative cases. However, I cannot rule out the influence of context. Future research might replicate this study with groups within the same institution or across a larger number of institutions.

4. CONCLUSION

This study set out to understand what test scores cannot capture: the lived experience of learning geometry for teaching. The findings show that instructional design leaves a lasting mark. Pre-service teachers who learned geometry through van Hiele phase-based instruction described following where proofs came from, trusting that difficulty would eventually make sense, growing gradually over time, and leaving with a clear teaching model. Those who learned through traditional instruction described memorisation without understanding, disengagement when lost, and uncertainty about how they would teach.

The study does not claim that van Hiele instruction is a perfect solution. Some participants waited passively rather than engaged actively. A few traditionally taught participants could articulate proof logic more clearly than some in the van Hiele group. The picture is more complicated than a simple "van Hiele works, traditional does not" narrative. This complexity matters. It reminds us that instructional design shapes learning outcomes but does not determine them entirely. Individual differences, including prior knowledge, motivation, and disposition toward reflection, also matter.

Nevertheless, the pattern across the data is consistent enough to draw a practical conclusion. When geometry instruction makes reasoning visible and proceeds step by step, more pre-service teachers develop the kind of understanding they will need to teach the subject. When instruction hides the reasoning, more pre-service teachers leave with fragile knowledge and constrained ideas about teaching. This is not just an academic finding. It has direct implications for how geometry is taught in teacher education programmes. The current study shows that how geometry is taught to pre-service teachers matters not just for their content knowledge but for their reasoning, persistence, and teaching identity. This offers a practical framework for teacher education reform.

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REFERENCES

[1] P. Akayuure, R. A. Oppong, D. A. Addo, and D. O. Yeboah, "Geometric thinking behaviours of undergraduates on-entry and at-exit of online geometry course," *Science Journal of Education*, vol. 10, no. 5, pp. 155-163, Oct. 2022. doi: 10.11648/j.sjedu.20221005.12.

[2] E. Asemani, S. K. Asiedu-Addo, and R. A. Oppong, "The Geometric Thinking Levels of Senior High School students in Ghana," *International Journal of Mathematics and Statistics Studies*, vol. 5, no. 3, pp. 1-8, June 2017.

[3] R. B. Armah and P. S. Kissi, "Use of the van Hiele Theory in Investigating Teaching Strategies used by College of Education Geometry Tutors," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 15, no. 4, em1694, Apr. 2019. doi: 10.29333/ejmste/103562.

[4] S. A. Tachie, "The challenges of South African teachers in teaching Euclidean geometry," *International Journal of Learning, Teaching and Educational Research*, vol. 19, no. 8, pp. 297-312, Aug. 2020. doi: 10.26803/ijlter.19.8.16.

[5] P. M. van Hiele, *Structure and Insight: A Theory of Mathematics Education*. Orlando, FL, USA: Academic Press, 1986.

[6] J. K. Alex and K. J. Mammen, "Students' understanding of geometry terminology through the lens of van Hiele theory," *Pythagoras*, vol. 39, no. 1, pp. 376-384, Oct. 2018. doi: 10.4102/pythagoras.v39i1.376.

[7] S. Hartono, R. Ramadhona, S. Irawati, D. Frentika, and H. T. N. Rizki, "Enhancing students' van Hiele geometric thinking levels through the integration of Geometer's Sketchpad (GSP) in geometry

learning," *Qubahan Academic Journal*, vol. 5, no. 2, Article a1639, Apr. 2025. doi: 10.48161/qaj.v5n2a1639.

[8] A. Gambini and I. Lénárt, "Basic geometric concepts in the thinking of in-service and pre-service mathematics teachers," *Education Sciences*, vol. 11, no. 7, Article 350, Jul. 2021. doi: 10.3390/educsci11070350.

[9] V. Kumazah and D. D. Agyei, "Enhancing high school geometry learning with inquiry-based teaching: Impact on student understanding, performance, and attitudes," *African Journal of Educational Studies in Mathematics and Sciences*, vol. 21, no. 1, pp. 1-14, Apr. 2025. doi: 10.4314/ajesms.v21i1.1.

[10] E. F. Akosah, F. O. Boateng, J. Arhin, and B. A. Obeng, "Assessing the readiness of pre-service mathematics teachers to learn Euclidean geometry," *International Journal of Didactical Studies*, vol. 5, no. 1, Article 24863, Apr. 2024. doi: 10.33902/ijods.202424863.

[11] D. L. Ball, M. H. Thames, and G. Phelps, "Content knowledge for teaching: What makes it special?" *Journal of Teacher Education*, vol. 59, no. 5, pp. 389-407, Nov. 2008. doi: 10.1177/0022487108324554

[12] S. T. Nazarovich and A. Kurudirek, "An overview of existing problems in teaching the science 'fundamentals of geometry,'" *International Journal of Social Sciences & Educational Studies*, vol. 11, no. 2, pp. 91-106, May 2024. doi: 10.23918/ijsses.v11i2p91.

[13] A. H. Schoenfeld, "On having and using geometric knowledge," in *Conceptual and Procedural Knowledge*. New York, NY, USA: Routledge, 2013, pp. 225-264.

[14] H. Gal and L. Linchevski, "To see or not to see: analyzing difficulties in geometry from the perspective of visual perception," *Educational Studies in Mathematics*, vol. 74, no. 2, pp. 163-183, Feb. 2010. doi: 10.1007/s10649-010-9232-y.

[15] T. M. Awado et al., "Impact of teaching style on perceived mathematics achievement of elementary education pre-service teachers: The mediating roles of attitude and math self-concept," *Discover Education*, vol. 3, Article 287, Dec. 2024. doi: 10.1007/s44217-024-00388-0.

[16] M. L. Crowley, "The van Hiele Model of development of geometric thought," in *Learning and Teaching Geometry*, K-12, 1987 Yearbook, M. M. Lindquist and A. P. Shulte, Eds. Reston, VA, USA: National Council of Teachers of Mathematics, 1987, pp. 1-16.

[17] Z. Usiskin, *van Hiele Levels and Achievement in Secondary School Geometry: Cognitive Development and Achievement in Secondary School Geometry Project*. Chicago, IL, USA: University of Chicago Press, 1982.

[18] M. N. Hassan, A. H. Abdullah, and N. Ismail, "Effects of integrative interventions with van Hiele phase on students' geometric thinking: A systematic review," *Journal of Critical Reviews*, vol. 7, no. 13, pp. 1133-1140, Jul. 2020. doi: 10.31838/jcr.07.13.194.

[19] T. D. Howse and M. E. Howse, "Linking the van Hiele theory to instruction," *Teaching Children Mathematics*, vol. 21, no. 5, pp. 305-313, Dec. 2014.

[20] F. Erdogan, "Prospective middle school mathematics teachers' problem posing abilities in context of van Hiele levels of geometric thinking," *International Online Journal of Educational Sciences*, vol. 12, no. 2, pp. 132-152, Mar. 2020. doi: 10.15345/iojes.2020.02.009.

[21] W. Burger and J. M. Shaughnessy, "Characterizing the van Hiele levels of development in geometry," *Journal for Research in Mathematics Education*, vol. 17, no. 1, pp. 31-48, Jan. 1986.

[22] D. Deslis, "Digital storytelling in teaching and learning mathematics: Synthesizing research on student voices and reflection," *Education Sciences*, vol. 15, no. 11, Article 1548, Nov. 2025. doi: 10.3390/educsci15111548.

[23] P. S. Källberg and H. Roos, "Meaning(s) of a student perspective in mathematics education research," *Educational Studies in Mathematics*, vol. 119, no. 2, pp. 367-392, Jan. 2025. doi: 10.1007/s10649-024-10374-w.

[24] E. Machisi, "Grade 11 students' reflections on their Euclidean geometry learning experiences," *EURASIA Journal of Mathematics, Science and Technology Education*, vol. 17, no. 2, Article em1938, Feb. 2021. doi: 10.29333/ejmste/9672.

[25] S. Chikiwa and M. Graven, "Exploring the development of South African pre-service teachers' reflective practice," *Pythagoras*, vol. 44, no. 1, Article a678, May 2023. doi: 10.4102/pythagoras.v44i1.678.

[26] I. K. Guce, "Investigating college students' views on mathematics learning through reflective journal writing," *International Journal of Evaluation and Research in Education*, vol. 6, no. 1, pp. 38-44, Mar. 2017.

[27] G. Kesebir and C. Oksuz, "The effects of student mathematics diaries on achievement, motivation and metacognitive awareness: mixed methods design," *International Journal for Mathematics Teaching and Learning*, vol. 23, no. 2, pp. 59-83, 2022.

[28] V. Negrea, "Exploring career changers' experiences in a school-based initial teacher education programme for science teachers in England," *International Journal of Educational Research*, vol. 125, Article 102342, 2024. doi: 10.1016/j.ijer.2024.102342.

[29] M. Graven and E. Heyd-Metzuyanim, "Mathematics identity research: The state of the art and future directions," *ZDM*, vol. 51, no. 3, pp. 361-377, Apr. 2019. doi: 10.1007/s11858-019-01050-y.

[30] J. M. Langer-Osuna and I. Esmonde, "Insights and advances on research on identity in mathematics education," in *Compendium for Research in Mathematics Education*, J. Cai, Ed. Reston, VA, USA: National Council of Teachers of Mathematics, 2017, pp. 637-648.

[31] R. B. Armah, P. O. Cofie, and C. A. Okpoti, "Investigating the effect of van Hiele phase-based instruction on pre-service teachers' geometric thinking," *International Journal of Research in Education and Science*, vol. 4, no. 1, pp. 314–330, Jan. 2018. doi 10.21890/ijres.383201

[32] J. W. Creswell and V. L. Plano Clark, *Designing and Conducting Mixed Methods Research, 3rd ed*. Thousand Oaks, CA, USA: Sage Publications, 2017.