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From Learning Obstacles to Conceptual Understanding: Developing GeoGebra-Assisted Teaching Materials for Elementary Fractions through Educational Design Research

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ABSTRACT

Learning fractions remains a significant challenge in elementary education, often hindered by specific learning obstacles. This study aims to develop GeoGebra-assisted teaching materials integrated with a Hypothetical Learning Trajectory (HLT) to bridge the gap between these obstacles and conceptual understanding. Adopting an Educational Design Research (EDR) approach, the study progressed through three iterative phases: analysis and exploration, design and construction, and evaluation and reflection. The development was grounded in an initial pedagogical analysis of student learning obstacles, which informed the HLT design. Data were collected through expert validation, teacher interviews, and field testing with 161 elementary students (divided into small-scale and large-scale trials), then analyzed qualitatively using retrospective analysis and constant comparative methods.The quantitative results indicate that the GeoGebra-based materials are highly valid (92.75% from media experts) and practical (81.75% in large-scale trials), effectively transforming the HLT into a verified learning path. However, qualitative findings revealed a critical nuance: while students engaged enthusiastically with dynamic visualizations, some faced challenges in transitioning from visual area models to formal symbolic notation. This underscores the necessity of teacher-led instrumental orchestration to scaffold mathematical generalizations. This research concludes that the synergy between dynamic geometry software and a structured learning trajectory provides a robust framework for overcoming cognitive barriers, provided it is integrated within a dialogic instructional environment.

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1. INTRODUCTION (12 PT)

The learning process is closely related to the competencies that teachers and prospective teachers must develop. Mastery of broad and deep content knowledge is a fundamental component of professional competence [1], [2]. However, previous studies have shown that many elementary school teachers and prospective mathematics teachers still experience difficulties in understanding mathematical concepts, particularly those requiring relational and conceptual reasoning [3] – [6]. Such limitations may affect their ability to design learning experiences that support meaningful mathematical understanding. Therefore, teachers need instructional materials that are systematically designed to facilitate productive interactions between learners and mathematical concepts.

For elementary school students, conceptual understanding is especially important in foundational topics. One topic that consistently generates learning barriers is fractions. Kilpatrick [7] emphasizes that number concepts form the basis for subsequent mathematical learning, yet fractions remain one of the most challenging topics for students [8]–[10], prospective teachers, and practicing teachers [11], [12]. A major source of difficulty is the tendency to introduce fractions through static and overly simplified representations, which often leads students to perceive fractions as abstract, counterintuitive, and disconnected from whole-number reasoning [13]. As a consequence, students frequently develop epistemological obstacles such as whole-number bias, where whole-number logic is incorrectly applied to fractional situations.

To address these obstacles, mathematics instruction requires teaching materials that not only present concepts accurately but also anticipate students' misconceptions and support conceptual progression [14]. One promising approach is the use of a Hypothetical Learning Trajectory (HLT), which predicts possible student thinking and organizes learning activities in a coherent sequence. When combined with dynamic mathematics software such as GeoGebra, HLT can provide opportunities for students to visualize, manipulate, and compare fractional quantities interactively, thereby making abstract relationships more accessible to elementary learners.

Addressing such epistemological obstacles requires instructional support that not only presents mathematical representations but also guides students through a coherent process of conceptual reconstruction. A Hypothetical Learning Trajectory (HLT) provides a framework for anticipating students' thinking, identifying potential misconceptions, and sequencing learning activities toward increasingly sophisticated understandings. At the same time, dynamic technology can make otherwise invisible relationships between fractional quantities more accessible. GeoGebra, in particular, enables learners to visualize and manipulate fractional units, compare equivalent fractions, and explore partitioning dynamically, thereby supporting conceptual reasoning beyond static textbook representations.

Although previous studies have reported the benefits of GeoGebra in mathematics learning and others have employed HLT to support conceptual development, these approaches have generally been examined separately. Few studies have integrated GeoGebra-based dynamic representations with an HLT framework within an Educational Design Research (EDR) process specifically to address elementary students' epistemological obstacles in learning fractions. Existing research tends to focus either on the effectiveness of technology use or on the design of learning trajectories, without systematically refining both through iterative classroom-based validation. This gap is important because overcoming fraction learning obstacles requires not only technological visualization but also a carefully designed progression of student thinking that is empirically tested in authentic learning contexts.

Accordingly, this study adopts an Educational Design Research approach to develop, refine, and validate a GeoGebra-assisted teaching material grounded in a structured HLT for elementary fraction learning. The novelty of the study lies in the integration of GeoGebra, HLT, and EDR to transform identified epistemological obstacles—particularly whole-number bias and difficulties in understanding fractional relationships—into opportunities for conceptual progression. The purpose of this research is therefore to develop teaching materials that are both valid and practical while providing an empirically grounded learning trajectory that supports elementary students' conceptual understanding of fractions.

2. METHOD

This study employs Educational Design Research (EDR), which focuses on developing solutions to complex educational problems while simultaneously advancing theoretical knowledge through a series of iterative cycles [15]. The study focused on developing, validating, and refining GeoGebra-assisted teaching materials based on a Hypothetical Learning Trajectory (HLT) for elementary students' fraction learning.

The EDR implementation involved two stages of field testing. The small-scale trial involved 31 students from one class and was conducted to identify technical problems, evaluate the clarity of learning activities, and refine the prototype before broader implementation. After revisions were completed, the large-scale trial involved 130 students from four additional fourth-grade classes to evaluate the practicality and classroom implementation of the revised teaching materials. To establish the quality of the instructional prototype, the study also involved two mathematics education experts (content validation), two educational technology experts (media validation), and three experienced elementary mathematics teachers who evaluated the pedagogical feasibility and classroom applicability of the developed materials.

The research followed an iterative cycle consisting of [15]: (1) Analysis and Exploration: Identifying epistemological obstacles in fraction concepts through diagnostic tests and literature review. This phase resulted in the initial HLT. (2) Design and Construction: Developing the GeoGebra-assisted prototype, focusing on dynamic representations to mitigate identified obstacles. (3) Evaluation and Reflection: A formative evaluation involving expert validation and micro-cycle testing. The implementation phase served as a "teaching experiment" to observe student-material interaction [16], [17].

The first phase involved a systematic investigation of the current teaching and learning context regarding fractions in elementary schools. The primary objective was to identify the epistemological obstacles that hinder students' conceptual understanding. Following [14] framework, the researcher conducted a diagnostic survey and preliminary interviews with Phase B students. The findings revealed that students struggled significantly with the concept of partitioning and the density of fractions, often misapplying whole-number logic to fractional parts.

Furthermore, an analysis of existing instructional materials was conducted to identify gaps that could be addressed through technology. This exploration served as the basis for developing the Initial Hypothetical Learning Trajectory (HLT). According to [16] and [17], an HLT consists of the learning goal, the planned instructional activities, and a prediction of the students' thinking and learning processes. In this study, the HLT was designed to anticipate potential student errors and provide GeoGebra-based scaffolds to transform these obstacles into conceptual milestones. This analytical foundation ensured that the subsequent design of GeoGebra-assisted materials was pedagogically grounded and purpose-driven.

In the second phase, the initial HLT was translated into a series of interactive GeoGebra-assisted tasks. The design focused on addressing the specific obstacles identified earlier, such as the 'whole-number bias' in fractions. Using an iterative process, a prototype was developed and subjected to expert judgment. Validators assessed the materials based on content validity, pedagogical alignment with the HLT, and technical usability. Feedback from experts led to the refinement of the GeoGebra applets, particularly in simplifying the dragging functions to ensure students remained focused on mathematical properties rather than technical manipulation.

The final phase involved a teaching experiment to evaluate the effectiveness of the design in a real classroom setting. Data from observations, field notes, and student interviews were analyzed through retrospective analysis [18]. This process involved a constant comparison between the predicted learning path (HLT) and the actual learning process observed. The reflection phase focused on identifying how the dynamic features of GeoGebra facilitated the transition from fragmented knowledge to conceptual understanding. The result of this phase was a refined learning trajectory and a set of teaching materials proven to be practical and effective for elementary fraction instruction. The instruments used in this research are validation sheets and learner response questionnaires which are complemented by observations, field notes, and interviews. The observation focused on: student engagement, interaction with GeoGebra, participation in discussions, occurrence of learning obstacles, teacher facilitation. Semi - structured interviews explored: students' reasoning, conceptual changes, difficulties encountered, perceptions of GeoGebra, effectiveness of learning activities. Field notes documented classroom events, unexpected learning behaviors, implementation fidelity, and design revisions throughout the teaching experiments.

Researchers developed a prototype which was then validated by media experts, material experts, and elementary school teachers. The prototype was refined based on the validation results. Next, the implementation process was carried out. In the practice of learning in the classroom, researchers collected qualitative data with observation instruments, field notes, and interviews. The data collection process can be illustrated in the figure 1.

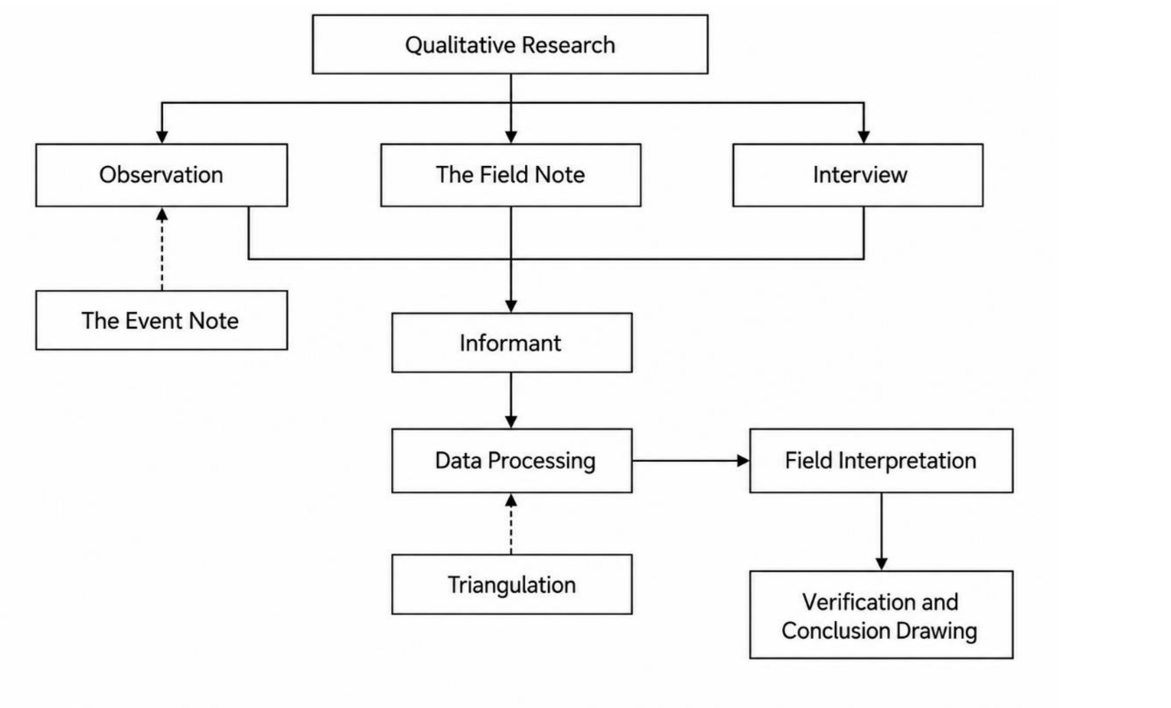

Figure 1. Qualitative Data Collection Process Chart

Data Analysis

Data analysis was obtained from inductive and deductive analysis methods. Before being analyzed, the data was first tested for validity using triangulation techniques [19] analysis of the validity of teaching materials obtained from validation questionnaire data filled in by experts. The results of this analysis are in the form of a percentage calculated using the formula seen in Formula 1, and the percentage results can be interpreted using the product validation category Table 1 [20].

$$V_a = \frac{T_{se}}{T_{sh}} \times 100\% \qquad 1)$$

Table 1. Product Validity Category

Percentage	Category
$80\% < Va \leq 100\%$	Very valid, can be used
$60\% < Va \leq 80\%$	Valid, usable
$40\% < Va \leq 60\%$	Moderately valid, can be used with minor improvements
$20\% < Va \leq 40\%$	Not valid, recommended not to use
$Va \leq 20\%$	Invalid, cannot be used

Furthermore, we can obtain practicality data from the scores of student response questionnaires that have been distributed after learning by applying the teaching materials that have been developed. To calculate the average score for each indicator using the following Formula 2:

$$P = \frac{\sum f}{N} \times 100\% \qquad 2)$$

And the results of the percentages can be interpreted by means of the table of the categories. [20].

Table 2. Category of Product Practicability

Percentage	Category
80% < P ≤ 100%	Very practical
60% < P ≤ 80%	Practical
40% < P ≤ 60%	Moderately practical
20% < P ≤ 40%	Less practical
P ≤ 20%	Not practical

Qualitative data from observations, interviews, and field notes were analyzed using retrospective analysis [18]. The analysis consisted of four stages: organizing all qualitative data according to each EDR cycle; coding students' conceptual difficulties, learning behaviors, and interactions with GeoGebra; comparing the observed learning process with the predicted learning trajectory (constant comparison); identifying revisions required for both the HLT and the instructional prototype. To enhance trustworthiness, data triangulation was conducted by comparing findings from diagnostic tests, observations, interviews, and field notes [19].

3. RESULTS AND DISCUSSION

This study produced GeoGebra-assisted teaching materials developed through the iterative cycles of Educational Design Research (EDR). Unlike conventional instructional development, this design is anchored in a Hypothetical Learning Trajectory (HLT), which serves as a pedagogical roadmap for anticipating student responses throughout the learning process [16].

The initial phase revealed profound epistemological obstacles among fourth-grade students, particularly regarding partitioning concepts and the nature of denominators. Field findings indicated a prevalent whole-number bias, where students perceived a fraction's value as increasing in tandem with the magnitude of the denominator. Furthermore, curriculum analysis showed that a reliance on static media hindered students from visualizing dynamic unit partitioning. Consistent with Brousseau’s [14] framework, these obstacles are inherent to the mathematical concepts but can be mitigated through carefully designed didactical situations in this study, realized through exploratory scenarios within GeoGebra.

Based on the identified obstacles, the researcher developed an instructional design integrating a Concept Map and the Mathematics Iceberg. This structure ensured a seamless transition from situational (contextual) problems to formal mathematics through GeoGebra’s schematic representations. The media design focused on dynamic partitioning features, allowing students to manipulate divisors in real-time. According to Fitriani, Darhim, and Juandi [21], the role of technology in this stage is to strengthen concept embedding through visualization, enabling students to conduct mental experiments before reaching formal mathematical conclusions.

The first stage in teaching material development research is analysis. At this stage, the researcher concluded that there were 2 steps, namely curriculum analysis and learner analysis. Curriculum analysis begins with analyzing school learning outcomes. Next is the analysis of learning obstacles. learning obstacles that arise show that there is still a lack of accuracy in providing the concept of fraction material in elementary schools. This was followed by an analysis of mathematics learning in one of the Elementary schools in Bandung. One of the important points obtained is: limited learning resources for fractions used in learning. Learner analysis showed that elementary school students who are still at the concrete operational thinking stage need learning that starts from contextual problems and need interactive media to strengthen the visualisation of a concept. Previous research states that one of the didactical functions of involving technology in learning is embedding concepts by strengthening the visualisation provided [21].

The next stage is the design or design of teaching materials. The design of teaching materials begins with the preparation of concept maps. The concept map is prepared to better understand the prerequisite material that students should already have, to the main material and developer material that must be mastered. Furthermore, the iceberg of mathematics learning and hypothetical learning trajectory were also developed. Based on these things, the next step is to design teaching materials. Researchers sorted the teaching materials and learning media aids according to curriculum metrics and objectives. The purpose of the material is designed to make it easier for students to understand the learning material taught. Therefore, the presentation of the material is designed in such a way that the material presented does not look complicated and more interesting. To make it more interesting in learning, researchers use Geogebra software which is adjusted to the learning material, namely fractions. In addition, at this stage, the researcher made a validation sheet to assess the adequacy of the development of geogebra-assisted teaching materials. Validation data from subject matter experts, media specialists, and practitioners categorized the teaching materials as "Highly Valid" and "Highly Practical." However, the core of this phase was the Retrospective Analysis [18], where the researcher compared the HLT with the students' actual learning paths in the classroom.

Observation data demonstrated that GeoGebra’s interactive visualization significantly aided students in reconstructing their understanding of fractional units. Students who initially struggled with equivalent fractions showed marked progress when utilizing the "overlapping area" feature in GeoGebra to verify that different symbols could represent the same value. This reflection indicates that technology integration, guided by a robust learning trajectory, can successfully transform learning obstacles into milestones of solid conceptual understanding.

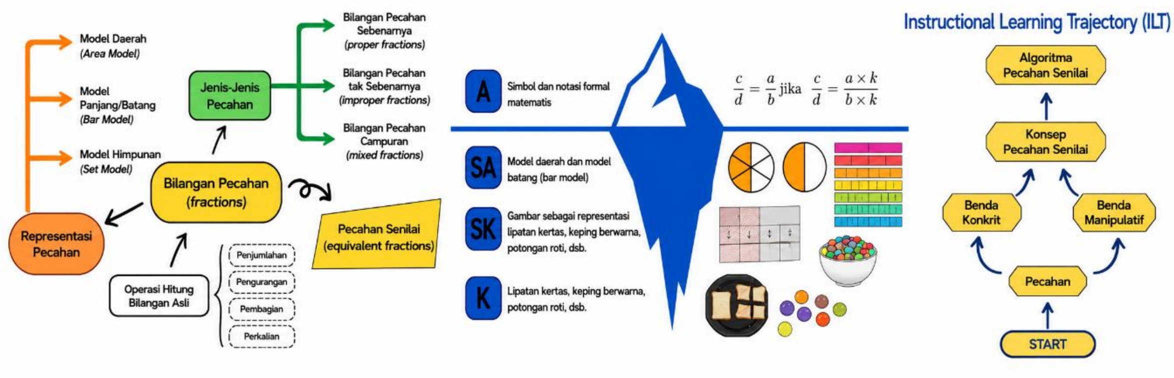

Figure 2. Design of Teaching Materials

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Figure 2 presents the structured design of the Geogebra-assisted teaching materials. The Concept Map outlines the essential components of fraction learning, ensuring a structured approach in material sequencing. And then, The Iceberg Model illustrates the progression from concrete to abstract understanding, highlighting the importance of visualization in mathematics learning. Meanwhile, the Instructional Learning Trajectory (ILT) provides a systematic roadmap guiding students from real-world objects to abstract fraction algorithms. These interconnected components ensure that students grasp fraction concepts effectively, in alignment with their cognitive development stages.

After designing the next teaching materials enter the development stage or development. The teaching materials to be developed are geogebra-assisted teaching materials on fraction material. Researchers modify geogebra software so that students better understand the material presented, as for some parts contained in the learning media in geogebra software as follows:

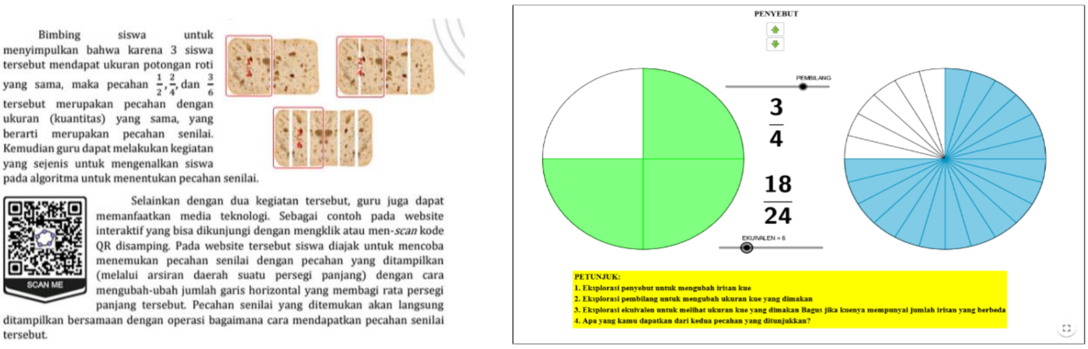

Figure 3. Geogebra-assisted teaching material design

After completing designing the development of teaching materials and before this teaching material is tested on a small scale, researchers conduct a validation assessment of the teaching materials. This validation assessment was carried out by media experts, material experts, and mathematics teachers. The evaluation received from media experts, material experts, and mathematics teachers aims to correct deficiencies in the teaching materials developed.

Validation was conducted by media experts, namely to determine the feasibility of teaching materials assisted by geogebra software. In this case, the aspects that will be assessed by media expert validators are content validation, presentation component validation, graphic component validation, and language validation. The results of the assessment of media experts are presented in Table 3.

Table 3 Media Expert Validation Results

No	Aspects	Score	Percentage	Category
1.	Content Validation	3.60	90%	Very valid
2.	Presentation Component Validation	3.75	93.75%	Very valid
3.	Graphic Component Validation	3.70	92.5%	Very valid
4.	Linguistic Validation	3.80	95%	Very valid
Total score		3.71	92.75%	Very valid

From the table 3 above, it can be seen that the content validation aspect obtained a percentage of 90%, a percentage of 93.75% for the presentation component, the graphic component 92.5%, and the linguistic validation 95%. A percentage value of 92.75% with a very valid category from media experts. The high validation scores from media experts indicate that the teaching materials are well-structured in terms of readability, visual appeal, and language clarity. The linguistic validation aspect received the highest score (95%), which suggests that the instructional language is clear and free from ambiguities. However, while the graphic component received a high score (92.5%), similarly, content experts highlighted that while the teaching materials align with the curriculum. These insights demonstrate that, although the materials are highly valid, there is room for further refinement to optimize student engagement and comprehension.

In addition, the content experts also assessed the suitability of the content for geogebra-assisted teaching materials. The assessment aspects include content feasibility, presentation feasibility, and language feasibility. The results of the assessment from content experts are presented in Table 4.

Table 4. Content Expert Validation Results

No	Aspects	Score	Percentage	Category
1.	Content Validation	3.25	81.25%	Very valid
2.	Presentation Component Validation	3.6	90%	Very valid
3.	Linguistic Validation	3.5	87.5%	Very valid
Total score		3.45	86.25%	Very valid

The table above shows that the content validation aspect obtained a percentage of 81.25%, while the presentation and linguistic feasibility aspects obtained a percentage of 90% dan 87.5%. The results of the percentage value given by the content expert is 86.25% with a very valid category. In addition, trials were also conducted on elementary school mathematics teachers to determine the feasibility of fraction material in geogebra-assisted teaching materials. The aspects to be assessed are user-friendliness, presentation attractiveness, and benefits. The assessment results of the validation are presented in Table 5.

Table 5. Maths Teacher Validation Results

No	Aspects	Score	Percentage	Category
1.	User-friendliness	3.5	87.25%	Very valid
2.	Attractiveness of Presentation	3.75	93.75%	Very valid
3.	Benefits	3.5	87.5%	Very valid
Total score		3.58	89.5%	Very valid

In Table 5 above, it can be seen that the user-friendliness aspect and the presentation attractiveness aspect obtained a percentage value of 87.25% and 93.75%, and 87.5% for the percentage value of the benefit aspect. The percentage value obtained from the mathematics teacher is 89.5% in the very valid category. Based on the results of the recapitulation of the validation score, a value of 89.25% was obtained with a very valid category, this can be interpreted that all aspects assessed are in accordance with the learning activities on fraction material. So it can be concluded that geogebra-assisted teaching materials are suitable for

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use by students even though there is little input from validators. Media experts, materials and teachers provide input regarding this geogebra-assisted teaching material, namely, the illustrations used need to be more contextual by the characteristics of students and the problems given also need to be contextual.

After completing the validation process and having been corrected in accordance with the input that has been given, then a small-scale test and a large-scale test are carried out to evaluate the usefulness of the teaching materials developed. This evaluation was carried out by distributing response questionnaires that were corroborated with field notes and observation and documentation sheets to several students to find out their responses to the teaching materials developed. In this study, the developed teaching materials were applied in five of the elementary schools in, Bandung City with 63 learners participating for the small-scale trial and 98 learners for the large-scale trial. The following is a recap of the results of students' responses to the small and large-scale tests.

Table 6. Recap of Learner Response Results on Small and Large-Scale Trials

No	Aspects	Small-Scale Trials			Large-Scale Trials		
		Score	Percentage	Category	Score	Percentage	Category
1.	User-friendliness	3	75%	Practical	3.15	78.75%	Practical
2.	Attractiveness of Presentation	3.25	81.25%	Very practical	3.25	81.25%	Very practical
3.	Benefits	3.5	87.5%	Very practical	3.4	85%	Very practical
Total score		3.25	81.25%	Very practical	3.27	81.75%	Very practical

As seen in the table above, the results of the total score in the small-scale trial obtained a percentage of 81.25% with a very practical category, while the results of the total score in the large-scale trial obtained a percentage of 81.75% with a very practical category. The integration of GeoGebra in this study was designed to bridge the gap between concrete experiences and abstract fraction concepts, following the Iceberg Model. Quantitative results from the large-scale trial indicated a high level of practicality (81.75%). However, qualitative findings revealed a critical nuance: while students engaged enthusiastically with the dynamic visualizations, some struggled to translate these visuals into formal symbolic notation.

Although both small-scale and large-scale trials resulted in a 'very practical' category, there were slight differences in student responses. The large-scale trial showed a marginal increase in the total percentage (81.75% compared to 81.25%), indicating a more consistent acceptance of the teaching materials when implemented in a larger group. However, a detailed review of student feedback revealed that while most students found the Geogebra-assisted materials engaging and easy to use, some still faced challenges in understanding certain fraction concepts, particularly in transitioning from visual representations to symbolic notation. Additionally, field notes indicated that a few students required additional scaffolding, such as guided problem-solving sessions, to fully grasp the concepts. These findings highlight the need for further refinement of instructional strategies to support students with varying levels of mathematical understanding.

For the next stage, namely the evaluation stage, which in this case will be carried out a review of the evaluation of responses from media and material experts, teachers, and students. Based on the assessment of Geogebra-assisted teaching materials, the following information was found. Evaluation results are summarised qualitatively starting from the validation process to the trials carried out. Based on the evaluation results, it was found that geogebra-assisted teaching materials can be used in learning, to help students in constructing their knowledge.

This research tries to develop Geogebra-based teaching materials on fraction material. This research is based on the results of research which states that effective learning requires teachers and prospective teachers to have knowledge in terms of mastery of mathematical content knowledge, pedagogical knowledge, and the ability to understand students as learners [22]. One form of being able to see these four things is by compiling effective learning resources according to the characteristics of the material and students. the involvement of technology or in this context is geogebra didactically, technology can help students learn mathematics, especially to encourage conceptual development [23], [24]. Geogebra is software that can be used in learning mathematics for all levels. Geogebra was chosen because it is considered quite effective in strengthening students' understanding of the material studied with effective visualisation given to students. This finding aligns with Duval's theory on semiotic representation, suggesting that access to dynamic visualization does not guarantee conceptual understanding unless explicitly connected to mathematical symbols. While the developed HLT successfully guided students through the initial 'model of' stage (using slice visualizations), the transition to the 'model for' stage (algorithms) required additional teacher scaffolding, as noted in the field notes. This supports Drijvers' [23] argument on 'instrumental orchestration,' where the teacher's role is pivotal in guiding students to synthesize digital observations into mathematical generalizations. Thus, the GeoGebra materials serve not as a replacement for instruction but as a powerful heuristic tool to visualize the otherwise abstract nature of fractions.

The results of data analysis show that Geogebra-assisted teaching materials can be used in mathematics learning to strengthen student understanding. Geogebra-assisted teaching materials developed with interactive media provide a visualisation picture needed by students. Presentation of material starts from the most basic to in-depth [25]. The presentation of material in this teaching material is in accordance with the curriculum and the packaging of the material is adjusted to the character of the students. Effective material, in accordance with the curriculum and student needs is the preparation of good learning resources for student learning.

The primary objective of this study was to develop GeoGebra-assisted teaching materials to overcome learning obstacles in fraction concepts. The quantitative results demonstrated that the developed materials are highly valid (average 92.75% from media experts) and practical for elementary classrooms (81.75% in large-scale trials). These high scores indicate that the integration of the *Iceberg Model* into the ADDIE development process successfully created a structured learning path. Specifically, the use of GeoGebra effectively served the "Model of" stage in the iceberg theory, providing dynamic visualizations that replaced static textbook images, thereby enhancing students' initial engagement and visualization capabilities.

However, a critical finding from the field trials reveals a nuanced challenge. While students excelled in interacting with the digital manipulatives, qualitative feedback indicated difficulties in transitioning from visual representations (e.g., area models in GeoGebra) to symbolic notation (formal algorithms). This phenomenon aligns with the "cognitive gap" often observed in mathematics education when technology is introduced. Although the *Instructional Learning Trajectory (ILT)* was designed to guide students from real-world objects to abstract algorithms, the results suggest that visualization alone is insufficient for deep conceptual mastery without explicit bridging. The lower score in content validation (81.25%) compared to linguistic feasibility (87.5%) further corroborates the complexity of designing digital content that perfectly aligns with abstract mathematical rigor.

The finding that some students required "additional scaffolding" and "guided problem-solving sessions" to fully grasp the concepts underscores the limitations of standalone digital tools. This study confirms that GeoGebra should be viewed as a *didactical instrument* rather than a replacement for instruction. The "very practical" category achieved in the large-scale trial was likely mediated by the teacher's active role in interpreting the dynamic changes on the screen. This supports the notion of *Instrumental Orchestration*, where the teacher must orchestrate the classroom discussion to ensure that the "technical" manipulation of the software translates into "mental" mathematical schemes. Thus, the developed material is most effective when used as a platform for dialogic teaching, rather than for solitary drill-and-practice.

The prepared teaching materials are presented in accordance with the characteristics of students to overcome learning obstacles in fraction materials. Learning materials are arranged based on the level of thinking of students. this can be seen from the responses given by students, and in accordance with the results of previous research that the use of technology (including geogebra) can help learning to create a pleasant learning environment [21]. The teaching materials developed are expected to be able to improve the motivation and learning outcomes of students.

4. CONCLUSION

This study developed GeoGebra-assisted teaching materials based on a Hypothetical Learning Trajectory (HLT) using an Educational Design Research (EDR) approach to support elementary students' learning of fractions. The iterative design process resulted in instructional materials that were evaluated as very valid by media experts (92.75%) and content experts (86.25%), and very practical based on student responses during classroom implementation (81.75%). These findings indicate that the developed materials are feasible for use in elementary mathematics instruction.

The study also demonstrates that integrating GeoGebra with a structured HLT provides meaningful opportunities to represent fraction concepts dynamically while anticipating common learning obstacles, such as whole-number bias. Nevertheless, the findings suggest that dynamic visualization alone is insufficient to promote conceptual understanding. Its educational value depends on how teachers guide students in connecting visual representations with formal mathematical concepts and symbolic notation.

From a practical perspective, the developed teaching materials can assist elementary teachers in designing more interactive and conceptually oriented fraction lessons. Rather than functioning as a substitute for instruction, GeoGebra should be used as a pedagogical tool that supports teacher-led discussion, questioning, and conceptual exploration.

Future studies are recommended to examine the effectiveness of the developed materials in improving students' conceptual understanding and learning outcomes through experimental or longitudinal research involving broader educational contexts.

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