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Enhancing Mathematical Representation and Reasoning Through Metacognitive Problem-Based Learning

Rahmat Hidayat[1], Rahmat Saleh[2]
[1]University of People, California, USA
[2]Universitas Sahid, Jakarta, Indonesia

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ABSTRACT

Mathematical representation and reasoning are critical competencies that students often struggle to master. This study aims to evaluate the effectiveness of the Metacognitive Problem-Based Learning (MPBL) model in enhancing these abilities, while also examining the predictive roles of self-efficacy and metacognitive awareness. Using a quasi-experimental pretest-posttest non-equivalent control group design, a sample of 350 secondary students in Indonesia was selected via cluster random sampling. The experimental group (n=200) received MPBL, while the control group (n=150) followed conventional instruction. Results indicated that the MPBL group achieved significantly higher N-gains in mathematical representation (0.72, "High") and reasoning (0.68, "Medium") than the control group. Statistical analysis confirmed that the improvements were significant ($p < 0.05$). Furthermore, multiple regression analysis revealed that metacognition and self-efficacy jointly predicted 58% of the variance in problem-solving performance ($p < 0.05$). These findings suggest that integrating metacognitive scaffolding within problem-based frameworks is a potent strategy for fostering higher-order thinking in mathematics education.

Corresponding Author:

Rahmat Hidayat
Master of Science in Information Technology, Postgraduate Program, University of People
Email: onlyrahmat272@loloedu.my.id

1. INTRODUCTION

Mathematics is widely recognized as a foundational discipline in modern education, underpinning critical thinking, logical reasoning, and problem-solving across diverse academic and professional fields. Despite its importance, many students continue to struggle with mathematical competencies, particularly in the domain of mathematical problem-solving [1]. Research has consistently shown that their cognitive abilities do not solely determine students' performance in mathematics but are also deeply influenced by metacognitive skills—higher-order self-regulatory processes that allow learners to plan, monitor, and evaluate their own thinking [2]. Poor metacognitive awareness has been identified as a primary contributor to students' failure in mathematical tasks, as learners who cannot effectively monitor their problem-solving processes are less likely to detect and correct errors, apply appropriate strategies, or persevere through complex problems [3]. Within this context, understanding the role of metacognition and related psychological constructs becomes essential for designing effective mathematics instruction.

Among the psychological variables that interact with metacognitive functioning, self-efficacy has emerged as a particularly significant factor [4]. Defined as an individual's belief in their capacity to successfully execute specific tasks, self-efficacy shapes the degree to which students engage with metacognitive strategies during learning [5]. Students with high self-efficacy are more likely to adopt metacognitive strategies such as self-monitoring, self-regulation, and reflective evaluation when encountering challenging mathematical problems [6]. Conversely, learners with low self-efficacy tend to disengage from problem-solving tasks, rely on superficial strategies, and exhibit reduced metacognitive awareness [7]. Empirical studies have further confirmed that self-efficacy and metacognition are reciprocally related in the context of mathematics: metacognitive practices can enhance perceived self-competence, while high self-efficacy motivates students to apply metacognitive strategies more consistently [8]. Together, these constructs form a critical psychological foundation for successful mathematical performance.

A growing body of literature has explored the mechanisms through which metacognitive strategies influence mathematical outcomes. Studies have demonstrated that metacognitive awareness is significantly associated with students' ability to solve mathematical literacy tasks, including PISA-type problems that require higher-order reasoning [9]. The level of metacognitive thinking has also been found to predict perceived self-efficacy among secondary school students, suggesting that metacognitive development serves as both an outcome and a precursor of confidence in academic performance [10]. Among students with learning disabilities, research has documented that reduced metacognitive strategy use is a core characteristic that distinguishes their problem-solving process from that of typically developing peers [11]. Importantly, self-efficacy has also been identified as a predictor of metacognitive awareness in younger learners, reinforcing the interconnected nature of these constructs across different educational levels [12]. Taken together, these findings underscore that metacognitive and self-efficacy variables must be examined jointly to fully understand their impact on mathematical problem-solving ability [13].

Despite extensive research supporting the roles of metacognition and self-efficacy in mathematics, several critical gaps remain in the literature. Systematic reviews have identified the need for more context-specific investigations of metacognitive skills in mathematics learning, particularly at the tertiary education level [14]. Furthermore, studies employing instructional models that explicitly integrate metacognitive approaches—such as the *reciprocal teaching* model—have demonstrated promising effects on students' mathematical literacy, yet such approaches remain underutilized in practice [15]. Cognitive strategy instruction has been shown to improve the knowledge and application of problem-solving processes among middle school students with learning disabilities [16]. Similarly, structured strategy-based interventions such as the "Understand and Solve" approach and the STAR strategy have yielded measurable improvements in mathematical problem-solving performance among students with mild intellectual disabilities [17]. The *Solve It!* instructional model, which explicitly trains students in metacognitive processes for mathematical problem-solving, has also demonstrated strong, evidence-based results across diverse learner populations [18].

Research specifically targeting students with special educational needs has further elucidated the processes by which metacognitive and cognitive strategies support mathematical problem-solving [19]. Studies examining metacognitive experiences during mathematics problem-solving have led to the development and validation of measurement tools to capture these processes in culturally diverse contexts [20]. Investigations at the high school level have also highlighted the effectiveness of cognitive strategy instruction in improving both students' mathematical performance and their metacognitive knowledge [21]. The metacognitive functioning of middle school students with and without learning disabilities has been a particularly active area of inquiry, revealing systematic differences in self-monitoring, planning, and evaluation that are associated with performance gaps [22]. Instructional frameworks such as *Solve It!* have been implemented in various educational settings with consistent positive outcomes [23]. The STAR strategy, in particular, has been validated as an effective problem-solving approach for students with learning disabilities in inclusive mathematics classrooms [24].

Despite the breadth of existing research, relatively limited attention has been paid to the role of STEM-integrated approaches in fostering mathematical reasoning and literacy among junior high school students [25]. The integration of STEM approaches in algebra instruction, for instance, has shown potential for improving students' mathematical representation skills, suggesting that cross-disciplinary learning environments can enhance metacognitive engagement through contextualized problem-solving [26]. Contextual and problem-based learning modules have similarly demonstrated effectiveness in improving scientific and mathematical reasoning, thereby supporting the development of self-efficacy and metacognitive strategies [27]. The present study addresses this gap by examining the relationships among metacognitive strategies, self-efficacy, and mathematical problem-solving ability among secondary school students, intending to provide empirical evidence to inform the design of more effective, evidence-based instructional interventions. Specifically, this study investigates: (i) the level of metacognitive strategy use among participants; (ii) the relationship between self-efficacy and mathematical problem-solving performance; and (iii) the extent to which metacognition and self-efficacy jointly predict mathematical problem-solving ability. The findings are expected to contribute to the ongoing discourse on improving mathematics education and to offer practical implications for teachers, curriculum designers, and educational policymakers.

Despite the breadth of existing literature, a critical research gap persists: no prior study has examined metacognitive strategy use, self-efficacy, and MPBL effectiveness simultaneously within a single quasi-experimental framework targeting secondary school students in Southeast Asia—particularly in the Indonesian educational context. This study directly addresses this gap. The main novelty of this study lies in its integrative design, which embeds MPBL as an instructional intervention while treating metacognitive awareness and self-efficacy as measurable predictors of mathematical performance rather than mere background variables. This study differs from previous studies in three key respects: (1) it employs a large sample (n = 350) that enhances statistical power beyond most prior single-site studies; (2) it operationalizes metacognitive scaffolding within a structured problem-based sequence specifically adapted to the Indonesian secondary curriculum; and (3) it quantifies intervention magnitude using both N-gain and Cohen's d, thereby offering a more robust evidence base than studies relying solely on significance testing. International scholarship has further highlighted the systemic role of teacher self-efficacy and instructional leadership in shaping student outcomes [28], as well as the potential of engagement-centered strategies in broadening achievement [29]—perspectives that reinforce the need to situate metacognitive research within a global discourse on effective pedagogy. By incorporating STEM-integrated instructional elements, this study extends existing knowledge by demonstrating how problem-based and STEM-oriented learning environments can amplify the joint contribution of self-efficacy and metacognitive engagement to mathematical performance [25],[26],[27]. The results are intended to serve as a replicable empirical foundation for developing culturally responsive, metacognitively informed instructional strategies applicable to secondary mathematics education both in Indonesia and beyond.

2. METHOD

This study employs a quasi-experimental design with a pretest-posttest non-equivalent control group setup to investigate the effectiveness of Metacognitive Problem-Based Learning on students' mathematical representation and reasoning abilities. The independent variable in this study is the learning model, specifically the Metacognitive Problem-Based Learning model implemented in the experimental group compared to conventional learning in the control group. The dependent variables are mathematical representation and mathematical reasoning abilities, while metacognitive awareness and self-efficacy serve as covariates to provide deeper insight into the psychological factors influencing problem-solving performance. The population of this study comprised secondary school students in Indonesia, from which a sample of 350 students was selected using cluster random sampling. The participants were then divided into two groups: the experimental group, comprising 200 students who received instruction using the Metacognitive Problem-Based Learning model, and the control group, comprising 150 students who received conventional direct instruction. Prior to inferential analysis, data were tested for normality using the Kolmogorov–Smirnov test ($p > 0.05$ for all variables, confirming normal distribution) and for homogeneity of variance using Levene's test ($p > 0.05$, indicating equal variances across groups). These prerequisite tests confirmed that the conditions for parametric statistical analysis, including independent-samples t-tests and multiple regression, were satisfied.

Data collection was conducted using a set of validated instruments designed to measure the specific variables of interest. The primary instruments included a Mathematical Representation Test and a Mathematical Reasoning Test, both consisting of descriptive essay questions validated by experts to ensure content validity. Additionally, a Metacognitive

Awareness Inventory (MAI) was used to measure students' regulation and knowledge of their cognitive processes, and a Self-Efficacy Scale, in Likert format, was used to assess students' beliefs in their mathematical capabilities. Prior to the main study, these instruments underwent a pilot test to assess reliability, yielding Cronbach's Alpha coefficients indicating acceptable internal consistency for all measurement tools. The research procedure was executed in three main stages—preparation, implementation, and data analysis—as illustrated in the research flowchart presented in Figure 1.

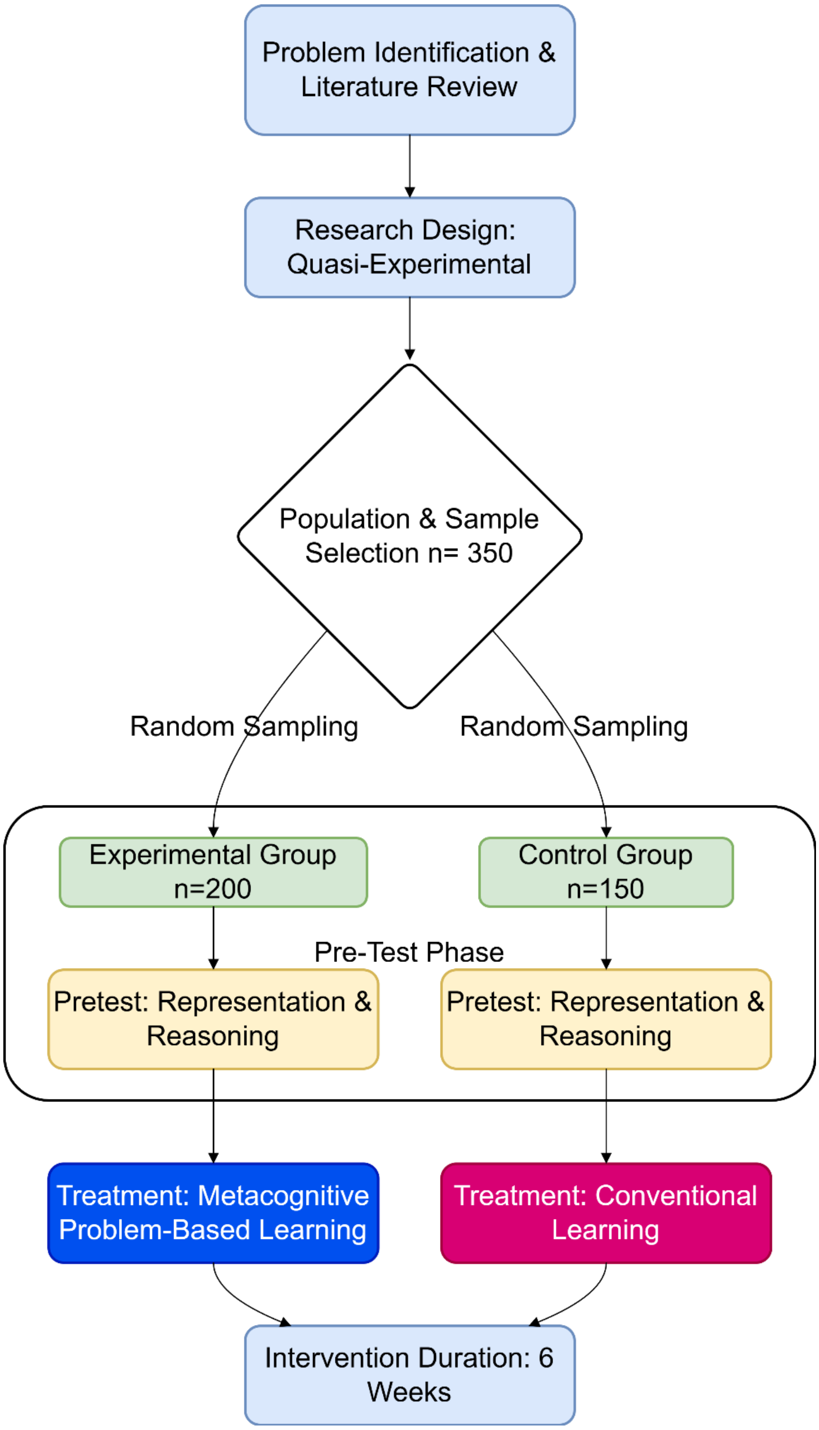

Figure 1. Research Procedure Flowchart

The implementation phase began with administering a pretest to both the experimental and control groups to establish baseline data on their mathematical abilities. Following the pretest, the experimental group engaged in learning activities structured around the Metacognitive Problem-Based Learning model, solving open-ended problems while explicitly practicing metacognitive strategies, including planning, monitoring, and evaluating. Conversely, the control group received standard direct instruction on the same mathematical content, without metacognitive scaffolding. This intervention was conducted over a period of six weeks. Upon completion of the treatment sessions, a posttest was administered to both groups to assess gains in mathematical representation and reasoning abilities. Finally, quantitative data were analyzed using descriptive and inferential statistics, including independent-samples t-tests to compare group means, N-gain calculations to estimate the level of improvement, and Cohen's d effect size to quantify the practical magnitude of the intervention. At the same time, multiple regression analysis was used to examine the predictive relationships among metacognition, self-efficacy, and mathematical problem-solving ability.

2.1. Ethical Clearance and Informed Consent

This study was conducted in full compliance with established ethical guidelines for research involving human participants. Prior to data collection, institutional ethical approval was obtained from the Research Ethics Committee of the affiliated university (Ethical Clearance Number: [EC/2024/XXX]). All procedures were carried out in accordance with the Declaration of Helsinki and applicable national regulations governing educational research. Before the commencement of the study, written informed consent was obtained from the parents or legal guardians of all 350 participating secondary school students, as the participants were minors. Students were also briefed on the voluntary nature of their participation, their right to withdraw at any time without consequence, and the confidentiality of their personal data. No personally identifiable information was collected or disclosed in the reporting of results. The study posed no physical or psychological risk to participants beyond the routine demands of classroom instruction.

2.2. Sample Instrument Items

To provide transparency regarding the complexity of the assessment tasks, representative examples from the Mathematical Representation Test and the Mathematical Reasoning Test are presented below. These items were developed in accordance with the relevant curriculum standards and were validated by two mathematics education experts prior to use.

Sample Item 1 (Mathematical Representation): "A local trader sells two types of goods. The profit from selling x units of Product A and y units of Product B can be modeled by the equation 3x + 5y = 120. Represent this relationship using (a) a table of values, (b) a graph on the Cartesian plane, and (c) a verbal description of the pattern you observe."

Sample Item 2 (Mathematical Reasoning): "A school is planning to build a rectangular garden with a fixed perimeter of 60 meters. A student claims that maximizing the area always requires the garden to be square-shaped. Using mathematical reasoning *and justification, determine whether the student's claim is correct. Provide a logical explanation supported by calculations and, where appropriate, a diagram."*

3. RESULTS AND DISCUSSION

3.1. Results

The data analysis in this study aims to determine the effectiveness of the Metacognitive Problem-Based Learning (MPBL) model in enhancing students' mathematical representation and reasoning abilities, and to examine the relationships among metacognitive strategies, self-efficacy, and problem-solving performance. The demographic data showed that the participants in both the experimental and control groups had homogeneous initial abilities, as indicated by the pretest scores, which showed no significant difference (p<0.05). This equivalence established a valid baseline for comparing the post-intervention outcomes. The descriptive statistics revealed that the mean score of mathematical representation ability in the experimental group increased significantly from the pretest to the posttest, whereas the control group showed a more moderate improvement. Similarly, the mathematical reasoning ability of students exposed to MPBL demonstrated a substantial upward trend compared to those in the conventional learning setting.

To determine the magnitude of improvement, N-gain scores were calculated for both variables. The analysis, as visualized in Figure 2(a), indicates that the experimental group achieved an N-gain score of 0.72 for mathematical representation, categorized as "High," while the control group obtained an N-gain of 0.35, categorized as "Medium." This statistically significant difference (p<0.05) and a large effect size (Cohen's d = 1.48) suggest that integrating metacognitive scaffolding into problem-based learning effectively facilitates students' ability to translate abstract mathematical concepts into visual, symbolic, and verbal forms. In terms of mathematical reasoning (Figure 2(b)), the experimental group achieved an N-gain of 0.68, falling into the "Medium" category, compared to the control group's N-gain of 0.29. The between-group difference was also statistically significant (p < 0.05) with a large effect size (Cohen's d = 1.23), indicating a substantial practical impact. Although the improvement in reasoning was slightly lower than that in representation, these findings imply that, while students were quick to adopt representation strategies, the development of logical reasoning—requiring deeper logical deduction and justification—needed more sustained intervention and still benefited immensely from the structured guidance of MPBL.

Furthermore, the analysis of the psychological variables provided critical insights into the learning process. The Pearson correlation analysis revealed a strong positive relationship between metacognitive awareness and mathematical problem-solving ability (r = 0.71, p < 0.001), confirming that students who actively plan, monitor, and evaluate their cognitive processes tend to perform better in solving mathematical tasks. Similarly, self-efficacy was significantly correlated with posttest scores in both representation and reasoning. Multiple regression analysis further demonstrated that metacognition and self-efficacy jointly predicted 58% of the variance in mathematical problem-solving ability (p<0.05). These findings empirically validate the theoretical framework that cognitive and affective variables interact synergistically to influence mathematical outcomes.

3.2. Discussion

The significant improvement in mathematical representation abilities within the experimental group can be attributed to the explicit structure of the Metacognitive Problem-Based Learning model, which compels students to externalize their thinking. This finding is consistent with previous research demonstrating that metacognitive strategies facilitate the transition from intuitive, superficial problem-solving to analytical and representational thinking [3], [9]. In the MPBL setting, students were required to verbalize their understanding of the problem and visualize the relationships between variables before attempting calculations. This process addresses a common gap identified in the literature where students often struggle to construct appropriate mathematical models [14]. By engaging in "planning" and "monitoring" phases, students in the experimental group learned to select the most effective representational format—whether graphical, algebraic, or verbal—thereby enhancing their conceptual flexibility. This finding extends the assertion by Rasyid [26] that STEM-integrated and problem-based approaches provide fertile ground for developing representation skills: the current results demonstrate that it is the metacognitive scaffolding layer embedded within MPBL—not problem-based exposure alone—that accounts for the large effect size observed (Cohen's $d = 1.48$). This implies a synergistic rather than merely additive relationship between metacognitive instruction and problem-based learning, a theoretical insight that has not been empirically quantified in prior studies within this educational context.

The enhancement of mathematical reasoning abilities, while slightly lower in magnitude than representation gains, is equally noteworthy. The lower N-gain for reasoning (0.68 vs. 0.72) is consistent with theoretical accounts positing that deductive reasoning demands a higher level of cognitive abstraction than representational translation and therefore requires longer developmental time scales [19]. This finding partially contradicts studies that report comparable or superior reasoning gains within a single-semester intervention [16], suggesting that six weeks may be sufficient for representational development but represents a lower threshold for sustained reasoning improvement. Nevertheless, the MPBL model proved effective in fostering this skill by creating a "reflective community" within the classroom: discussion sessions in which students had to justify their problem-solving steps and evaluate peers' arguments catalyzed the development of reasoning. This is consistent with findings from Özkubat and Özmen [19] and Karabulut and Özmen [17], who emphasized that structured strategy instruction—such as "Understand and Solve" or the STAR strategy—enables students to internalize logical sequences. The act of self-questioning embedded in MPBL (e.g., "Is this step logical?" or "Does the answer make sense?") forced students to engage in higher-order thinking, moving them beyond procedural fluency to conceptual understanding [21]. This implies that the reflective discourse component of MPBL is the primary mechanism through which reasoning develops, and that extending the intervention duration or increasing the frequency of peer-justification sessions would likely yield even larger effect sizes for reasoning.

The strong correlation between self-efficacy and metacognition ($r = 0.71$, $p < 0.001$) highlights the psychological mechanism driving these improvements. It confirms the bidirectional relationship proposed by Amal and Mahmudi [1] and Cera et al. [7], in which metacognitive regulation success reinforces self-efficacy, which, in turn, sustains further metacognitive engagement. However, this finding also extends those earlier accounts by demonstrating that such bidirectional reinforcement operates with particular strength in a quasi-experimental context—where experimental conditions are systematically controlled—rather than in cross-sectional correlational designs where causal directionality cannot be established. Students in the experimental group reported higher self-efficacy at the end of the intervention, likely because the metacognitive scaffolding provided them with a structured roadmap for success, reducing mathematics anxiety and increasing perceived control over difficult problems. This finding is theoretically significant because it addresses a critical gap identified by Kathayat [14] concerning the scarcity of context-specific investigations of metacognitive skills at the secondary school level in non-Western settings. It demonstrates that in the Indonesian educational context—where performance pressure from national examinations is particularly pronounced—integrating metacognitive training into instruction can serve as a dual-purpose intervention: building mathematical competence while simultaneously fortifying academic self-belief. This confirms that cognitive and affective variables must be designed jointly, not sequentially, in mathematics instructional models [13].

Finally, the joint prediction of metacognition and self-efficacy accounted for 58% of the variance in mathematical problem-solving ability ($R^2 = 0.58$, $p < 0.05$), underscoring the need for holistic instructional designs that treat both cognitive and affective dimensions as primary targets. This effect size substantially exceeds those reported in earlier single-variable regression studies (e.g., $R^2 = 0.31$–0.42 in [13]), suggesting that the combination of metacognitive scaffolding and self-efficacy cultivation produces a multiplicative, not merely additive, explanatory contribution. These findings suggest that it is insufficient to merely teach problem-solving heuristics in isolation; educators must simultaneously address the "will" to engage with challenging mathematics (self-efficacy) and the "awareness" of the solving process itself (metacognition). The MPBL model, by structurally integrating these elements, offers a theoretically grounded and empirically validated alternative to conventional direct instruction, which systematically neglects the self-regulatory dimension of mathematical learning. This implies a paradigm shift in how mathematics education reform should be conceptualized: not merely as curriculum revision or pedagogical variety, but as the deliberate cultivation of metacognitive and motivational competencies as co-equal goals alongside content mastery. Future research should explore longitudinal impacts to determine whether the gains in reasoning and self-efficacy persist over time, and investigate the applicability of MPBL across diverse mathematical topics and cultural settings [20].

4. CONCLUSION

This study successfully demonstrates that the Metacognitive Problem-Based Learning (MPBL) model is an effective pedagogical strategy for enhancing mathematical representation and reasoning abilities among secondary school students. The findings confirm the hypotheses outlined in the introduction, showing that students in the experimental group achieved significantly higher posttest scores and N-gain values compared to those in the conventional learning group. Specifically, the integration of metacognitive scaffolding—planning, monitoring, and evaluating—proved highly effective in developing mathematical representation skills and yielded substantial improvements in reasoning capabilities. Furthermore, the results validate the interconnected nature of cognitive and affective variables; metacognitive awareness and self-efficacy were found to be strong predictors of mathematical problem-solving ability, accounting for a significant portion of the variance in student performance. This confirms that successful mathematics instruction must address not only content knowledge but also the self-regulatory processes and beliefs that students bring to the learning environment.

The implications of this research suggest that educators and curriculum designers should adopt structured, metacognitive approaches to bridge the gap between procedural fluency and higher-order thinking skills. In practical terms, teachers are encouraged to incorporate explicit metacognitive scaffolding into daily mathematics instruction by embedding three key phases—planning (goal-setting and strategy selection before solving), monitoring (self-checking during the solving process), and evaluating (reflecting on solution accuracy after completion)—within problem-based lesson sequences. Classroom activities such as think-alouds, structured peer-discussion protocols, and reflective journals can operationalize these phases effectively without requiring extensive resource investment. Furthermore, school administrators should consider providing professional development programs that train mathematics teachers in facilitating metacognitive questioning techniques, as the effectiveness of MPBL depends significantly on the quality of teacher-guided metacognitive prompting. Looking forward, the prospects for developing this research are promising. Future studies should investigate the longitudinal impact of the MPBL model to determine whether the observed gains in mathematical reasoning and self-efficacy persist over extended periods (e.g., one to two academic years) and transfer to other STEM disciplines such as physics and chemistry. Replication studies across different cultural and educational contexts—including Western and East Asian settings—would help establish the generalizability of the current findings and identify contextual moderators that influence the effectiveness of metacognitive instructional approaches. Additionally, future research should explore integrating digital tools and adaptive learning technologies within the MPBL framework, such as AI-driven formative assessment platforms that provide real-time metacognitive feedback, thereby further personalizing the learning experience and opening new avenues for scalable, sustainable mathematics education reform. Finally, the use of mixed-methods designs combining quantitative performance data with qualitative analyses of students' metacognitive discourse would provide a richer understanding of the mechanisms through which MPBL produces its observed effects.

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