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Reconceptualising The Geometry Gap: Theoretical Insights From Van Hiele And Constructivist Perspectives On Triangle Theorem Instruction In South Africa

Shakespear Chiphambo
Walter Sisulu University, South Africa

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ABSTRACT

Geometry forms a fundamental component of secondary school mathematics because it supports the development of spatial reasoning, logical thinking, and deductive proof skills. However, persistent learner difficulties in understanding geometric concepts, particularly triangle theorems, remain a significant concern in many education systems. This conceptual paper examines the factors contributing to the geometry gap in South African schools, with a particular focus on the teaching and learning of triangles and their associated theorems. The paper adopts a conceptual research approach that synthesises recent literature and theoretical perspectives in mathematics education, including the van Hiele theory of geometric thinking, constructivist learning theory and socio-cultural perspectives on learning. Through an integrative analysis of recent scholarship, the paper identifies structural, pedagogical and cognitive factors that collectively influence learners' geometry learning outcomes. The study further proposes a conceptual framework explaining how these dimensions interact to produce persistent gaps in learners' geometric reasoning and problem-solving abilities. The interconnected nature of these challenges offers theoretically grounded insights that may inform curriculum development, teacher professional development and instructional innovation aimed at improving geometry education. The findings contribute to ongoing debates on mathematics education equity and provide a foundation for future empirical research on strategies for strengthening geometry instruction in South African schools.

Corresponding Author:

Shakespear Chiphambo
Faculty of Education, Mathematics, Science and Technology Education Department, Walter Sisulu University, South Africa
Email: schiphambo@wsu.ac.za

1. INTRODUCTION

Geometry education plays a foundational role in the development of mathematical reasoning, spatial visualisation, and deductive thinking. Within the secondary mathematics curriculum, the study of triangles and their associated theorems serves as a critical gateway to formal mathematical proof and logical reasoning. Learners' ability to understand properties such as congruence, similarity, and angle relationships is central to higher-order mathematical thinking and to success in advanced mathematical domains [1], [2]. Contemporary mathematics education research increasingly emphasises geometry as a cognitive tool that supports analytical reasoning, problem solving, and conceptual abstraction [3]. Consequently, effective geometry instruction is widely recognised as an essential component of quality mathematics education systems worldwide. However, persistent gaps in learners' conceptual understanding of geometric principles continue to challenge educators, particularly in contexts where systemic educational inequalities affect instructional quality and learner engagement [4].

Globally, the teaching and learning of geometry remain a persistent challenge across diverse educational contexts. Studies consistently report that learners struggle to transition from procedural manipulation to conceptual reasoning when engaging with geometric theorems and proofs [5]. The abstract nature of Euclidean geometry requires learners to interpret diagrams, construct logical arguments, and connect visual and symbolic representations, tasks that many learners find difficult without appropriate pedagogical support [6]. Furthermore, international assessments and comparative studies reveal that inadequate instructional strategies, limited use of dynamic visualisation tools, and insufficient emphasis on conceptual reasoning often contribute to poor learner performance in geometry [7]. Researchers, therefore, argue that addressing geometry learning difficulties requires a combination of improved pedagogical approaches, teacher professional development, and the integration of technological resources capable of enhancing spatial reasoning and conceptual understanding [8].

Within the South African educational landscape, these global challenges are compounded by systemic inequalities that affect teaching and learning conditions. Many schools, particularly those serving historically disadvantaged communities, face significant structural constraints, including overcrowded classrooms, inadequate instructional materials, and limited access to technological resources [2], [9]. Such conditions often hinder effective mathematics instruction and contribute to disparities in learner achievement across socio-economic contexts. Empirical studies indicate that learners from low-income communities frequently encounter additional barriers such as limited exposure to mathematical discourse, insufficient academic support, and reduced opportunities to develop higher-order mathematical reasoning [10]. These factors collectively shape a learning environment in which geometry, and particularly triangle theorems, becomes difficult for many learners to master.

Evidence of a persistent geometry gap is increasingly visible in South African mathematics performance trends. National and regional assessments reveal that learners consistently demonstrate weak performance in geometry-related tasks requiring reasoning, proof construction, and theorem application [1], [11]. Researchers attribute these difficulties to a combination of curriculum complexity, limited teacher content knowledge, and the dominance of procedural teaching approaches that prioritise memorisation over conceptual understanding [12]. In many classrooms, learners are expected to reproduce geometric theorems without fully understanding their logical foundations or real-world relevance [13].

As a result, geometry is frequently perceived as one of the most difficult areas of the mathematics curriculum, contributing to learner anxiety, disengagement, and declining mathematics achievement.

The persistence of these challenges highlights a critical research problem: the widening gap between the intended geometry curriculum and learners' actual conceptual understanding of triangles and related theorems in South African schools. Despite curriculum reforms and increased emphasis on mathematical reasoning, many learners continue to struggle with the logical structures and visual representations required for successful geometry learning [4], [14]. This gap raises important questions regarding the effectiveness of current pedagogical approaches, teacher preparedness, and the availability of resources that support meaningful engagement with geometric concepts. Without targeted interventions, the geometry gap may continue to reinforce broader educational inequalities and limit learners' access to advanced mathematical learning pathways.

Addressing this research problem requires a theoretically grounded understanding of how learners develop geometric reasoning. This study draws on constructivist perspectives of learning and the Van Hiele theory of geometric thinking, which emphasises the progressive development of learners' conceptual understanding through distinct cognitive levels [15]. According to this theoretical framework, learners move from basic visual recognition of shapes to more advanced analytical and deductive reasoning about geometric relationships. However, effective progression through these levels depends heavily on appropriate instructional strategies, scaffolded learning experiences, and opportunities for active engagement with geometric concepts [16]. Integrating these theoretical perspectives allows the study to critically examine how current classroom practices either support or hinder learners' development of geometric reasoning.

Building on this theoretical foundation, the present study seeks to contribute to the ongoing discourse on improving mathematics education in South Africa by examining the underlying causes of the geometry gap and identifying potential strategies for improvement. The primary objectives of this research are: **(1)** to investigate the key factors contributing to learners' difficulties in understanding triangles and their theorems; **(2)** to examine the role of teacher preparedness, instructional practices, and curriculum implementation in shaping geometry learning outcomes; and **(3)** to explore pedagogical and institutional strategies that may help bridge the geometry gap in South African schools. Through this analysis, the study aims to generate insights that can inform more effective teaching approaches and support equitable access to quality mathematics education.

Ultimately, this research seeks to provide evidence-based recommendations that may strengthen geometry instruction and improve learner achievement. By identifying the structural, pedagogical, and cognitive factors that contribute to persistent geometry learning difficulties, the study hopes to contribute to the development of more inclusive and effective mathematics education practices. The findings are expected to support educators, curriculum developers, and policymakers in designing interventions that enhance conceptual understanding, promote mathematical reasoning, and ultimately narrow the geometry gap within South African schools [3], [8]. In doing so, the research aims to advance broader efforts toward educational equity and improved mathematics outcomes for all learners.

2. PROBLEM STATEMENT

Despite the recognised importance of geometry in developing mathematical reasoning and logical thinking, learner performance in Euclidean geometry continues to present significant challenges in many education systems. Research consistently shows that learners struggle with conceptual understanding of geometric relationships, particularly when required to apply theorems, construct proofs, and interpret spatial representations [1], [6], [18]. These difficulties are especially pronounced in topics involving triangles and their associated theorems, where learners must integrate visual interpretation, deductive reasoning, and formal mathematical language. As a result, geometry is frequently perceived as one of the most cognitively demanding components of the secondary mathematics curriculum.

In the South African context, these challenges are exacerbated by systemic inequalities that shape teaching and learning environments. Many schools operate under conditions characterised by limited instructional resources, overcrowded classrooms, and insufficient access to technological tools that support conceptual mathematics learning [2], [9]. Such structural constraints often restrict opportunities for interactive learning and exploratory problem-solving, which are essential for developing geometric reasoning. Additionally, disparities in socio-economic conditions continue to influence educational outcomes, with learners from historically disadvantaged communities experiencing greater barriers to accessing quality mathematics instruction [10], [14].

Another critical factor contributing to the geometry gap relates to teacher preparedness and pedagogical practices. Studies indicate that many mathematics teachers experience difficulties in teaching Euclidean geometry due to limited content knowledge, insufficient training in proof-based instruction, and reliance on procedural teaching methods [12], [21]. Consequently, geometry lessons often emphasise memorisation of theorems and step-by-step procedures rather than fostering deep conceptual understanding and logical reasoning. When learners are unable to meaningfully connect geometric concepts to underlying principles, they are more likely to experience confusion and disengagement, which ultimately contributes to poor performance in national assessments [11], [13].

Furthermore, curriculum demands and assessment practices may unintentionally widen the geometry gap. While the South African mathematics curriculum emphasises deductive reasoning and formal proof, many learners have not yet developed the cognitive foundations necessary to engage with abstract geometric arguments [4], [5]. This misalignment between curriculum expectations and learners' conceptual readiness creates significant learning barriers, particularly in topics involving triangle theorems, congruence, and similarity. Without effective pedagogical interventions, these challenges may continue to reinforce persistent disparities in mathematics achievement across the education system.

Given these concerns, there remains a need for comprehensive research that examines the underlying causes of learners' difficulties with geometry and identifies strategies that can support improved teaching and learning practices. Addressing the geometry gap is essential not only for enhancing mathematics performance but also for ensuring that learners develop the analytical and problem-solving skills necessary for participation in STEM-related fields. This study, therefore, investigates the factors contributing to persistent challenges in teaching and learning triangle theorems in South African schools, to identify pedagogical and systemic interventions that may help bridge the geometry gap.

3. RESEARCH AIM

The primary aim of this study is to critically examine the persistent challenges associated with the teaching and learning of triangle theorems in South African schools. By synthesising recent scholarly literature and theoretical perspectives in mathematics education, the study seeks to identify structural, pedagogical, and cognitive factors that contribute to learners' difficulties in mastering geometric reasoning. Ultimately, the paper aims to propose theoretically grounded insights that can inform improved instructional practices, curriculum implementation, and policy interventions aimed at strengthening geometry education and promoting equitable mathematics learning outcomes in South African schools [1], [2], [14]. In addition, to develop a conceptual framework that explains the underlying causes of the geometry gap.

4. RESEARCH OBJECTIVES

To achieve the aim of the study, the following objectives guide the conceptual analysis:

a. to investigate the key factors contributing to learners' difficulties in understanding triangles and their theorems.
b. to examine the role of teacher preparedness, instructional practices, and curriculum implementation in shaping geometry learning outcomes; and
c. to explore how theoretical perspectives can explain learners' challenges in developing geometric reasoning.
d. to propose a conceptual framework that informs strategies for improving geometry teaching and learning in South African schools.

5. RESEARCH QUESTIONS

The following research questions guide the study:

a. What factors contribute to persistent learner difficulties in understanding triangle theorems in South African schools?
b. How do teaching practices, teacher preparedness, and curriculum implementation influence geometry learning outcomes?
c. What theoretical perspectives can explain learners' challenges in developing geometric reasoning?
d. How can a conceptual framework inform strategies for improving geometry teaching and learning in South African schools?

6. LITERATURE REVIEW

6.1. Importance of geometry in mathematics education

Geometry plays a central role in mathematics education because it supports the development of spatial reasoning, logical thinking, and deductive proof skills. Scholars

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argue that geometry provides learners with opportunities to connect visual intuition with formal mathematical reasoning, thereby strengthening analytical thinking and problem-solving abilities [3], [6]. The study of triangles and their associated theorems is particularly important as it introduces learners to fundamental concepts such as congruence, similarity, and angle relationships that underpin many advanced mathematical topics [18].

Furthermore, geometry contributes significantly to the development of learners' mathematical communication and reasoning abilities. When learners engage with geometric proofs and problem-solving tasks, they are required to justify their reasoning and construct logical arguments, skills that are essential for success in mathematics and STEM disciplines [7]. Consequently, effective geometry instruction is widely recognised as an essential component of quality mathematics education systems.

6.2. Global Challenges in Geometry Learning

Despite its importance, the teaching and learning of geometry remains challenging in many educational contexts worldwide. Studies consistently show that learners struggle to understand geometric concepts beyond basic visual recognition of shapes [5]. The transition from intuitive spatial reasoning to formal deductive proof often presents significant cognitive difficulties for learners, particularly when instruction emphasises memorisation rather than conceptual understanding [6].

International research also highlights the role of instructional approaches in shaping learners' geometry achievement. Traditional teacher-centred teaching methods frequently limit opportunities for learners to explore geometric relationships or engage in meaningful reasoning activities [8]. As a result, many learners develop fragmented knowledge of geometric theorems without understanding the logical relationships that connect them. These challenges have prompted calls for more innovative teaching approaches that incorporate dynamic visualisation tools and inquiry-based learning strategies [19].

6.3. Geometry Learning in the South African Context

In South Africa, the challenges associated with geometry education are further complicated by systemic inequalities that affect the broader education system. Research indicates that many schools operate under conditions characterised by limited resources, overcrowded classrooms, and inadequate access to technological tools that support interactive mathematics learning [2], [9]. These structural constraints often hinder the effective teaching of abstract mathematical concepts such as geometry.

Additionally, disparities in socio-economic conditions contribute to unequal learning opportunities among learners. Studies show that learners from historically disadvantaged communities frequently experience limited exposure to high-quality mathematics instruction, which affects their ability to develop strong conceptual foundations in geometry [10]. These disparities contribute to persistent achievement gaps in mathematics performance across different school contexts.

6.4. Evidence of the Geometry Gap

Empirical evidence from national assessments and academic research indicates that South African learners consistently experience difficulties in geometry-related topics. Performance trends reveal that learners often struggle with questions requiring reasoning, proof construction, and the application of triangle theorems [11]. These challenges are frequently attributed to limited conceptual understanding and inadequate opportunities for learners to engage in meaningful geometric reasoning.

Furthermore, teacher preparedness plays a crucial role in shaping geometry learning outcomes. Studies indicate that some mathematics teachers experience difficulties in teaching Euclidean geometry due to limited subject content knowledge and insufficient training in proof-based instruction [12], [21]. As a result, geometry lessons may focus heavily on procedural problem solving rather than conceptual understanding. These challenges collectively contribute to what scholars increasingly describe as a geometry gap within the South African education system.

7. THEORETICAL FRAMEWORK

Understanding learners' difficulties in geometry requires a theoretical perspective that explains how geometric reasoning develops over time. This study is primarily informed by the van Hiele theory of geometric thinking, which proposes that learners progress through distinct levels of conceptual understanding when learning geometry [5], [15]. According to this theory, learners initially recognise geometric shapes based on visual characteristics (visualisation level), before gradually developing the ability to analyse properties, understand relationships between figures, and eventually construct formal deductive proofs. Progression through these levels is not automatic; rather, it depends on carefully structured instructional experiences that guide learners from intuitive visual reasoning toward more abstract mathematical thinking.

Within this framework, difficulties in learning triangle theorems can often be attributed to a mismatch between learners' cognitive development and the level of reasoning required by the curriculum. For instance, learners who are still operating at the visual or descriptive levels may struggle to understand formal proofs or logical deductions involving angle relationships and congruence criteria. Research suggests that many learners encounter geometry instruction that assumes higher-level reasoning before they have developed the necessary conceptual foundations [18]. Consequently, they resort to memorising theorems rather than understanding the logical relationships that underpin them.

The theoretical framework is further supported by constructivist learning theory, which emphasises that knowledge is actively constructed through meaningful engagement with learning experiences [20]. From a constructivist perspective, learners develop mathematical understanding by exploring relationships, discussing ideas, and solving problems collaboratively. Geometry learning, therefore, benefits from instructional approaches that encourage visual exploration, dynamic representations, and guided discovery. Tools such as dynamic geometry software have been shown to support learners in constructing and testing geometric conjectures, thereby strengthening conceptual understanding and spatial reasoning [8], [19].

In addition, socio-cultural perspectives on learning highlight the role of classroom interactions, language, and contextual factors in shaping mathematical understanding [22]. In multilingual educational environments such as South Africa, learners may face additional challenges when engaging with formal mathematical language used in geometry instruction. Effective teaching, therefore, requires pedagogical strategies that support mathematical communication, scaffold learners' reasoning processes, and connect geometric concepts to learners' prior knowledge and experiences.

By integrating the Van Hiele model, constructivist learning theory, and socio-cultural perspectives on mathematics education, this study provides a comprehensive framework for examining the geometry gap in South African schools. These theoretical perspectives help explain why learners struggle with triangle theorems and how instructional practices, curriculum design, and classroom environments influence the development of geometric reasoning. The framework also guides the analysis of potential interventions that can support more effective geometry teaching and learning.

8. METHODOLOGY

This paper adopts a conceptual research design, which focuses on developing theoretical insights through critical analysis and synthesis of existing scholarly literature. Unlike empirical studies that rely on primary data collection, conceptual research aims to advance knowledge by examining existing theories, identifying gaps in the literature, and proposing new frameworks that explain complex educational phenomena.

The methodological approach used in this study can be described as a systematic conceptual analysis combined with an integrative literature review. The purpose of this approach is to synthesise current knowledge on geometry education, particularly in relation to the teaching and learning of triangles and their theorems in secondary school mathematics. Recent scholarly publications from 2021 to 2024 were examined to ensure that the analysis reflects contemporary developments in mathematics education research, including post-pandemic educational reforms, technological integration in mathematics instruction, and emerging debates on educational equity.

The literature selection process focused on peer-reviewed journal articles, scholarly books, policy reports, and international assessment studies related to the following key themes:

a. geometry education and geometric reasoning
b. teaching and learning of triangle theorems
c. mathematics education inequalities
d. teacher knowledge and pedagogy in geometry
e. theoretical models of geometric thinking

The analysis involved identifying recurring themes, theoretical perspectives, and empirical findings across the selected literature. These insights were then synthesised to develop a conceptual explanation of the geometry gap in South African schools. By integrating multiple theoretical perspectives, including the Van Hiele model of geometric thinking, constructivist learning theory, and socio-cultural perspectives on mathematics education, the study proposes a conceptual framework that explains how structural, pedagogical, and cognitive factors interact to influence learners' geometry learning outcomes.

The conceptual methodology adopted in this study is particularly appropriate because the geometry gap represents a complex educational problem that cannot be fully understood through a single empirical investigation. Instead, a theoretically informed synthesis of existing research allows for a broader understanding of the systemic and pedagogical dynamics that shape geometry education. The resulting framework provides a foundation for future empirical research and offers insights that may inform curriculum development, teacher professional development, and instructional innovation in mathematics education.

9. Conceptual framework of the study

Based on the theoretical perspectives and literature reviewed, the geometry gap in South African schools can be understood as the result of the interaction between three major dimensions:

a. Structural Factors

- Socio-economic inequality
- Resource limitations
- School infrastructure disparities
- Access to technology and learning materials

b. Pedagogical Factors

- Teacher content knowledge
- Instructional approaches
- Professional development opportunities
- Use of technology in geometry instruction

c. Cognitive Factors

- Learners' level of geometric reasoning
- Conceptual understanding of triangle theorems
- Mathematical language and communication skills
- Learners' attitudes toward geometry

These three dimensions interact to influence learners' ability to develop geometric reasoning and successfully apply triangle theorems. The conceptual framework, therefore, suggests that addressing the geometry gap requires integrated interventions that simultaneously target instructional practices, teacher development, and structural inequalities within the education system.

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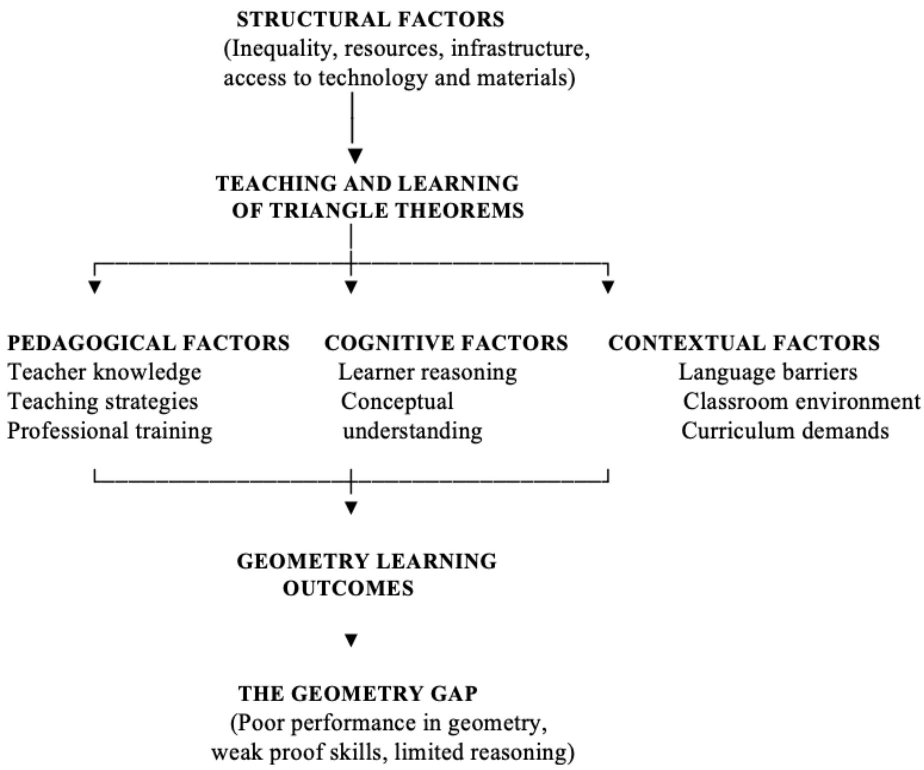

Figure 1: Conceptual Framework Explaining the Geometry Gap in South African Schools

The conceptual framework proposes that the geometry gap in South African schools emerges from the interaction of structural, pedagogical, and cognitive factors within the mathematics learning environment. Structural factors such as socio-economic inequality, limited educational resources, and disparities in school infrastructure create conditions that influence teaching and learning processes [2], [9]. These structural challenges often limit access to quality mathematics instruction and restrict opportunities for learners to engage with meaningful geometric learning experiences.

Pedagogical factors relate to the instructional practices used by mathematics teachers when teaching triangle theorems and other geometry topics. Research suggests that limited teacher content knowledge, inadequate professional development, and reliance on procedural teaching methods can reduce opportunities for learners to develop deep conceptual understanding of geometric principles [12], [21]. Consequently, learners may memorise theorems without understanding the logical reasoning that underpins them.

Cognitive factors refer to learners' levels of geometric reasoning and conceptual understanding. According to the Van Hiele model of geometric thinking, learners progress through hierarchical stages of understanding, from visual recognition of shapes to formal deductive reasoning [5], [15]. When instruction does not align with learners' cognitive readiness, learners may struggle to comprehend geometric proofs and relationships involving triangle theorems.

The interaction of these dimensions ultimately shapes learners' geometry learning outcomes. When structural constraints, pedagogical limitations, and cognitive challenges intersect, they contribute to the geometry gap, characterised by weak learner performance in geometry-related tasks, limited proof construction skills, and difficulties applying geometric theorems [11], [18].

10. Contributions of the study

This conceptual paper contributes to the field of mathematics education in several important ways. First, it provides a comprehensive synthesis of recent research on geometry learning challenges within the South African context. Second, it develops a conceptual framework that explains the underlying causes of the geometry gap by integrating multiple theoretical perspectives. Finally, the study offers practical insights that may inform curriculum reform, teacher professional development, and pedagogical innovation aimed at improving geometry instruction and learner achievement.

By highlighting the structural, pedagogical, and cognitive dimensions of geometry learning difficulties, the study provides a foundation for future research and policy initiatives aimed at strengthening mathematics education in South Africa.

11. CONCLUSION

This conceptual paper examined the persistent challenges associated with the teaching and learning of triangle theorems in South African schools. Drawing on recent literature and established theoretical perspectives in mathematics education, the study identified multiple factors that contribute to the geometry gap, including systemic inequalities, pedagogical limitations, and learners' difficulties in developing geometric reasoning. The analysis highlights that geometry learning challenges cannot be attributed to a single cause but rather emerge from the complex interaction between structural conditions, classroom practices, and cognitive development processes.

The findings emphasise the importance of strengthening geometry instruction in ways that promote conceptual understanding rather than procedural memorisation. In particular, the study highlights the relevance of the van Hiele theory of geometric thinking in explaining why many learners struggle with triangle theorems and geometric proof. When instruction does not align with learners' cognitive development levels, learners may experience difficulties progressing toward higher levels of geometric reasoning [5], [18].

Furthermore, the study underscores the need for improved teacher preparedness and professional development in geometry teaching. Teachers play a critical role in facilitating meaningful mathematical learning experiences, yet research indicates that some educators may lack the specialised knowledge required to teach Euclidean geometry effectively [12], [21]. Addressing these challenges requires targeted interventions that support teachers in developing both content knowledge and pedagogical strategies for teaching geometry.

Ultimately, closing the geometry gap requires a coordinated approach that addresses structural inequalities, enhances instructional practices, and supports learners in developing a deeper conceptual understanding of geometric relationships. By proposing a conceptual framework that integrates these dimensions, this study contributes to ongoing efforts to strengthen mathematics education and promote equitable learning opportunities in South African schools.

12. RECOMMENDATIONS

Based on the conceptual analysis presented in this study, several recommendations can be proposed to improve geometry education and reduce the geometry gap in South African schools.

12.1. Strengthening teacher professional development

Teacher professional development programmes should place greater emphasis on geometry instruction, particularly in relation to triangle theorems and geometric reasoning. Professional training should focus on enhancing teachers' conceptual understanding of geometry as well as equipping them with pedagogical strategies that support inquiry-based and learner-centred instruction [21].

12.2. Integrating technology in geometry instruction

The use of dynamic geometry software such as GeoGebra and other visualisation tools can significantly enhance learners' understanding of geometric relationships. These technologies allow learners to explore geometric concepts interactively and test mathematical conjectures, thereby supporting deeper conceptual learning [8], [19].

12.3. Promoting conceptual teaching approaches

Mathematics instruction should prioritise conceptual understanding rather than rote memorisation of theorems. Teachers should encourage learners to investigate geometric relationships, justify their reasoning, and construct logical arguments when solving geometry problems [3].

12.4. Addressing structural inequalities in education

Educational policymakers should continue to address systemic inequalities that affect teaching and learning conditions in South African schools. Improving access to learning materials, technological resources, and supportive learning environments is essential for strengthening mathematics education outcomes [2], [9].

12.5. Supporting learners' development of geometric reasoning

Instructional practices should be aligned with learners' cognitive development levels as proposed by the Van Hiele theory. Teachers should design learning activities that gradually guide learners from visual recognition of geometric figures to higher levels of analytical and deductive reasoning [5], [15].

13. IMPLICATIONS FOR FUTURE RESEARCH

While this conceptual study provides theoretical insights into the geometry gap, further empirical research is needed to investigate how the proposed framework operates in real classroom contexts. Future studies may explore how specific instructional strategies, teacher training programmes, or technological interventions influence learners' understanding of triangle theorems and geometric reasoning. Such research would provide valuable evidence to support the development of more effective mathematics education policies and practices.

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