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Didactic Transposition Analysis on Circle Material in High School

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ABSTRACT

Circle material is a fundamental topic in geometry that often causes misconceptions in high school students due to the difference between scientific knowledge and knowledge taught in school. This study aims to analyze the external didactic transposition of circle material by comparing the conceptual structure of scientific knowledge (scholarly knowledge) to the knowledge to be taught in high school mathematics textbooks. This study uses a qualitative descriptive approach with a document analysis method, comparing university-level geometry textbooks as a representation of scholarly knowledge with high school mathematics textbooks as a representation of knowledge to be taught. The results of the study show that external didactic transposition occurs in almost all circle concepts, characterized by simplification of language, adjustment of abstraction levels, reduction of concept classification, and addition of pedagogical and visualization contexts. The simplification and addition of pedagogical context can affect students' conceptual understanding, especially in the relationships among elements of circles, such as arcs, central angles, circumferential angles, and tangents. These findings open up further research opportunities into internal didactic transposition and other mathematics topics and can serve as a reference for teachers and book developers to improve the quality of coherent, conceptual mathematics learning.

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1. INTRODUCTION

Mathematics is a science taught at every level of education, from elementary school to college, and is closely tied to human life, as many daily activities involve mathematical concepts such as calculating, measuring, and making transactions [1]. In formal education, mathematics is a subject taught at every level and plays a strategic role in shaping students' thinking skills [2]. In addition to being related to numbers, formulas, and symbols, mathematics plays a strategic role in developing students' logical, critical, systematic, and reasoning skills, because, through the process of learning mathematics, these skills can be trained and developed in a directed manner [3]–[8]. In essence, mathematics has an abstract object of study and develops through a process of abstraction that produces formal concepts systematically arranged. At the high school level, learning mathematics becomes the foundation for understanding abstract concepts for advanced study and real problem-solving.

Mathematics learning in high school acts as a link to higher education while equipping students with academic and professional competencies [10]. However, in learning practice, it is still found that students tend to be procedural and have difficulty understanding concepts in depth, so that mathematical understanding does not develop optimally [11]. This condition shows the need for a learning approach that connects mathematics as scientific knowledge with mathematics taught in schools. Various studies show that there is a difference between the scholarly *knowledge* that develops in academic literature and the knowledge presented to students through the curriculum and school teaching materials, which is a consequence of the didactic transposition process, but has the potential to be a source of misconceptions and learning barriers if the conceptual structure is not reconstructed appropriately [12]. These differences can occur not only in certain topics but also across various mathematical materials, including geometry, which have strong connections between concepts. One of the geometric materials that requires a complete conceptual understanding is the material of circles, because the concepts in it such as radius, diameter, arc, center angle, circumference, and tangent are structurally interconnected, so that the process of transforming the concept of the circle from a scientific form to a teaching form needs to be systematically analyzed so that mathematics learning in high school can take place effectively and meaningfully.

Scholarly *knowledge* is knowledge that is produced and developed in the scientific community, formally arranged and organized in a mathematical concept system. This knowledge is not intended for teaching purposes, but undergoes a didactic transposition so that it can be adapted for teaching in schools [13], [14]. The mathematics knowledge taught in schools results from the transformation of scientific knowledge, and the quality of learning is greatly influenced by how the concept is reconstructed into teaching materials and practices [15]. In the context of school education, scientific knowledge cannot be presented in its entirety to students due to its high level of abstraction and complexity. Therefore, a transformation process is needed to adapt scientific knowledge to students' characteristics through curriculum and teaching materials [16]. Thus, didactic transposition can be understood as a process of adapting scientific knowledge to the school learning context so that the knowledge becomes meaningful and can be taught to students [17]. In the framework of didactic transposition, this process involves the selection, simplification, and reconstruction of scientific knowledge into *knowledge to be taught*, institutionalized in learning practice [18]. Inaccuracies in this transformation process have the potential to distort the meaning of mathematical concepts, which can further hinder students' conceptual understanding as a whole. There has been no study that explicitly analyzes the incompatibility between the conceptual structure of the circle material and scholarly knowledge, as presented in high school mathematics textbooks as the main source of learning. The problem this research addresses is the potential incompatibility between the conceptual structure of the circle material and its presentation in high school mathematics textbooks, which risks leading to misconceptions and hindering student learning.

The risk of misinterpretation is even greater when the scientific structure of the concept is not properly maintained during didactic transposition. The study confirms that misconceptions are often rooted in mismatches between the scientific structure of the concept and the form it is presented in schools, especially in areas of geometry that require visual representation and precise definition [19]. In the context of high school, the mathematical knowledge taught should still be based on *scholarly knowledge* that has been academically validated. Such inconsistencies can cause students to misinterpret the concept of a circle, such as an arc or tangent, thus hindering advanced geometry problem-solving skills [20]. Didactic transposition consists of two main stages, namely internal didactic transposition and external didactic transposition. Internal didactic transposition occurs at the stage of implementing learning in the classroom, when *knowledge to be taught* is realized into *knowledge actually taught* through teacher teaching practices and didactic interaction with students [13].

Furthermore, external didactic transposition occurs when curriculum designers and textbook writers reconstruct scientific knowledge into *knowledge to be taught,* which is outlined in the curriculum and school teaching materials [21]. Inaccuracies at the external level, especially in simplifying concepts and selecting representations, can lead to shifts in the meaning of mathematical concepts. As a result, this condition has the potential to cause misconceptions in students because the material presented no longer represents the formal mathematical structure in its entirety [22]. The two stages of didactic transposition are epistemological in nature and directly impact conceptual understanding.

Circle material is one of the fundamental topics in geometry, with a close relationship among concepts such as radius, diameter, arc, central angle, circumference, and related concepts such as arc length and tangent. The relationship between these concepts shows that understanding circles is structural and hierarchical, so mastery of prerequisite concepts is very important for learning circles [23]. However, recent research shows that high school students still hold many misconceptions about circles, especially in classifying circle elements, understanding the relationship between arcs and angles, and applying these concepts in problem-solving. The misconception is not only caused by erroneous preconceptions, but also by the presentation of material that emphasizes procedural aspects and the use of formulas without strengthening conceptual meaning [11], [22], [24], [25]. The didactic transposition study examines how *the scholarly knowledge* circle is reconstructed into teaching materials presented in schools through curriculum and textbooks. A number of studies show that, in this process, concepts are simplified, the level of abstraction is adjusted, and pedagogical context is added to align the material with students' cognitive characteristics. However, it is still necessary to maintain the core conceptual structure so that learning is mathematically meaningful [13], [15]. However, studies in Indonesia that specifically analyze the relationship between *scholarly knowledge* circles, their presentation in high school textbooks, and their implications for potential student misconceptions are still limited. Winarji's previous research emphasized the didactic transposition of the circle concept at the junior high school level and did not study in depth the comparison of conceptual structures between school textbooks and scientific knowledge sources at the high school level [23]. This condition shows that there is a research gap related to the analysis of external didactic transposition of circle material at the high school level which emphasizes the suitability of the conceptual structure between scientific sources and textbooks. Therefore, this study aims to analyze the external didactic transposition of circular material in high school by comparing the conceptual structure of scholarly knowledge and knowledge to be taught in mathematics textbooks.

Research on the Analysis of Didactic Transposition of Circle Materials in High School is an academic and practical urgency. Theoretically, this research contributes to strengthening the study of didactic transposition in the context of geometry, which, in recent years, has been widely recommended by mathematics education researchers [15], [19]. In practice, this research is important for curriculum developers, textbook writers, and teachers to ensure that the material taught in the circle is not only procedural but also reflects accurate scientific concepts, thereby minimizing misconceptions and improving students' conceptual understanding.

2. METHOD

This research method uses a qualitative descriptive approach with an analytical-descriptive type because the focus is on describing and analyzing how the concept of mathematics in circle material is transformed from scientific knowledge (*scholarly knowledge*) specified at the university level to knowledge to be *taught* in school textbooks. Analysis is a qualitative data collection technique that involves collecting and analyzing documents related to the research subject [26], [27]. The qualitative approach was chosen because it emphasizes understanding the meaning of the text and the context of presenting concepts in depth, without quantitative testing, and is suitable for the study of documents such as textbooks as well as for the process of transposing knowledge from one form to another. In mathematics education research, didactic transposition is understood as the process of transforming academic knowledge into an organized and simplified form to make it suitable for teaching in schools. This idea has been discussed in various studies in mathematics education to explain the transformation of scientific content into learning content (didactic transposition) and its relationship to the preparation of textbooks and curricula [22].

This study analyzed two data sources, namely textbooks as written documents. The first book is a university-level geometry textbook used in Geometry studies at the Mathematics Education Study Program of the University of Muhammadiyah Bengkulu, and is positioned as a representation of scholarly knowledge because its presentation is based on formal, abstract knowledge. In contrast, the second book is a textbook for the high school mathematics class XI Independent Curriculum, published by the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia, as a representation of the knowledge to be taught, as it is officially used as a learning material in schools. Both books were selected purposively based on content criteria relevant to the scope of the circular material to ensure a precise and informative focus of analysis. The selection of the comparative approach for these two books is consistent with the practice of didactic transposition research in the comparative study of textbooks that examine changes in the structure and organization of content between university and school books [12].

Data collection is carried out through document analysis, where the researcher observes, records, and maps the presentation of the concept of circles, including definitions, elements, properties, circumferential and wide formulas, symbolic representations, and examples of problems in the two books. The collected data is then reduced and presented in tables or matrices to facilitate the next stage of analysis. The data analysis technique used is content *analysis*, which aims to identify and describe didactic transposition patterns in circular material systematically. The analysis focuses on the simplification of concepts, changes in representation, and the rearrangement of material from *scholarly knowledge* to *knowledge to be taught*. Chronologically, the stages of analysis include: (1) Identification of the conceptual structure of the circle in the university book, (2) identification of the presentation of concepts in high school textbooks, (3) mapping of differences in conceptual structure, and (4) interpretation of the form of external didactic transposition that occurs.

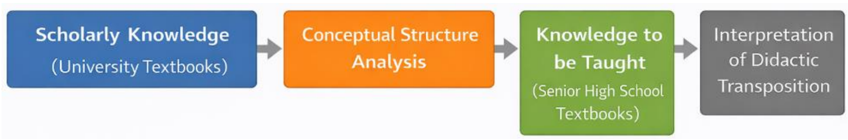

Figure 1. Stages of External Didactic Transposition

This approach is consistent with previous research that used content analysis to examine the transposition of mathematical concepts from academic sources into school textbooks [22]. To ensure the validity of the data (*trustworthiness*), this study applies qualitative validity through *an external audit, in which mathematics education experts with competence in didactic transposition theory and teaching material development review the data*. The qualitative validity of this kind of research emphasizes the importance of examining interpretations and analytical categories by parties outside the researcher to increase the credibility and consistency of the findings [12].

3. RESULTS AND DISCUSSION

The didactic transposition of knowledge about circles is analyzed through a comparison of knowledge in each unit of transposition, namely, scholarly *knowledge* and circle knowledge presented in school mathematics textbooks (*knowledge to be taught*). This analysis aims to identify how formal and theoretical mathematical knowledge is transformed into knowledge that can be taught to students, taking into account cognitive characteristics and school learning objectives.

According to the didactic theory of transposition put forward by Chevallard, the knowledge taught in schools is not identical to scientific knowledge, but instead results from a transformation that involves simplification, selection, and adjustment of the context [13]. Therefore, the difference between *scholarly knowledge* and *knowledge to be taught* is a logical consequence of pedagogical needs, but it still needs to be studied critically to avoid distorting conceptual meaning. The focus of analysis in this study is the concepts of elements and definitions in the material of circles, because these concepts play a fundamental role in building students' relational understanding of circle geometry. The following presents the findings on line and angle knowledge in each transposition unit.

Scientific Knowledge About Circles (*Scholarly Knowledge*)

Scientific knowledge about circles is obtained through the analysis of geometry textbooks and journal articles of research results that represent the formal structure of mathematics. This knowledge is seen as a priori, compiled in a deductive, systematic manner, and based on definitions and axiomatic relationships between concepts [28]. In the context of didactic transposition, scientific knowledge serves as the primary source (reference knowledge) for preparing learning materials in schools.

In scientific knowledge, a circle is defined as the set of all points on a plane that are equally spaced from a fixed point called a center. This definition places a circle as a geometric object based on a *set of points (locus of points*), thus emphasizing conceptual precision and mathematical formality. From this basic definition, derivative concepts such as radius, diameter, arc, arc string, juring, tembereng, center angle, circumferential angle, and tangent are developed, all of which are explained through precise and interrelated definitions.

Scientific knowledge of circles contains a relatively complete classification of concepts. For example, circular arcs are classified into small arcs, large arcs, and semicircles based on their size relative to the circle. The rules for naming and symbolizing circular objects are also formally explained, including the use of arc symbols and the naming provisions based on the number of dots. This presentation shows that scientific knowledge focuses not only on the recognition of concepts, but also on the formation of conceptual structures that support deductive reasoning and mathematical proofs.

With these characteristics, scientific knowledge about circles is highly abstract and relatively free of everyday-life context. Therefore, this knowledge requires a didactic transposition to adapt it to the learning needs of schools.

Table 1. Summary of Analysis of Scientific Textbooks Circle Material

	Definition	***Scholarly Knowledge***
1.	Definition of a Circle	A circle is a structure consisting of points with equal distances from the central point. (page 118).
2.	Definition of Radius	Radius is any part of a line with one endpoint at the center of the circle and the other end at the circle. (Hal 118)
3.	Definition of an Arc	A circular arc is a part of a circle (page 122)
4.	Types of Arcs	There are 3 types of bows, namely: ▪ semicircle: an arc whose endpoints are the endpoints of a diameter, ▪ Small arc: an arc that is less than half a circle. ▪ Large Arc: An arc that is more than half a circle (page 122)
5.	Names and Symbols of Arcs	The symbol ⌒ is used to indicate a bow symbol written over the endpoints that make up the arc. The small bow is named using only two points of the end of the bow. The large bow is named 3 points. The first and third points are the endpoints, and the midpoint is each point on the arc between the endpoints (page 122)

6.	Definition of Central Angle	The center corner is the angle formed by the two radius of the circle. The corner point is the center of the circle (page 121)
7.	Definition of Inscribed Angle	Circumferential angle: An angle formed by two bowstrings with an angle point on a spiral. (page 126)
8.	Definition of a Chord	A bowstring is any part of a line whose ends are on a circle. (page 119)
9.	Definition of Tangent and Point of Tangency	A tangent is any line that is flat with a circle that cuts the circle at only one point. A tangent point is a tangent between a tangent and a circle (page 119)
10.	Definition of Common Tangent	▪ A common tangent is a line that intersects two circles that are equal. ▪ An external common tangent is a common tangent that does not intersect the line segment joining the centers of the two circles. ▪ An internal common tangent is a common tangent that intersects the line segment joining the centers of the two circles.

Circle Knowledge in School Math Textbooks (*Knowledge to be taught*)

The circle knowledge in the school mathematics textbook analyzed in this study is a form of *knowledge to be taught*, namely, scientific knowledge that has been transformed to align with the learning objectives, the curriculum, and students' characteristics. The circle material in the textbook is presented in a special chapter and serves as the main learning resource for students at school.

In contrast to scientific knowledge, the definition of a circle in school textbooks is presented in simpler and more communicative language, for example, as "the position of points that are equal in distance from a certain point". Although mathematically equivalent to the scientific definition, this redaction shows the simplification of formal language into pedagogical language. Concepts such as radius, diameter, arc, and center angle are introduced gradually and reinforced with visual illustrations to aid students' understanding.

In addition, school textbooks tend to simplify the classification of circle concepts. Some categories of scientific knowledge are not always presented in full but are selected based on learning needs. On the other hand, school textbooks use contextual and visual approaches, such as images, examples of circular objects, and illustrations of everyday phenomena, to relate abstract concepts to students' concrete experiences. This approach aligns with the constructivist view, which emphasizes the importance of context in developing mathematical understanding.

Thus, the circle of knowledge in school mathematics textbooks results from the selection, simplification, and adjustment of scientific knowledge. This process aims to make the material easier for students to learn, but at the same time has the potential to reduce the completeness and depth of the conceptual structure.

The circular material is presented in Chapter 2. The following is a description of the line and angle material contained in the textbook.

Table 2. Summary of School Textbook Analysis Circle Material

	Definition	*Knowledge to be taught*
1.	Definition of a Circle	A circle is the location of points that are equal to a certain point (called the center of the circle). (page 49)
2.	Definition of Radius	That same distance is called the radius. The line segment connecting the center of the circle to one of its points is also called a radius. (page 49)
3.	Definition of an Arc	The part of the circle is called the arc of the circle (page 50)
4.	Types of Arcs	There are 2 types of bows, (The smaller arc is called the minor arc, and the larger part is called the major arc. (page 50)
5.	Names and Symbols of Arcs	Arc BC is written as $\widehat{BC}$. Magnitude $\widehat{BC}$. Determined by the magnitude $\angle BAC = \alpha$ (point A is the center of the circle) (page 50)
6.	Definition of Central Angle	The angle α is called the central angle facing the $\widehat{BC}$. The center corner is the angle whose corner point is located at the center of the circle, and the angular legs are the radius of the circle. (page 50)
7.	Definition of Inscribed Angle	The angle θ is called the circumferential angle facing the $\widehat{BC}$. The circumferential angle is the angle where the angular point is located on the spiral, and the angular legs are in the form of a bowstring (page 50)
8.	Definition of a Chord	An arc string is a line segment connecting two points on a circle (page 50). An example and picture of an arrow is given, where the string of arrows connecting two points in a circle is called a bowstring (page 70)
9.	Definition of Tangent and Point of Tangency	In school textbooks, a picture of the train wheel touching the train at one point. Mathematically, it is said that the rails are the tangents of the wheel, and their contact points are referred to as tangents. (page 61)
10.	Definition of Common Tangent	A common tangent is a tangent that is the tangent of two circles. $\overline{CD}$ is the outer federal tangent for Circle A and Circle B (page 66), along with a picture of 2 circles and tangents. $\overline{EF}$ is the inner federal tangent for Circle A and Circle B (page 67), along with an example of a picture of two circles and tangents. The tangent of the federation is also depicted through contextual examples such as bicycle chains, the sun, the moon, and the Earth.

Based on tables 1 and 2 above, the comparison between scholarly *knowledge* books and school mathematics textbooks (*knowledge to be taught*) shows that there are variations in the form of didactic transposition in the circular material. In the material on the definition of the circle, an external didactic transposition was found. The transposition is characterized by the simplification of language, the addition of conceptual information, and adjustments to the level of abstraction in school textbooks. This adjustment aims to help students understand the concept of circles more concretely, without losing their mathematical essence as sets of points equally distant from the center.

In the material on the definition of radius, it was found that there was an external didactic transposition in the form of simplifying terms and adjusting the level of formality of the definition. School textbooks tend to emphasize the meaning of distance from the center of the circle rather than formal geometric structures. This transposition is pedagogical and aims to make it easier for students to relate the concept of fingers to the concept of circles as a whole.

In contrast to the previous material, the definition of the arc lacks external didactic transposition. The difference between scientific knowledge books and school textbooks is editorial and does not show any change in the level of abstraction, simplification of concepts, or pedagogical adjustments. Therefore, these differences cannot be categorized as a form of didactic transposition. However, in the material on the types of circular arcs, it was found that a simplification of the classification of concepts characterized an external didactic transposition. School textbooks present a smaller number of classifications than scientific knowledge books. This simplification aims to adapt the material's complexity to schools' learning needs without eliminating the basic concept of circular arcs.

External didactic transposition is also found in the material on names and the writing of bow symbols; it appears in the form of simplifying notation rules and shifting the focus of discussion. School textbooks emphasize the relationship between the arc and the center angle more than the accuracy of formal symbol conventions. This transposition is pedagogical and aims to introduce arc notation in a contextually appropriate manner, tailored to students' levels of understanding.

In the material on the definitions of central and circumferential angles, external didactic transposition is evident through the addition of conceptual contexts in school textbooks. The center angle and the circumferential angle are not only defined based on their geometric elements, but are also explicitly associated with the arc of the circle with which they face. Adding this context aims to help students understand the relationship among the circle's elements without altering the mathematical meaning of each concept.

Furthermore, in the definition of the bowstring, there is an external didactic transposition characterized by the simplification of the definition and the concretization of concepts through the use of examples and visual illustrations. School textbooks present a bowstring as a line segment explicitly connecting two points on a circle. This transposition aims to make it easier for students to distinguish the bowstring from other circular elements. The same thing was also found in the material on the definition of tangents: there was an external didactic transposition, in the form of the use of real context and visual illustrations in school textbooks. This approach replaces the presentation of formal definitions contained in scientific knowledge books. The transposition is pedagogical and aims to help students build an intuitive understanding of the concepts of tangents and tangents.

In the material on the definition of communal tangents, there is an external didactic transposition characterized by the simplification of formal definitions and the use of illustrations and real context in school textbooks. School textbooks postpone the presentation of formal criteria and emphasize more conceptual visualization. This adjustment aims to make it easier for students to understand the concept of the federal tangent without changing its mathematical meaning.

Based on these findings, there is an external didactic transposition process in the circle material, characterized by simplification of language, adjustment of the abstraction level, reduction of concept classification, and the addition of pedagogical context. These changes are mainly found in the concepts of the definitions of circles, radii, types of arcs, writing arc symbols, center angles, circumferential angles, arcstrings, tangents, and common tangents. Meanwhile, in the concept of the circular arc, no significant conceptual changes were found, so the structure of scientific knowledge was maintained in the knowledge to be taught.

The simplification and addition of pedagogical context to some circle concepts have implications for how these concepts are represented in learning. Based on the findings of the analysis, the simplification of the classification of bows in school textbooks causes not all types of bows discussed in scientific knowledge books to be introduced to students, so that the formal relationship between types of bows is not fully displayed. In addition, in the writing of bow symbols, school textbooks emphasize more on the relationship of the bow to the central angle, while the rules of formal notation, as presented in scientific knowledge books, are not explicitly discussed. In the concept of tangent and federal tangent, the delay in presenting formal criteria, such as the requirement that the fingers be straightened to the tangent line, leads school textbooks to emphasize more visual and illustrative examples. As a result, students' understanding is more focused on recognizing the forms and visual features of the concept, while the mathematical basis underlying it has not been introduced in depth at this stage. The findings have undergone validation by mathematics education experts, and the validation results show that the categories and interpretations of the identified didactic transposition forms are in accordance with the characteristics of the material and the objectives of the analysis set.

These differences in conceptual structure can affect how students build relationships among circle concepts. Simplification that is not balanced with conceptual reinforcement can lead to misconceptions, especially in understanding the relationships among arcs, central angles, circumferential angles, and other elements of a circle. These findings are in line with previous research indicating that simplification of concepts and changes in presentation during didactic transposition can trigger misconceptions if they are not accompanied by adequate conceptual explanations [22], [23].

The findings of this study also support the view that didactic transposition is an integral part of mathematics learning [16]. However, this process can create learning barriers if concepts are simplified without maintaining the coherence of the mathematical knowledge structure [24]. Therefore, caution is needed when transposing external didactics, so that the pedagogical adjustments made continue to support the development of a complete conceptual understanding in students.

4. CONCLUSION

This study shows that the material of circles in high school undergoes external didactic transposition from scientific knowledge (*scholarly knowledge)* to knowledge *to be taught* in school textbooks. This process shows a change in the conceptual structure as a form of adaptation to the school learning context, which has the potential to affect the coherence of concept understanding if it is not accompanied by strengthening the relationship between concepts.

The implications of this study emphasize the importance of maintaining harmony between the scientific structure of mathematical concepts and their presentation in teaching materials. Practically, these findings guide textbook writers, curriculum developers, and teachers to pay closer attention to the integrity of concepts when presenting circular materials, so that learning is not only procedure-oriented but also supports the development of students' conceptual understanding.

This study has limitations because it analyzes only external didactic transposition in circle material in high school mathematics textbooks, without examining classroom implementation and its direct impact on student understanding. In addition, this study is limited to one geometry topic, so the results cannot be generalized to other mathematics materials.

Based on these limitations, further research is recommended to examine internal didactic transposition and expand studies on other mathematical topics. This research is expected to improve the quality of mathematics learning and teaching materials in schools, enabling students to build a more conceptual, coherent, and meaningful understanding of mathematics.

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