

Students' Analogical Reasoning in Solving Number Pattern Problems

Intan Juniarmi¹, Rohati², Duano Sapta Nusantara³

^{1,2,3}Universitas Jambi, Jambi, Indonesia

Article Info

Article history:

Received 2025-09-26

Revised 2025-11-20

Accepted 2025-11-20

Keywords:

Analogical reasoning

Junior high school

Mathematical problem solving

Number patterns

Student answers

ABSTRACT

This study aimed to analyze students' answers in solving number pattern problems in terms of analogical reasoning ability. Six seventh-grade students with varying academic levels participated in the study. Data were collected through written tests and semi-structured interviews, then analyzed using the analogical reasoning framework, which includes four indicators: encoding, inferring, mapping, and applying. The results showed that the total scores of the six participants ranged from 18 to 30. Student 1 obtained the lowest score (18), indicating weaknesses particularly in applying pattern rules to determine the required terms correctly. In contrast, Students 3, 4, and 5 achieved the maximum score of 30, demonstrating consistency in recognizing pattern rules, mapping structural similarities, and applying their reasoning accurately across tasks. The average total score was 27.5, suggesting that most students demonstrated relatively strong analogical reasoning skills, although some individuals still experienced difficulty in the applying stage, which demands higher precision and conceptual understanding. The study contributes to understanding how analogical reasoning influences students' mathematical problem-solving and offers pedagogical insights for enhancing instruction of non-routine tasks through reasoning-based learning strategies.

This is an open-access article under the [CC BY-SA](#) license.



Corresponding Author:

Rohati

Program Studi Magister Pendidikan Matematika, Universitas Jambi

Email: rohati.fkip@unja.ac.id

1. INTRODUCTION

In the classroom, students' ability to solve math problems often shows contrasting patterns [1], [2]. When faced with routine problems where the steps for solving are clear and similar to the examples given by the teacher, most students can solve them well [3]. However, a different situation arises when the problem is presented in a non-routine or contextual form that demands deeper understanding [4]. In this condition, many students appear confused, unable to connect concepts learned with new situations, and even produce

erroneous or incomplete answers [5]. This phenomenon suggests that procedural mastery alone is insufficient to guarantee students' success in solving math problems as a whole [6].

The answers students write are not just the result of calculations, but also a reflection of how they think and understand concepts [7]. Through these answers, teachers and researchers can track the cognitive processes that students undergo, including the strategies they employ and the difficulties they encounter [8]. Analysis of the errors provided valuable information about misconceptions, reasoning errors, or weaknesses in linking relevant concepts [9]. This information is important because it can help teachers understand students' weak points more specifically, so that the learning strategies designed are not only focused on achieving results, but also improving students' thinking processes to make them more meaningful.

One of the thinking skills that plays an important role in solving math problems is analogical reasoning [10]. This ability refers to the process of finding similar structures or relationships between two different situations [11]. In the context of learning mathematics, students are required not only to remember procedures, but also to be able to transfer knowledge from familiar problems to new problems that have similar patterns [12]. The analogical reasoning process can be reviewed through several stages, namely encoding information from the given problem, inferring relevant relationships, mapping the relationship between problem elements, and applying this knowledge to produce the right solution [13]. It is through this stage that we can see how students build bridges between old understandings and new challenges, so the quality of their answers is an important indicator of the depth of their reasoning.

Number patterns are one of the basic materials in mathematics taught at the junior high school level. This material not only demands numeracy skills, but also students' ability to recognize regularities, find relationships, and predict the continuation of a pattern [14]. In this process, students are actually trained to see the similarity in structure between the familiar pattern and the new pattern given. This activity is closely related to analogical reasoning, as students must encode information from visible patterns, infer hidden rules, map the relationship between old and new patterns, and apply this knowledge to determine the continuation of the pattern. Thus, number pattern material is the right context to examine students' analogical reasoning skills through analyzing the answers they produce [15], [16].

So far, many studies in the field of mathematics education have focused more on the achievement of student learning outcomes, such as the level of mastery of material, test scores, or achievement of competencies [17], [18]. Such a focus is important, but it often overlooks the thought processes behind students' answers. In fact, a deeper understanding of how students reason can provide a more complete picture of the quality of their learning. In particular, studies that use the analogical reasoning framework as an analytical knife are still very limited. However, although several studies have explored how students think in solving number pattern problems, only a few have specifically examined how analogical reasoning is reflected through students' written solutions and thought processes. This gap underscores the need for research that examines students' reasoning processes through a structured framework of analogical reasoning. This leaves room for research that not only

captures the correctness of answers but also examines how students arrive at these answers through the process of analogical reasoning.

Considering these conditions, it is essential to gain a deeper understanding of students' thinking profiles when solving math problems. Analysis of students' answers will not only reveal the correctness or incorrectness of the results, but can also indicate the extent to which they can apply analogical reasoning in processing information [19]. The results of this analysis can serve as a basis for teachers to develop learning strategies that emphasize the reasoning process, enabling students to be not only skilled in working on routine problems but also flexible in dealing with new ones.

In addition, this research is expected to contribute to the development of the literature on the relationship between students' answers and analogical reasoning ability, which is still rarely discussed in depth. Thus, the purpose of this study is to analyze students' answers to mathematics problems in terms of their analogical reasoning ability, thereby obtaining a comprehensive picture of students' thinking patterns and understanding the implications for learning mathematics in the classroom. Therefore, this study aims to describe and analyze students' analogical reasoning processes, including encoding, inferring, mapping, and applying when solving number pattern problems. Through this analysis, this study aims to provide a deeper understanding of how students comprehend, connect, and apply mathematical concepts in the context of number patterns.

2. METHOD

2.1. Research Design

This study used a descriptive qualitative research design. This design was chosen because the research objective was to describe in depth the students' answers in solving math problems on number pattern material in terms of analogical reasoning ability. The focus was not only on the correctness of the answers but also on how students encoded information, inferred rules, mapped relationships, and applied pattern structures to generate solutions.

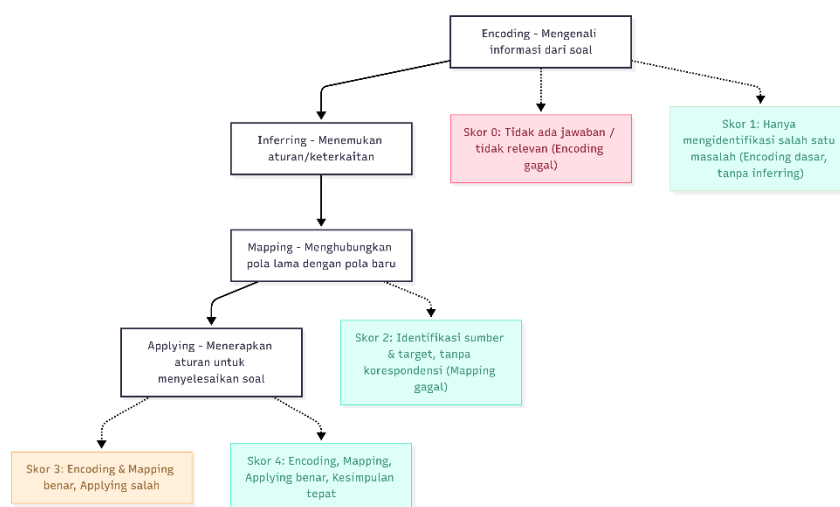


Figure 1. Process Analogical Reasoning

2.2. Participants

The subjects of this research were seventh-grade students who had studied material on number patterns. The selection of subjects was conducted using a purposive sampling technique, as researchers required participants who could provide information in line with the research focus. From all students in the class, six participants were selected based on variations in ability levels, specifically high, medium, and low. The selection of six participants with varying ability levels aims to enable the researcher to obtain a more comprehensive understanding of how analogical reasoning is employed in the problem-solving process. Thus, the analysis conducted not only represents one particular ability group but also describes the tendency of students' thinking patterns more broadly, especially in linking familiar problems with new problems in the context of number patterns. This study followed standard ethical procedures, and all personal information was anonymized.

2.3. Instruments

Table 1. Alignment of Test Items with Analogical Reasoning Indicators in Number-Pattern Tasks

Cognitive Process	Explanation	Item
Encoding	Students identify and extract relevant information from a number pattern, such as recognizing the constant difference or the structure of the sequence.	• Task 1(a): "Explain the rule of the number pattern 2, 4, 6, 8," • Task 4(a): "State the rule of the pattern 4, 7, 10, 13,"
Inferring	Students infer or formulate the underlying rule based on observed regularities in the pattern.	Task 2(a): "State the rule used in Pattern A (3, 6, 9, 12, ...) and Pattern B (5, 10, 15, 20, ...)."
Mapping	Students compare two patterns to identify similarities and differences, showing how they relate to the structures of the sequences.	• Task 2(b): "What are the similarities and differences between Pattern A and Pattern B?" • Task 4(c): "Is the pattern 2, 5, 8, 11, ... similar to the pattern 4, 7, 10, 13, ...? Explain your reason."
Applying	Students apply the inferred rule to new situations, such as determining the n -th term or generating a new sequence.	• Task 1(b): "Determine the 10th term of the pattern 2, 4, 6, 8," • Task 3: "Create a new pattern with the same rule (+5), but start from the number 7." • Task 4(b): "Determine the 12th term of the pattern 4, 7, 10, 13," • Task 4(d): "Create a new pattern with the same rule, but start from 6."

The main instrument was a written test consisting of four number-pattern problems, each aligned with one indicator of analogical reasoning (encoding, inferring, mapping, and

applying). The tasks involved simple contextual number patterns, requiring students to identify rules, compare patterns, and extend sequences. The scoring rubric was adapted from Loc & Uyen [20] by adjusting the descriptors to match the four indicators of analogical reasoning used in this study. The rubric included a 0–4 scale for each indicator, with clearer operational definitions tailored to number-pattern tasks. To ensure content validity, the test and rubric were reviewed by two experts. Revisions were made based on their suggestions regarding clarity, cognitive demand, and alignment between items and indicators.

2.4. Data Collection

Data were collected through written tests followed by semi-structured interviews to explore students' reasoning in greater depth. Each participant completed the written test individually and was then interviewed to clarify their thought processes, selected strategies, and interpretation of the tasks.

2.5. Data Analysis

Data analysis in this study was conducted through three main stages: data reduction, data presentation, and conclusion. At the data reduction stage, student answers obtained from written tests and interviews were selected, organized, and categorized according to the research focus, allowing for more focused analysis of relevant information. Furthermore, at the data presentation stage, the results of the analysis were presented in the form of narratives and tables based on indicators of analogical reasoning ability, allowing students' reasoning patterns to be observed more clearly. The last stage is conclusion drawing, where the analyzed data is interpreted to identify trends, differences, and weaknesses in students' thinking processes.

The analysis of students' answers uses the analogical reasoning ability framework, which includes four main indicators. First, encoding, which is how students recognize and represent information from number pattern problems. Second, inferring, which is how students find the rules or relationships that underlie number patterns. Third, mapping, which is how students build relationships between familiar patterns and new patterns given. Fourth, applying, which involves students using the results of their reasoning to solve problems effectively.

To maintain data validity, this study used triangulation techniques. Triangulation was carried out by comparing data from written tests and interview results, so as to obtain a deeper understanding of how students solve problems. In addition, peer debriefing was conducted to ensure that the interpretation of the analysis results was consistent and unbiased. In this way, the validity of the research findings can be better guaranteed while strengthening the arguments built in the discussion.

3. RESULTS AND DISCUSSION

After all data is collected through written tests and interviews, the next step is to analyze student answers using the analogical reasoning framework. This analysis aims to examine how students recognize patterns, draw connections, create maps, and apply their

knowledge to solve problems. The findings of this analysis process will be presented in the research results section, beginning with the presentation of student answer scores obtained through the assessment rubric, followed by a description of sample answers that represent variations in students' thought processes.

3.1. Results

A more in-depth picture of students' analogical reasoning ability was obtained through the analysis of six students' answers to the number pattern problems given. The analysis process was conducted using a modified scoring rubric from [20], which comprises four main indicators: encoding, inferring, mapping, and applying. Through this rubric, each student's answer is assessed and scored between 0 and 4 based on the quality of reasoning demonstrated. Furthermore, the scores of each student are presented in a table to show the differences in achievement on each question. After that, some representative examples of students' answers, both correct and incorrect, are presented to provide a real illustration of students' thinking patterns and common errors. In this way, readers can more easily understand the profile of students' analogical reasoning on number pattern material, as well as see the variety of ways they solve mathematical problems.

Table 1 presents the scores from the analysis of six students' answers based on the analogical reasoning rubric. The scores shown cover each subtask, starting from the identification of pattern rules, comparison between patterns, and the application of new patterns. This table illustrates the variation in students' abilities to carry out the encoding, inferring, mapping, and applying processes. The total score of each student also provides an overview of the overall analogical reasoning profile, making it easier to compare the strengths and weaknesses of each individual in solving number pattern problems.

Table 2. Scores for Each Component of Analogical Reasoning

Student Label	Score
S1	18
S2	29
S3	30
S4	30
S5	30
S6	28

Based on the results, the total scores of the six participants ranged from 18 to 30. Student 1 obtained the lowest score, 18, which indicates that there are still many weaknesses, especially in applying pattern rules to the appropriate answer form. In contrast, Students 3, 4, and 5 achieved the maximum score of 30, showing consistency in recognizing rules, mapping similarities between patterns, and applying the results of their reasoning appropriately. The average total score was 27,5, indicating that most students demonstrated relatively strong analogical reasoning skills. This difference in scores indicates that most students have been able to demonstrate the encoding, inferring, mapping, and applying processes well; however, there are still individuals who face difficulties, especially in the applying stage, which requires higher accuracy. In general,

this data indicates that the majority of students possess good analogical reasoning skills, although there are individual variations that require attention during the learning process.

In addition to the total score, a more comprehensive understanding of students' analogical reasoning ability can be obtained by directly examining the answers they give. The analysis of representative answers aims to illustrate exactly how students understand pattern rules, map between patterns, and apply their reasoning to solve problems. Some answers were selected as representative examples, both those that showed correct solutions and those that still contained errors. In this way, students' thinking patterns and potential difficulties they face can be seen more clearly, and provide an in-depth picture of the analogical reasoning process that takes place in the classroom.

Figures 2–7 present representative examples of correct and incorrect responses across different ability levels, illustrating how students demonstrated encoding, inferring, mapping, and applying these concepts in their written solutions. The analysis begins with Problem 1, which requires students to recognize the rule in a simple number sequence and determine the 10th term of the pattern. This problem is designed to test encoding and applying skills, i.e., how students understand the regularity in number patterns and use it to find numbers in certain positions. The students' answers varied from those who correctly wrote the rule and calculated the 10th term according to the procedure, to those who only wrote part of the continuation of the pattern without achieving the required result. The selection of examples of students' answers to this problem is intended to show the differences in the level of accuracy and application of the rules on which their analogical reasoning skills are based.

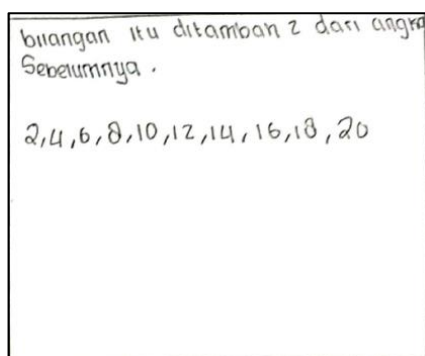


Figure 2. Answer S4

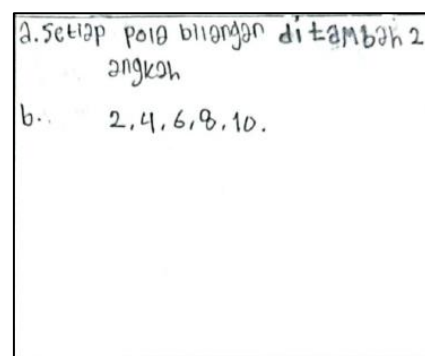


Figure 3. Answer S2

In part (a), Student 4 wrote the rule clearly that the pattern increased by 2 from the previous number. This demonstrates good encoding skills, as students can recognize the regularity in the sequence. Furthermore, in part (b), Student 4 continues the sequence until it reaches the 10th number, which is 20, as requested by the question. This process demonstrates that students not only understand the rule (inferring) but are also able to apply it consistently to obtain the correct result (applying). This answer reflects a complete analogical reasoning profile, from pattern recognition to the correct application of the rule. To further clarify S4's reasoning during the task, an excerpt from the interview is presented below:

"I saw that each number increases by two, so I just kept adding two until I reached the tenth term. It was easy because the pattern is consistent."

Unlike Student 4, Student 2 did write the rule in part (a) correctly, namely "each number pattern plus 2 numbers". However, in part (b), there was a misunderstanding: the student wrote 2, 4, 6, 8, 10, so it stopped when the number 10 appeared, not finding the 10th term of the sequence. This error indicates that although the encoding was done correctly, the applying stage did not proceed according to the requirements of the question. Students failed to interpret the question correctly, so the procedure used stopped before reaching the answer. This illustrates that the weakness lies not in understanding the rule, but in applying it in the context of a more specific question. To better understand the source of S2's error, the following interview excerpt illustrates the student's interpretation of the question:

"I thought the question was asking for the number 10 in the sequence, not the tenth term. So I stopped when I reached 10."

Problem 2 requires students to identify the rules of two different number patterns: Pattern A, which involves multiples of 3, and Pattern B, which involves multiples of 5. Then, they must compare the similarities and differences between the two. Through this problem, encoding and inferring skills are tested when students determine the rules of each pattern, while mapping is tested when they are asked to find the relationship between the two patterns. The variety of students' answers showed a difference in the level of rigor: some were able to correctly mention the rules while identifying the similarities and differences of the patterns, while others only mentioned some of the information without adequate comparative analysis. To illustrate this difference, examples of students' correct and incorrect answers are provided.

a. Pola A harus ditambah 3 angka
 Pola B harus ditambah 5 angka
 b. Sama harus ditambah beberapa angka.
 dan kalau perbedaannya kalau pola
 A. ditambah 3 angka, kalau pola
 B. ditambah 5 angka.

Figure 4. Answer S4

In Problem 2, most students correctly recognized the rules in each pattern: Pattern A increases by 3 and Pattern B increases by 5, so the encoding and inferring stages were generally not an obstacle. The main difference appeared at the mapping stage, when students were asked to compare the two patterns. As shown in Figure 4, S4 correctly stated the rule for both Pattern A and Pattern B and was also able to identify the structural similarity between the two sequences. S4 demonstrated strong mapping skills by recognizing that both patterns share the number 15, indicating an understanding of how the

sequences relate to one another. To provide deeper insight into how S4 identified the structural similarity between the two patterns, the following interview excerpt illustrates the student's reasoning:

"Both patterns increase by the same amount each time, but Pattern A adds three and Pattern B adds five. I noticed they both have the number fifteen, so I thought that means the patterns overlap at that point."

Similar answer patterns were also exhibited by Students 2 and 5, who thoroughly mentioned both the similarities and differences. In contrast, Students 3 and 6 only wrote some information, for example, only the similarities or added differences that were not precise, thus showing a partial mapping. Meanwhile, Student 1 failed to display relationship analysis by only writing the abbreviation "A.B." without further explanation. This finding shows that although the majority of students are quite good at recognizing pattern rules, the ability to map relationships between patterns still varies and is a weak point for some students.

Problem 3 asked students to create a new number pattern with the same rule as the previous pattern, i.e., always plus 5, but starting from 7. This question primarily assessed the application ability, as students were not only asked to recognize the rule but also to apply it to a new context with a different starting point. The answers that emerged showed variation: some students were able to write the pattern correctly, namely 7, 12, 17, 22, and so on, while others experienced calculation errors, resulting in inconsistent results with the recognized rule. By examining students' correct and incorrect representative answers, we can see the extent to which students can transfer their understanding of number patterns to new situations.

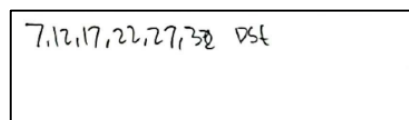
A rectangular box containing the handwritten text "7, 12, 17, 22, 27, 32,". The handwriting is in black ink on a white background.

Figure 5. Answer S5

As shown in Figure 5, Student S5 successfully constructed a new number pattern beginning at 7 while applying the same +5 rule. This indicates a strong ability to transfer the known structure to a new context. Student 5 answered by writing the sequence 7, 12, 17, 22, 27, 32, This answer shows that students can understand the meaning of the question well (encoding), identify the rule of adding 5 from the previous pattern (inferring), and then apply it correctly to the new starting point (applying). The strength of Student 5's answer is evident from his consistency in adding the number 5 sequentially without counting errors, ensuring that the formed pattern is correct. This confirms that students do not just memorize patterns, but actually transfer knowledge from the initial pattern to the new pattern according to the requested context. Thus, Student 5 demonstrated a complete ability in analogical reasoning in this problem, particularly in the application stage, which was the main focus. To further clarify how S5 constructed the new number pattern, the following interview excerpt provides insight into the student's reasoning process:

"I knew the pattern always adds five, so when the question said to start at seven, I just kept adding five to make the new sequence. It is the same rule, only the starting number changes."

Problem 4 consists of several parts that are more complex than the previous problems, because it not only requires students to find the pattern rule and determine the n th term, but also to compare two different patterns and form a new pattern with a similar rule. Thus, this problem tested almost all aspects of analogical reasoning: encoding when recognizing patterns, inferring when determining the rule of addition, mapping when comparing similarities in structure between patterns, and applying when applying rules to find specific terms and construct new patterns. Students' answers showed a wide variation: some were able to answer correctly by mentioning the rule, calculating the 12th term, and forming a new pattern correctly, while others made mistakes in applying the rule, failed to write down the required 12th term, or provided general explanations without accurate calculations. A representative analysis of the correct and incorrect answers to this question will clearly show students' strengths and weaknesses in each stage of analogical reasoning.

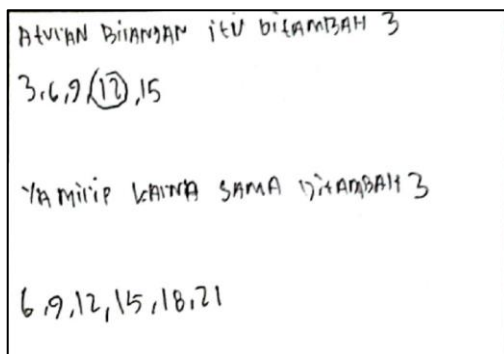


Figure 6. Answer S5

In part (a), Student 5 wrote the rule clearly that the number pattern increases by 3. This demonstrates the correct encoding and inference skills, as students can explicitly recognize the regularity of the pattern. In part (b), the student wrote the sequence 3, 6, 9, 12, 15, with the number 12 marked with a circle. This answer does not match the requested pattern, because the pattern should start from 4, not from 3, and what is requested is the 12th number, not just continuing the pattern until 12 appears. This error reveals a misunderstanding of the question's meaning. From the analogical reasoning point of view, the student had actually encoded the rule, but failed in the applying stage because she was unable to apply the rule correctly to find the 12th term. In part (c), the student wrote that the pattern "yes, it is similar, because it always adds 3". Although the explanation is brief, this answer demonstrates an awareness of the structural similarity between patterns. This reflects good mapping skills, although the reasoning given is still very simple. In part (d), the student constructed a new pattern starting from 6, namely 6, 9, 12, 15, 18, 21. This answer is correct and demonstrates that the student can apply the rule effectively in a new context. This process demonstrates that although the student made an error in determining the 12th term in part (b), he was still able to apply the correct method when asked to create

a new pattern from a different starting point. Overall, Student 5's answer demonstrates that encoding, inferring, and mapping are quite strong; however, applying these skills is still weak when the problem context requires calculations involving specific terms. This confirms that some students need to be trained not only in recognizing patterns, but also in understanding the meaning of the question and applying the rules consistently as the question demands.

To clarify the reasoning behind Student 5's error in determining the 12th term, while still being able to apply the rule correctly in a different context, the following interview excerpt provides insight into the student's thinking:

"I knew the pattern always adds three, so I just continued it until I saw the number twelve. I thought that was what the question wanted. I did not realize it was asking for the twelfth term. However, when it asked me to make a new pattern starting from six, that was easier because I just had to keep adding three from the new starting number."

Overall, the analysis shows consistent performance in encoding and inferring, whereas the applying component led to the most frequent errors, especially when determining the n -th term. Students with lower total scores tended to misunderstand the intent of questions or fail to apply rules systematically, while high-performing students completed all steps of analogical reasoning more accurately.

3.2. Discussion

The findings of this study indicate that students generally demonstrated strong abilities in encoding and inferring number patterns but experienced noticeable difficulty in the applying stage, especially when required to determine the n -th term of a sequence. This suggests that the early stages of analogical reasoning, specifically understanding basic information and identifying regularities, are not the primary obstacles for most seventh-grade students. While several students were able to map structural similarities between patterns, only a few provided complete or conceptually accurate explanations. Errors were concentrated mainly in tasks that required transferring the known rule into a more abstract or computational context. This suggests misconceptions in interpreting the question's intention, as well as weaknesses in connecting recognized rules with the specific instructions of the question. In other words, the problem lies not in recognizing the rule, but in applying it in a more formal numerical context.

Additionally, variations were observed during the mapping stage. Some students were able to clearly show the similarities between patterns, even presenting numerical evidence, such as identifying the same number in two different sequences. However, some students only mention similarities in general without providing in-depth reasons, or even make mistakes in mentioning differences. This suggests that the ability to establish structural correspondence between two patterns remains limited for some students, necessitating further training. These results suggest that students' analogical reasoning in the context of number patterns is uneven across the four components, with applying emerging as the weakest link.

The difficulty in applying rules appears to stem from challenges in transferring understanding from familiar examples to more abstract tasks, such as determining an n -th term. This aligns with transfer of learning theory, which states that learners often struggle when the new context demands flexible adaptation of previously learned structures [21]. Students who relied heavily on surface-level continuation tended to misinterpret the meaning of “the 12th term,” suggesting limited metacognitive regulation in monitoring the demands of the question.

Consistent with Sternberg’s theory of analogical reasoning, the mapping and applying stages require higher-order abstraction, which many students were unable to achieve fully [13]. Prior research also shows that learners often succeed in recognizing rules (encoding/infering) but struggle to transfer them to symbolic or generalized contexts [22]. The present findings support these studies, reinforcing that the gap between recognizing a pattern and applying it structurally is a significant cognitive hurdle for early adolescents. The brief explanations provided by some students also indicate limited metacognitive awareness, particularly in justifying the similarities between patterns. This echoes findings by Richland et al [23], who emphasize that explicit comparison and structural mapping must be scaffolded for students to articulate meaningful analogies.

This finding reinforces the importance of mathematics learning that emphasizes not only the result, but also the reasoning process that students use [24], [25]. Analysis of the answers showed that students who were weak at the applying stage often actually understood the rule, but failed to adapt it to the demands of the problem. The causal factor may lie in the low habit of students in dealing with non-routine problems that demand higher reasoning [26], [27], as well as the tendency of mathematics learning in the classroom that still focuses on routine procedures.

The specialty of this study lies in the way of analysis used. Instead of only assessing correct or incorrect answers, this study examines the students’ reasoning process in more detail using the analogical reasoning framework, which includes encoding, infering, mapping, and applying. This approach allows for a more comprehensive profile of student thinking to be revealed, as well as identifying points of weakness that are not apparent when only looking at the final results [28], [29]. Thus, this research makes an important contribution from both theoretical and practical perspectives. On the one hand, it enriches the study of analogical reasoning in learning number patterns. On the other hand, it serves as a reflection material for teachers to develop learning strategies that emphasize students’ thinking processes.

The results highlight the need for instructional practices that support a deeper structural understanding, rather than relying on procedural repetition. Teachers can integrate analogy-based instructional strategies, such as guided comparison, explicit mapping activities, and structured discussion of similarities and differences between patterns. These strategies can strengthen students’ ability to transfer pattern rules into new contexts and improve their performance in the applying stage. Additionally, incorporating metacognitive prompts, such as asking students to explain why a sequence follows a certain rule or how one pattern relates to another, may help students regulate their reasoning and avoid misinterpreting the problem statement.

Overall, this discussion confirms that students' analogical reasoning skills on number pattern material are quite good, especially in the early stages of recognizing rules and finding regularities. However, there are still weaknesses in applying rules to answer more specific questions and in building correspondences between patterns in depth. These findings suggest that mathematics learning should provide more opportunities for students to practice dealing with non-routine problems, carefully interpreting the meaning of questions, and comparing pattern structures across various contexts. Thus, teachers can help students not only to be skilled in solving routine problems, but also to develop higher-order thinking skills that are at the core of mathematical reasoning.

This study has several limitations. The sample size was small ($n = 6$), limiting the generalizability of the findings to broader populations. The tasks focused solely on arithmetic patterns, which may not fully represent students' analogical reasoning across other mathematical domains. Furthermore, the study relied heavily on written responses and brief interviews; richer data, such as think-aloud protocols, may provide deeper insights into students' cognitive processes. Future research should involve larger samples and diverse pattern types to more comprehensively capture developmental differences in analogical reasoning.

4. CONCLUSION

This study found that students were generally able to accurately encode and infer number pattern rules; however, difficulties emerged when tasks required determining specific terms or applying rules in more abstract contexts. While mapping skills appeared moderately strong, indicated by students' recognition of structural similarities, applying remained the weakest stage of analogical reasoning. These findings highlight an uneven development across the four components of analogical reasoning in solving number pattern problems. The results suggest that teachers should incorporate analogical reasoning exercises into non-routine mathematical contexts to strengthen students' ability to transfer and apply pattern rules. Integrating guided comparisons, explicit mapping discussions, and structured opportunities for students to justify similarities and differences between patterns can help improve the applying stage, which remains a critical challenge for many learners. This study was limited to six participants from a single grade level, which constrains the generalization of the findings to broader populations. Future research could involve larger samples, encompass a wider range of mathematical domains beyond number patterns, or utilize richer data sources, such as think-aloud protocols, to capture students' reasoning more comprehensively.

ACKNOWLEDGEMENTS

The authors express their gratitude to the Directorate of Research and Community Service (DPPM), Ministry of Education, Culture, Research, and Technology, for providing financial support through the institutional research grant under Contract Number 28/UN21.11/PT.01.05/SPK/2025. The authors also extend appreciation to the partner school, the mathematics teacher, and all participating students for their cooperation during

data collection. Finally, sincere thanks are given to all individuals and parties who contributed to and supported the completion of this research.

REFERENCES

- [1] X.-Q. Dao and N.-B. Le, "Investigating the Effectiveness of ChatGPT in Mathematical Reasoning and Problem Solving: Evidence from the Vietnamese National High School Graduation Examination," 2023, *arXiv*. doi: 10.48550/ARXIV.2306.06331.
- [2] J. Nilimaa, "New Examination Approach for Real-World Creativity and Problem-Solving Skills in Mathematics," *Trends in Higher Education*, vol. 2, no. 3, pp. 477–495, July 2023, doi: 10.3390/higheredu2030028.
- [3] T. Keleş and Y. Yazgan, "Gifted eighth, ninth, tenth and eleventh graders' strategic flexibility in non-routine problem solving," *The Journal of Educational Research*, vol. 114, no. 4, pp. 332–345, July 2021, doi: 10.1080/00220671.2021.1937913.
- [4] N. Ozrecberoglu, S. Aydın, and O. Aydın, "Students' Skills In Solving Non-Routine Mathematical Problems," *eqr*, vol. 5, no. 2, June 2022, doi: 10.31014/aior.1993.05.02.504.
- [5] H. D. A. D. Umbara, A. Jupri, and J. A. Dahlan, "Mathematical Critical Thinking Ability of Grade IX Students in Solving Ill-Structured Problems," *Form.J.II.P.MIPA*, vol. 15, no. 1, Feb. 2025, doi: 10.30998/formatif.v15i1.27337.
- [6] J. Řičan, V. Chytrý, and J. Medová, "Aspects of self-regulated learning and their influence on the mathematics achievement of fifth graders in the context of four different proclaimed curricula," *Front. Psychol.*, vol. 13, p. 963151, Oct. 2022, doi: 10.3389/fpsyg.2022.963151.
- [7] B. P. Uyen, D. H. Tong, and N. T. B. Tram, "Developing Mathematical Communication Skills for Students in Grade 8 in Teaching Congruent Triangle Topics," *EUROPEAN J ED RES*, vol. volume–10–2021, no. volume–10–issue–3–july–2021, pp. 1287–1302, July 2021, doi: 10.12973/eu-jer.10.3.1287.
- [8] S. L. Chew and W. J. Cerbin, "The cognitive challenges of effective teaching," *The Journal of Economic Education*, vol. 52, no. 1, pp. 17–40, Jan. 2021, doi: 10.1080/00220485.2020.1845266.
- [9] A. A. Khasawneh, A. A. Al-Barakat, and S. A. Almahmoud, "The Effect of Error Analysis-Based Learning on Proportional Reasoning Ability of Seventh-Grade Students," *Front. Educ.*, vol. 7, p. 899288, July 2022, doi: 10.3389/educ.2022.899288.
- [10] Y. Qi, Y. Chen, X. Yu, X. Yang, X. He, and X. Ma, "The relationships among working memory, inhibitory control, and mathematical skills in primary school children: Analogical reasoning matters," *Cognitive Development*, vol. 70, p. 101437, Apr. 2024, doi: 10.1016/j.cogdev.2024.101437.
- [11] M. E. Gray and K. J. Holyoak, "Teaching by Analogy: From Theory to Practice," *Mind Brain and Education*, vol. 15, no. 3, pp. 250–263, Aug. 2021, doi: 10.1111/mbe.12288.
- [12] R. Darmayanti, R. Sugianto, and Y. Muhammad, "Analysis of Students' Adaptive Reasoning Ability in Solving HOTS Problems Arithmetic Sequences and Series in Terms of Learning Style," *Numerical: J.Mat and Pend.Mat*, pp. 73–90, May 2022, doi: 10.25217/numerical.v6i1.2340.
- [13] R. J. Sternberg, "Component processes in analogical reasoning.," *Psychological Review*, vol. 84, no. 4, pp. 353–378, July 1977, doi: 10.1037/0033-295X.84.4.353.
- [14] Adelia Indriani, Zahwah Zahwah, and Syutaridho Syutaridho, "Memahami Cara Belajar dan Kesulitan Siswa dalam Menyelesaikan Soal Pola Bilangan," *Pentagon*, vol. 3, no. 2, pp. 74–79, May 2025, doi: 10.62383/pentagon.v3i2.523.
- [15] D. Agusantia and D. Juandi, "Kemampuan Penalaran Analogi Matematis di Indonesia: Systematic Literature Review," *Symmetry*, vol. 7, no. 2, pp. 222–231, Dec. 2022, doi: 10.23969/symmetry.v7i2.6436.
- [16] T. Wulandari and I. U. Machromah, "Kemampuan Penalaran Matematis Siswa dalam Menyelesaikan Soal HOTS pada Materi Pola Bilangan," *Cendekia*, vol. 8, no. 1, pp. 689–700, Mar. 2024, doi: 10.31004/cendekia.v8i1.2014.
- [17] A. K. Ahmad, Ishak, and Afdalia, "Peningkatan Hasil Belajar Matematika melalui Model Pembelajaran Kooperatif Tipe Two Stay Two Stray," *IJME*, vol. 1, no. 2, pp. 79–87, July 2022, doi: 10.58917/ijme.v1i2.23.
- [18] N. S. R. Hasibuan, Y. Roza, and M. Maimunah, "Analisis Kesalahan Siswa dalam Menyelesaikan Masalah Matematika Berdasarkan Teori Kastolan," *j. paedagog. penelit. pengemb. pendidik.*, vol. 9, no. 3, p. 486, July 2022, doi: 10.33394/jp.v9i3.5287.
- [19] D. Deslis and D. Desli, "Does this Answer Make Sense? Primary School Students and Adults Judge the Reasonableness of Computational Results in Context-Based and Context-Free Mathematical Tasks," *Int J of Sci and Math Educ*, vol. 21, no. 1, pp. 71–91, Jan. 2023, doi: 10.1007/s10763-022-10250-0.

-
- [20] N. Loc and B. P. Uyen, "Using Analogy in Teaching Mathematics: An Investigation of Mathematics Education Students in School of Education - Can Tho University," 2014. Accessed: Sept. 26, 2025. [Online]. Available: <https://www.semanticscholar.org/paper/Using-Analogy-in-Teaching-Mathematics%3A-An-of-in-of-Loc-Uyen/e5a7d4c9177bfd429e541c5fc6b04d8cccd8c7dc>
- [21] J. D. Bransford and D. L. Schwartz, "Rethinking Transfer: A Simple Proposal with Multiple Implications," *Review of Research in Education*, vol. 24, p. 61, 1999, doi: 10.2307/1167267.
- [22] T. P. Carpenter, T. P. Carpenter, E. Fennema, M. L. Franke, L. Levi, and S. B. Empson, *Children's mathematics: cognitively guided instruction*, Second edition. Portsmouth, NH: Heinemann, 2015.
- [23] L. E. Richland, O. Zur, and K. J. Holyoak, "Cognitive Supports for Analogies in the Mathematics Classroom," *Science*, vol. 316, no. 5828, pp. 1128–1129, May 2007, doi: 10.1126/science.1142103.
- [24] S. Sachdeva and P.-O. Eggen, "Learners' Critical Thinking About Learning Mathematics," *INT ELECT J MATH ED*, vol. 16, no. 3, p. em0644, June 2021, doi: 10.29333/iejme/11003.
- [25] R. Marasabessy, "Study of Mathematical Reasoning Ability for Mathematics Learning in Schools: A Literature Review," *Indonesian J. Teach. Sci.*, vol. 1, no. 2, pp. 79–90, Dec. 2021, doi: 10.17509/ijotis.v1i2.37950.
- [26] K. J. C. Encio, "Mathematical Habits Of Mind In Solving Non-Routine Problems And Academic Performance Of Grade 9 Students ," *Int. J. Res. Publ. Rev.*, pp. 3473–3481, July 2022, doi: 10.55248/gengpi.2022.3.7.21.
- [27] Z. Kablan and S. S. Uğur, "The relationship between routine and non-routine problem solving and learning styles," *Educational Studies*, vol. 47, no. 3, pp. 328–343, May 2021, doi: 10.1080/03055698.2019.1701993.
- [28] L. E. Rivas and M. Trench, "Analogical reasoning during hypothesis generation: the effects of object and domain similarities on access and transfer," *Thinking & Reasoning*, vol. 31, no. 2, pp. 158–182, Apr. 2025, doi: 10.1080/13546783.2024.2409467.
- [29] F. G. Putra, A. Saregar, R. Diani, M. Misbah, S. Widyawati, and K. Imama, "Enhancing mathematical reasoning: role of the search, solve, create, and share learning," *EduLearn*, vol. 18, no. 3, pp. 961–969, Aug. 2024, doi: 10.11591/edulearn.v18i3.21399.
-