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Mapping the Mind: Exploring Students' Visual-Spatial Thinking in Transformation Geometry

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ABSTRACT

This study investigates high school students' visual-spatial thinking ability in the context of transformation geometry using a descriptive quantitative design. A total of 241 students from public and Islamic high schools in Jakarta participated in the study, responding to a validated 40-item multiple-choice test assessing four key indicators: imagining, problem-solving, pattern seeking, and conceptualising. Results reveal that students generally performed at a medium level across all indicators, with the highest scores in imagining (M=62.97) and the lowest in conceptualising (M=47.05). These findings suggest that while students demonstrate some spatial reasoning capability, their conceptual understanding remains limited. The study highlights the importance of incorporating visualisation-focused strategies in geometry instruction to strengthen spatial reasoning and deeper comprehension of transformation concepts. Future research should explore the longitudinal impact of visual-spatial training and its role in developing higher-order geometrical thinking.

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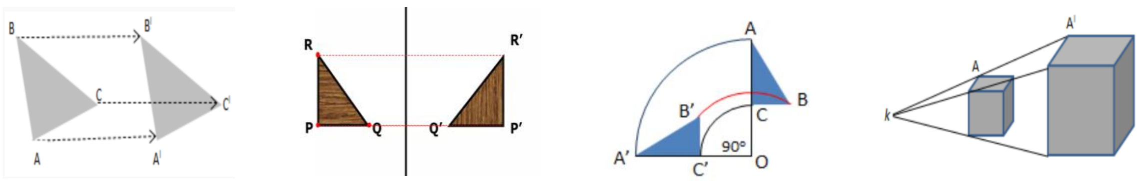
1. INTRODUCTION

Geometry ability is still interesting to study because many students in Indonesia still have difficulty understanding geometry [1]. It was found that causes students to find it hard to understand geometrical thinking, such as students' misconceptions in geometry [2], the low visual spatial ability of students in both 2D geometry and 3D geometry [2]; a low intuition ability of students in understanding geometry problems [1], a low understanding of geometry at the early level of formal education, while geometry is taught hierarchically, as stated in the education curriculum [3].

NCTM [4] and Mas'udah [5], said that students geometry abilities from levels 9-12 consist of: (1) can analyse the properties and character of two-dimensional and three-dimensional geometric objects, and then be able to build mathematical arguments about the relationship of geometry to others; (2) can determine the position of a point more accurately and specifically, then be able to describe spatial relationships through geometric coordinates and connect them with other systems; (3) can apply transformations and use them symmetrically to analyse mathematical situations; (4) can use visualisation, spatial reasoning, and geometric models to solve problems.

Furthermore, transformation geometry is one of the essential concepts in high school mathematics and is a branch of Geometry [6]. Transformation geometry is divided into four sub-contents: translation, reflection, dilation, and rotation [3]. While in the mathematics education curriculum, transformation geometry consists of five concepts: translation, reflection, rotation, dilation, and transformation composition. Students need to understand transformation geometry because it will benefit them in improving their spatial skills, geometry reasoning skills, and geometry problem-solving abilities [7]. Transformation geometry is not only encountered frequently in everyday life, but it can also serve as a fundamental concept required in other learning areas in mathematics. Knowledge of transformation geometry improves students' spatial ability and geometric reasoning skills and strengthens mathematical proofs [8]. Transformation geometry has many roles in support of students' cognitive development. It provides various opportunities for students to develop their spatial visualisation skills and geometric reasoning to gain mathematical proof skills [9].



(a) translation (b) reflection (c) rotation (d) dilation

Figure 1. Types of transformations in geometry

The concept can be explained as follows: *translation* determines the position of an object as a result of a shift. In the concept of translation, it appears that points A, B, and C shifted to the right into points A', B', and C' (Figure 1.a). *Reflection* is determining the position of an object as a result of reflection. In the concept of reflection, we must first determine the line of reflection, so that each point is reflected on the line of reflection (Figure 1.b) the result of reflection is that the distance from P, Q, R to the line of reflection is equal to the distance from P', Q', and R' to the line of reflection. *Rotation* determines the position of an object as a result of rotation. To perform rotational movements, we must first determine the angle of rotation, for example the angle of rotation 90^o, The results of the rotas show that points A', B' and C' are the result of rotations as far as 90o from points A, B and C (Figure 1.c). *Dilation* determines an object's position due to a change in scale (enlarged or reduced). It appears that the small box is enlarged by k to produce a large box (Figure 1.d). Meanwhile, what is meant by transformation composition is determining the position of an object by making two movements, such as rotation, translation, translation with reflection, and so on [3], [6], [10].

Previous researchers in various countries have studied spatial abilities. For example, Demir [3] researched the spatial abilities of elementary school students in the country of Turkey, Ndungo [8] examines the transformation geometry abilities of high school students in Uganda, Thohirudin [11] using MICA's transparency learning media to improve the spatial abilities of high school students in Indonesia, and Sukiyanto [2] analyse the visual-spatial intelligence of high school students in Indonesia.

Visualisation is one of the abilities that supports geometry knowledge. Moreover, Spatial thinking is a fundamental aspect of human cognition [12]. Van Hiele states that the ability to visualise is fundamental in geometry [3], [13]–[15]. Visual-spatial skills are closely related to using the sense of sight to see or find geometric patterns and then find solution strategies to solve geometric problems. Visual-spatial concepts can be found in the concepts of 2D geometry and 3D geometry [16], [17]. This study aimed to show that students in high school in Indonesia perform well in terms of spatial visual ability.

2. METHOD

This study uses a quantitative descriptive research design [18] because the data can be compiled numerically to describe a population accurately [19]. The 241 students from Islamic high schools and public high schools in the province of Jakarta in 2024/2025 were surveyed using a 40-question multiple-choice visual-spatial thinking test [20].

The instruments consist of four indicators: imagining, conceptualising, problem-solving, and pattern seeking [2], [21], [22]. In this research, Students are given 80 minutes to complete the 40 questions.

Table 1. Indicators of visual-spatial ability

No	Indicator	Description
1	imagining	Solve geometric problems using the senses of vision in perspective: translation, reflection, dilation, and rotation.
2	Conceptualising	Solve spatial geometry problems by collecting and constructing conceptual frameworks to show the relationship between facts and problems.
3	problem-solving	Solve geometric problems that involve analysing, interpreting, reasoning, predicting, evaluating, and reflecting.
4	pattern seeking	Solve spatial geometry problems by searching for patterns, then looking for relationships between patterns and mathematical concepts.

Five lecturers in Mathematics Education at UIN Syarif Hidayatullah Jakarta and six high school mathematics teachers have validated the instrument. The result is analysed by using Lawshe's CVR method [23], [24]:

$$CVR = \frac{N_e - \frac{N}{2}}{\frac{N}{2}} \quad 1)$$

Note:

CVR : Content Validity Ratio

Ne : Number of validators stating essential question items

N : Number of validators

Criterion: $-1 \leq CVR \leq 1$, when the value $CVR > 0$, it means that more than 50% of the validator states that the question item is essential [23], [24]. If the value CVR is greater than 0, it can be concluded that the statement/question assessed is more essential and has higher content validity. Based on the calculation CVR = 0,2, it was concluded that 40 questions were declared valid and suitable for measuring visual-spatial thinking skills.

Table 2. Categorise visual-spatial abilities [25]

Category	value	Score
High	x ≥ mean + SD	x ≥ 72.82
Medium	mean +SD > x ≥ mean -SD	72.82 > x ≥ 36.36
Low	x < mean - SD	36.36 < x

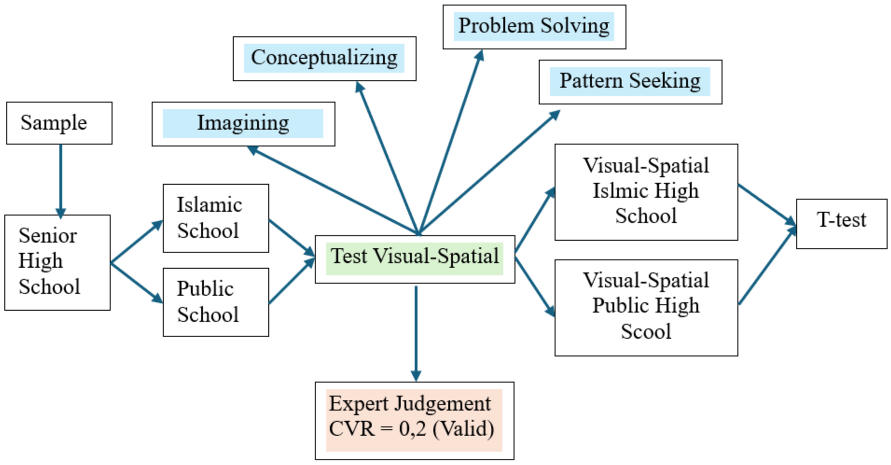


Figure 2. Flow of procedural steps in research

Figure 2 shows the technical procedures for collecting data. This study aims to analyse the visual-spatial abilities of high school students, so the respondents are high school students. Islamic and Public high schools follow the same teaching and learning procedures, but the curriculum distinguishes them. Public high schools use a curriculum from the Ministry of Education, while Islamic high schools use a curriculum from the Ministry of Education and the Ministry of Religion. So it is necessary to take from both schools to find out their differences.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1 Analysis of Data: Students' visual-spatial ability

Table 3 shows that the data of 241 respondents are valid and can be used for further analysis. The average score of high school students' visual-spatial thinking skills is 61.21, which is included in the medium category [25]. In other words, high school students have good visual-spatial abilities as a basis for understanding geometry concepts. Meanwhile, viewed from the visual-spatial thinking ability indicator, an average score was obtained for the imagining indicator = 62,97 (medium category); problem-solving indicator = 52,95 (medium category); patterns seeking indicator = 57,18 (medium category); and conceptualising indicator = 47,05 (medium category) [25]. So, the highest ability of high school students is found in the imagining indicator, and the lowest is found in the conceptualising indicator.

Table 3. Mean of students' visual-spatial ability

		Imagining	Problem-solving	pattern seeking	conceptualising
N	Valid	241	241	241	241
	Missing	0	0	0	0
Mean		62,97	52,95	57,18	47,05
St. Deviation		18.50	23.42	21.38	23

3.1.2 T-test for students' visual-spatial thinking viewed from school

The next analysis is to determine whether the school background affects the visual-spatial ability. The results of the t-test calculation using SPSS version 23 are presented in Table 4, which describes descriptive statistics for visual-spatial ability scores reviewed from school backgrounds.

Table 4. Visual-spatial ability scores reviewed from schools

Factor School		Statistic
Islamic Senior High School	N	118
	Mean	53.33
	St. Deviation	24.00
	Minimum	15.00
	Maximum	97.50
Public Senior High School	N	123
	Mean	55.79
	St. Deviation	9.94
	Minimum	22.50
	Maximum	85.00

Viewed from the school's background, students' visual-spatial scores from Islamic high schools average 53.33 (medium category) with a standard deviation of 24, and students' visual-spatial scores in high school average 55.7927 (medium category) with a standard deviation of 9.94 [25].

After determining whether the two average scores of students at Islamic and public schools differ, a hypothesis test must be carried out to determine whether the scores differ significantly. Before we use the t-test, we have to check the normality assumption using the chi-square. Based on Table 5, Sig. (2-tailed) = 0.00 0.05. We conclude that the two samples are normally distributed.

Table 5. Normal distribution test using chi-square

	Total
Chi-Square	196.763[a]
df	32
Asymp. Sig.	.000

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The next analysis uses a t-test with the proposed statistical hypothesis:

H_0 : There is no difference in the average score of visual-spatial ability between Islamic School students and public high school students

H_1 : There is a difference in the average score of visual-spatial ability between Islamic School students and public high school students

Table 6. Independent sample test for visual-spatial ability

		Levene's Test for Equality of Variances		t-test for Equality of Means		
		F	Sig.	t	df	Sig. (2-tailed)
Total	Equal variances assumed	148.962	.000	-1.041	239	.299

Because the mean score of students' visual-spatial ability from Islamic high schools is not significantly different from the mean students' visual-spatial ability from public schools, and Sig. (2-tailed) = 0.299 > 0.05, it is concluded that H_0 is accepted, which means that students' visual-spatial ability for students at Islamic High School is not different from students' visual-spatial ability for students at a public high school

3.1.3 Test of the between-subject Effect of students' visual-spatial ability

After we know that the school background does not affect students' visual-spatial ability, we can conclude that the ability itself influences the students' ability to solve pure geometry problems.

Interesting to conduct further analysis on whether imagining ability influences the other three abilities. Furthermore, we can see the relationship between indicator imagining and problem solving, imagining and pattern seeking, and imagining and conceptualising. As we know, imagining is a foundation for geometrical problem-solving [21].

A statistical hypothesis was proposed for further analysis:

Hypothesis 1

H_0 : There is no positive effect as significant from imagining to pattern seeking

H_1 : There is a positive effect as significant from imagining to pattern seeking

Hypothesis 2

H_0 : There is no significant positive effect of imagining on problem solving

H_1 : There is a positive effect as significant from imagining to problem solving

Hypothesis 3

H_0 : There is no significant positive effect from imagining to conceptualising

H_1 : There is a positive effect as significant from imagining to conceptualising

The results of the test of the between-subject effect calculation using SPSS are presented in Table 7.

Table 7. Test of between-subject Effect

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	73252.870[a]	129	567.852	7.104	.000
Intercept	311068.240	1	311068.240	3891.480	.000
PS	1150.586	5	230.117	2.879	.018
Pattern	10341.135	13	795.472	9.951	.000
Concept	1493.785	13	114.907	1.437	.154

Dependent Variable: Imagining

a. R Squared = .892 (Adjusted R Squared = .766)

Based on Table 7, we found that:

Hypothesis 1

(i) Sig. value for Problem solving S = 0.018 < 0.05, it is concluded that H_0 is accepted, which means there is a relationship between imagining and problem-solving.

Hypothesis 2

(ii) Sig. value for Pattern = 0.000 < 0.05, it is concluded that H_0 is accepted, which means that there is a relationship between imagining and pattern seeking; and

Hypothesis 3

(iii) Sig. value for Concept = 0.154 > 0.05, it is concluded that H_0 is accepted, which means there is a relationship between imagining and conceptualising.

3.2. Discussion

3.2.1 Instrumen Test Visual-Spatial

The steps that need to be taken to develop spatial visual abilities are (1) visual identification, (2) discovery and matching of patterns in 2D and 3D geometry, (3) information manipulation, (4) mental representation, (5) transformation, and (6) problem orientation [26]. The visual-spatial ability indicators used in this study refer to Haas [2], [21], which consists of imagining, conceptualising, problem-solving, and pattern-seeking.

Meanwhile, the material is geometric transformation, consisting of five sub-materials: translation, reflection, rotation, dilation, and Composition of Transformation. This concept plays a vital role in developing students' spatial [8]. General explanation about the instrument delivered at Table 8 below:

Table 8. Description of instruments of visual-spatial thinking

Transformation Geometry	Aspect of Visual-Spatial Thinking	Indicator	item
Translation	Imagining	• Imagine determining the translation results of an object by deception of the components of that object	1
	Problem-Solving	• Determining the position of the translation results of an object	2
		• Analyse and determine the steps of the strategy to solve a translation problem of an object	
	Pattern seeking	• Find patterns and determine the colour combinations that will form in a certain order on a colour pattern	1
Reflection	Imagining	• imagining and determining the form of the reflection of an object	5
	Conceptualising	• Analyse and determine shapes that are not the result of the reflection of an object	6
Dilation	Imagining	• Imagine and determine the dilation results of an object by deception of components on that object	1
	Problem-Solving	• Determine the dilation result by using the scale factor of an object	1
Rotation	Imagining	• Imagine and determine the rotation results of an object	2
	Conceptualising	• Analyse and determine shapes that are not the result of the rotation of an object	7
	Problem-Solving	• Analyse and determine the steps of the strategy to solve a rotation problem on a flat plane in the context of space	2
		• Analyse and determine the strategy steps to solve a rotation problem of an object	
	Pattern seeking	• find patterns and determine the results of rotation that form subsequently of a two-dimensional object	6
		• find patterns and determine the results of rotation that form subsequently of a three-dimensional object	
Composition of Transformation	Pattern seeking	• Find patterns and determine the results of the subsequent transformations of an object	6
		• Find patterns and determine combinations of transformation results that will be formed in a certain order on an object pattern	
Total item			40

3.2.2 Senior High School Visual-Spatial Ability

The high school students at public and Islamic schools in Jakarta Province, Indonesia, have visual-spatial abilities in the medium category (mean = 61,21 / max score =100). This means that high school students in Jakarta have quite a good understanding of level 1 geometry, Van Hiele [2]. In line with the results, Rensiana [27] found that the average score of the student showed that the student's ability to understand the concept of transformation geometry was on average 60 (medium category), and the lowest score on the indicator applied the concept of 2D transformation geometry logically to construct a mathematical argument about the relationship between geometry and ethnomathematics. Satriawati [28] shows that the ability of high school students to solve transformation geometry problems is mostly that 80% of students are still under mastery learning at school, namely KKM.

Visual-spatial abilities are the basis for understanding geometric concepts [29] or solving math problems [30], [31]. Spatial visualisation is the ability to manipulate object figures into alternate spatial arrangements. Although visual-spatial is seen as abstract, it is powerful for supporting geometry achievement and plays a critical role in mathematics learning, doing, and communication [32]. Therefore, high school students' visual-spatial abilities need to be retrained.

Viewed on indicators: imagining, problem solving, pattern seeking, and conceptualising, three abilities (imagining, problem solving, pattern seeking) show a medium category, it is good enough because the ability to imagine only relies on the thinking process of the brain, and students do not have too much difficulty in translating, rotating, reflecting, and dilating [33]. In the completion, it is only possible that some immediately find the answer quickly, and some students need tools such as pens and pencils, or maybe other objects to visualise it with concrete materials [34]. Visual-spatial may serve as a "mental blackboard" on which various mathematical concepts, relations, and operations can be modelled and visualised [32]. Meanwhile, conceptualising abilities are the lowest score. In line with Mas'udah [5] and Sukiyanto [2], high school students can solve internal visualisation problems, but cannot solve problems through the visual implementation of problem-solving (external visualisation problem).

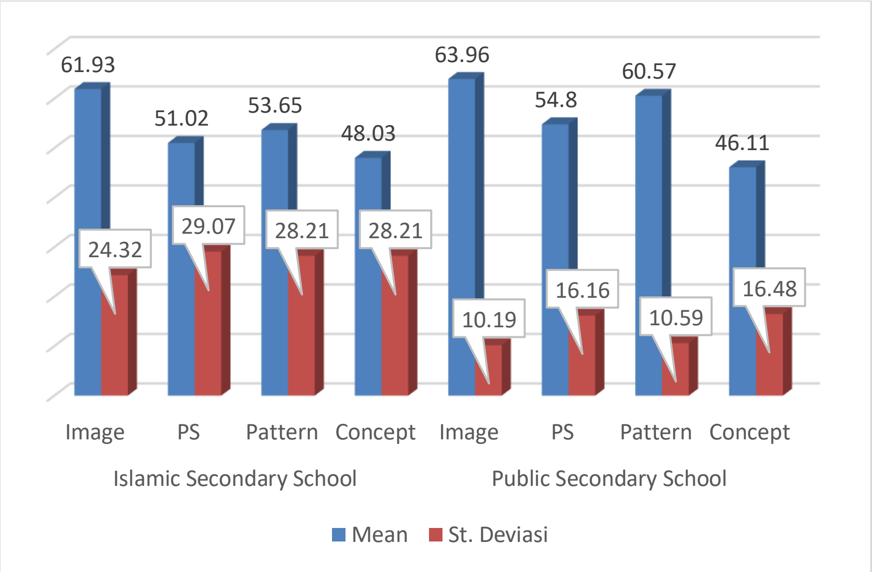


Figure 3. Students' Visual-Spatial Abilities

In Figure 3, the visual-spatial ability of students from public and Islamic schools shows equivalent abilities. The ability score based on the indicator is not significantly different.

This means that regardless of school background, the visual-spatial abilities of high school students in DKI Jakarta are similar in the medium category.

Interesting to know the effect of imagining on problem-solving, pattern seeking, and conceptualising. Imagining incorporates mental processes and the use of representational tools to imagine and transform objects and relations [29], mentally creating and manipulating images of objects in space, from fixed or changing perspectives on the objects, so that one can reason about the objects and actions on them, both when the objects are and are not visible [35].

In this section, each subject was further tested to determine the relationship between the subjects. The data analysis showed that imagining positively affected pattern seeking and problem-solving ability, but the imagining indicator did not affect conceptualising ability.

In other words, problem-solving and pattern seeking were still closely related to imagining ability [26], [36]. Conceptualising incorporates solving mathematical problems. It is not only influenced by the ability to imagine or visualise, but there are other competencies, such as understanding the Problem, understanding the concept and the formula, and solving it [3].

The mathematics curriculum in Indonesia has been made systematically, where geometry is taught from elementary to high school. At the elementary school level, geometry is taught in the introduction of the concept of plane and spatial figures, at the junior high school level geometry is taught on the properties of plane and spatial figures, while at the senior high school level geometry is taught for the concept of 3 dimensions which includes the concepts of points, lines, planes, distances, and angles. This systematic material allows us to teach geometry concepts according to Van Hiele. However, the reality is that mathematics teachers teach mathematical concepts directly at the analysis and problem-solving stages. Meanwhile, visual-spatial abilities receive less attention [15].

3.2.3 Exploring students' Visual-Spatial Ability viewed from the indicator

Viewed from the visual-spatial ability indicator, students find several questions difficult because the questions that give the correct answers are only shown to some students. For example:

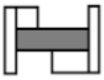
Question 1, indicator imagining:

The shadow formed from the translation of the image  *is….*

a.  b.  c.  d. 

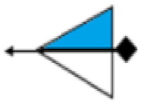
The scores for question 1 were the lowest, and only 29% of students could answer correctly. Students consider the Problem difficult because they experience difficulty forming mental representations of sequential patterns. They have difficulty making a "mental image" of the colour sequence found in the picture. They prefer to solve problems that can be directly visualised.

Question 2, indicator conceptualising:

Specify an image that is not a shadow that is generated from the reflection of the following image 

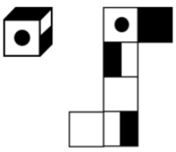
Only 51% of students could correctly answer an example of a conceptualising question (question 2), with the lowest score. Most students can not answer any of these questions because of the limitations in reflecting concepts, which require determining the axis of reflection. In this question, the axis of reflection is unknown, so there are many possibilities for where it should be located.

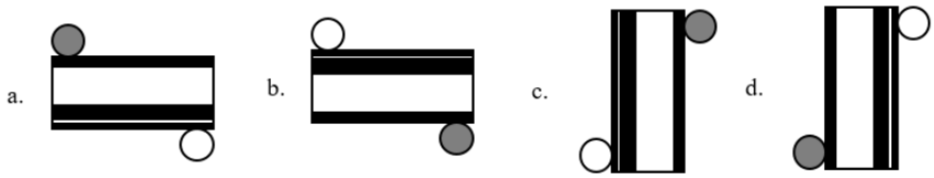
Question 3, indicator conceptualising:

Which of the images below is not a reflection of  *?*

For question number, 53% of students could correctly answer. This question is considered easy to solve because only the concept of reflection is required.

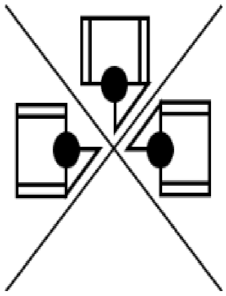
Question 4, indicator problem-solving :

 *look at the image, then*  *is processed to*

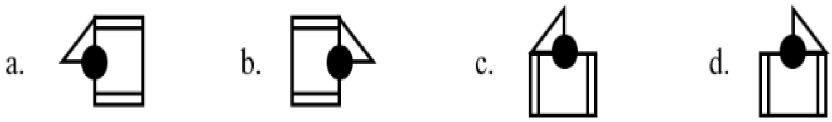


Only 50% of students can give the correct answer to question number 4. This Problem can be solved by using imagination first. Before solving it, students must be able to visualise a tube as a two-dimensional image (net of tube).

Question 5, indicator pattern seeking:



the correct image to fill in the blank is



Only 59% of students can give the correct answer to question number 5. This Problem can be solved by using the pattern.

4. CONCLUSION

This study concludes that public and islamic high school students in DKI Jakarta Province have visual-spatial abilities in the medium category. Visual-spatial abilities need to be further developed as a foundation in learning geometry or mathematics. Based on the mathematics curriculum in Indonesia, the material on geometry has been made systematic and hierarchical from elementary school to senior high school level. However, in its implementation in the classroom, teachers need to pay special attention to teaching visual-spatial skills using manipulatives or digital tools as an initial stage in learning geometry.

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