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A Conceptual Framework for Making Model Assumptions Explicit in Special Relativity Learning Resources

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ABSTRACT

Special relativity learning resources may present mathematically correct relations without making their physical assumptions or instructional purpose explicit. Existing work on relativity textbooks, scientific modelling and representations helps identify such issues but does not, by itself, prescribe a claim-level procedure for distinguishing an omission from a contradiction or a legitimate scope restriction. This conceptual study develops an evidence-based decision framework for reviewing undergraduate relativity resources. A purposive synthesis of 28 research works was organised into four conceptual strands, translated into seven analytical criteria, and operationalised through separate records of task relevance, information availability and physical consistency. An ordered procedure recommends one of five actions: retain, clarify, correct, defer or exempt. Sixteen author-constructed vignettes in eight contrasting pairs illustrate how changes in context, notation, approximation or purpose alter the warranted action. The main contribution is a traceable path from a bounded instructional claim and its supporting evidence to a proportionate revision, rather than a checklist equating brevity with error. Teachers and resource developers can use the record to justify targeted edits and discuss competing interpretations. The cases are illustrative; independent expert appraisal, authentic-resource coding and classroom evaluation are still needed.

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1. INTRODUCTION

Special relativity requires learners to distinguish events, clock readings, reference frames, and the physical systems represented by equations. A relation can be manipulated algebraically while the meanings of its time intervals or velocities remain uncertain. Reviews report persistent difficulties with reference frames, the postulates, and relativistic

phenomena [1], while research on learners' reasoning about light propagation shows how differently the same situation can be interpreted in words and diagrams [2]. An instructional statement such as 'moving clocks run slowly' can therefore be misread as a universal property of one clock rather than a comparison between specified event intervals and frames. Making these conditions explicit offers a way to examine what an explanation permits a learner to infer; it does not, by itself, demonstrate improved understanding.

Studies of special relativity textbooks have examined historical, epistemological, and didactic emphases [3], [4], and educational reconstruction has informed the design and evaluation of teaching sequences [5]. Related modelling and representation frameworks illuminate the purposes of models, the relation between mathematical expressions and physical situations, and the coordination of representations [6]–[20]. These approaches provide essential theoretical resources, but their original objectives are not to assign an ordered editorial action to a specific textbook claim after recording the available context. For a reviewer of a lesson, it consequently remains necessary to distinguish three different situations: an explicit physical contradiction, an explanation absent from the inspected passage but recoverable elsewhere, and an explanation that the stated learning task does not require. This is the operational gap addressed here; it is not a claim that previous approaches are generally inadequate or that none could be adapted for this task.

For example, a time-dilation formula may be accompanied by a clear definition of proper time in the preceding paragraph, may be printed in isolation without accessible definitions, or may explicitly identify an incorrect quantity as proper time. The visible equation can look identical in all three cases, yet the defensible actions differ. Similarly, a short reference sheet may legitimately omit a derivation that a self-contained derivation lesson must provide. The educational significance is that a learner's inference depends on the relation between the stated claim, the defined quantities, and the learning task; an omission should not be diagnosed as false physics merely because its explanation is not local.

This article develops a conceptual framework for evaluating a claim, or a relevant omission, and its bounded, recoverable context in undergraduate special relativity resources. It addresses three questions: (1) which criteria connect the represented system, assumptions, mathematics, and interpretation; (2) how can the location of explanatory information be recorded separately from the physical consistency of a claim; and (3) how can these judgments warrant a proportionate action for resource revision? The first question supports disciplined interpretation, the second guards against confusing missing information with error, and the third makes an editorial recommendation auditable. Deliberately constructed contrasts demonstrate the logic. The study neither estimates the quality of published textbooks, measures learning gains, nor proposes a new physical law.

1.1. Scientific Modelling and the Role of Purpose

Models select features of a target system for a particular explanatory or predictive task. Gilbert places models and modelling at the centre of authentic science education [6]. Grosslight and colleagues showed that students’ conceptions of models differ from experts', including in their understanding of models as constructed intellectual tools [7].

Schwarz and colleagues link modelling practices to knowledge of model purposes and their revision [8]. These contributions support examining what a resource says a model is intended to accomplish, rather than treating its equations as self-contained descriptions of all physical situations.

Zwickl and colleagues identify construction, prediction, interpretation, recognition of limitations, and revision in model-based reasoning within physics laboratories [9]. Their framework is relevant here as a conceptual guide to the connections that explanations need to make. Its empirical context was laboratory reasoning; it is not imported as an already validated coding instrument for relativity texts. Parker's account of adequacy for purpose further supports evaluating a model in relation to a task and conditions of use [10]. In the present framework, this requires recording the resource's declared audience, purpose, and scope before deciding that an omitted topic constitutes a defect.

Physical assumptions and mathematical approximations require separate attention. A physical assumption specifies the represented system or interaction, such as inertial motion during an interval or the absence of external energy transfer. A mathematical approximation simplifies a representation under those physical assumptions, such as truncating a series. The former restricts what situation is modelled; the latter changes the accuracy of a calculation within a model. The distinction is useful only when the analyst can identify which statement plays which role and explain the consequences of changing it.

1.2. Equations and Representations as Explanations

Sherin's account of symbolic forms connects equation structure to conceptual interpretation [11]. Tuminaro and Redish describe patterns in how students use knowledge during physics problem solving [12]. Together, these perspectives motivate asking what quantities mean and what reasoning an expression is being used to support. They do not imply that an equation's appearance establishes a learner's understanding.

The ternary perspective developed by Hansson and colleagues examines relationships among reality, theoretical models and mathematics [13]. A subsequent application extends this approach to physics textbooks and associated lessons [14]. More recent classroom research continues to examine how these relationships are communicated in teaching [15]. This existing work is especially close to the present concern with the connections that accompany mathematical expressions. Our framework adds a procedure for documenting the evidence behind a judgment and choosing among different revision actions in a bounded resource analysis.

Ainsworth's DeFT framework distinguishes design features, representational functions, and learner tasks [16]. Fredlund and colleagues explain why disciplinary representations may require unpacking before their meaning becomes accessible to students [17]. Empirical studies by Kohl and Finkelstein [18] and Meltzer [19] show that performance can depend on representational format. Work–energy instruction also provides examples of purposeful coordination among verbal, visual, and mathematical representations [20]. These findings support inspecting the links among representations, rather than rewarding a resource merely for containing many diagrams. The present article

examines the interpretive opportunity a resource provides; actual learner use requires separate evidence.

1.3. The Specific Contribution

The framework integrates model purpose, disciplinary representations and context-sensitive document analysis into a claim-level editorial protocol. Its specific contribution is not the seven topics considered separately, which have precedents in the cited literature, but their operational combination: the analyst identifies the task and a bounded evidence packet, records where information is available, tests an explicitly stated interpretation for consistency, and then follows an ordered action rule. An absent derivation can therefore prompt clarification without being mislabelled a physical contradiction; conversely, a demonstrably false claim cannot be excused solely because the resource is short. Table 1 contrasts this specification with selected neighbouring approaches. These comparisons describe complementary emphases, not claims of universal superiority.

Table 1. Relationship to selected existing approaches

Approach	Existing focus	Specification added here
Relativity textbook analysis [3], [4]	Historical epistemological and didactic interpretation	A documented path from a focal claim to a revision action
Modelling in physics laboratories [9]	Construction interpretation limitations and revision	Questions answerable from a learning resource and its context
Adequacy for purpose [10]	Evaluation relative to a use and its conditions	A relevance gate for judging omissions
Ternary analysis [13], [14]	Relations among reality models and mathematics	Separate information and consistency records linked to action
Representation frameworks [16], [17]	Functions, tasks and unpacking of representations	Evidence for the interpretation being attributed to a representation

The implications are educational but require careful delimitation. Conceptual-change research shows that correct statements alone are not sufficient evidence of learner understanding [21], and instruments such as the Relativity Concept Inventory illustrate the additional design and validation required to measure conceptual learning [22]. Here, the outcome is an explicit framework and an inspectable demonstration. The proposed classroom value is that teachers and students can discuss which assumptions and definitions justify an equation; whether this improves conceptual reasoning is a question for future empirical testing.

2. METHOD

2.1. Research Design and Boundaries

The research design is conceptual synthesis and theory adaptation, informed by methodological guidance for conceptual articles [23] and integrative reviews [24]. The research objects are published conceptual and methodological arguments, together with author-constructed instructional vignettes; there are no human participants, field sites, survey variables, or hypothesis tests. For that reason, respondent profiles, population-based

sampling, descriptive statistics, and human-subject ethics procedures are not applicable. The analytical unit is one claim or task-relevant omission with a defined context packet, rather than a student or textbook treated as a statistical observation.

Four linked operations were used. First, relevant constructs were mapped across relativity education, model purpose and assumptions, representations, and document-analysis methods. Second, each construct was translated into a question that an analyst could answer from a bounded passage: what is measured (C1), what system is included (C2), which physical interaction is assumed (C3), what is approximated (C4), whether the stated mathematics and physics cohere (C5), for what conditions and task the claim is applicable (C6), and what the prose, equations, and diagrams license a reader to infer (C7). Third, the criteria were combined with separate information-status and consistency records and ordered revision rules. Fourth, contrasting author-constructed cases tested whether a change in one focal condition produced the intended difference in action. The process specifies an auditable adaptation of theory, not an empirically validated scale or an independently reproduced coding exercise.

2.2. Literature Selection and Use

The purposive conceptual synthesis drew on 28 research works: 27 journal articles and one book chapter. The inclusion logic was conceptual relevance, not publication year or a statistically representative sample. Four strands were considered: (i) special relativity learning and textbook analysis, which motivated attention to frames and physical interpretation [1]–[5]; (ii) scientific modelling and adequacy for purpose, which informed assumptions, system boundaries, limitations and task relevance [6]–[10]; (iii) mathematical and visual representation research, which informed symbol meaning, approximations, consistency and interpretive links [11]–[22]; and (iv) guidance on conceptual synthesis, document analysis and reproducibility, which informed evidence recording and transparent rule checking [23]–[28]. Searches combined 'special relativity education', 'relativity textbooks', 'model assumptions', 'model evaluation', 'multiple representations', 'physics equations' and 'qualitative document analysis'. Targeted web searches, publisher pages and reference links were used to locate material. Bibliographic metadata were checked against DOI registration records and publisher or institutional pages on 23 October 2026.

A work was retained when its accessible content directly supported the definition of a criterion, a distinction from a neighbouring approach, or the rationale for the analytical procedure. Relevance was checked against its specific conceptual role, rather than inferred from a title alone; claims about methods or outcomes were limited to accessible abstracts or sections. No predetermined screening count, exhaustive database protocol, or claim to systematic-review coverage was used, and none is retroactively asserted. The criterion-to-strand mapping above documents the principal synthesis step; the accompanying literature-role matrix retains the reference-level rationale. An openly accessible textbook section [29] is used later as a limited application illustration, not as a twenty-ninth research work in this conceptual synthesis or as a sampled validation corpus.

2.3. Unit of Analysis and Evidence Record

Directed qualitative content analysis provides the methodological rationale for beginning with theoretically informed categories [25]. Document analysis requires attention to the document's provenance and context [26]. Guidance on meaning units and trustworthiness further supports preserving the text that supports an interpretation [27]. These methodological ideas are used to design a future document-analysis procedure; the demonstration here applies the procedure to constructed packets, not to sampled books.

The unit is a claim or relevant omission, together with a bounded-context packet. In a real resource, a packet should include the paragraph or equation, its caption or exercise instructions, directly relevant definitions, the declared learning objective, and explicit cross-references. An analyst records the edition or version, page or stable section, exact passage, context searched, criterion, relevance decision, information status, consistency judgment, reasoning, and proposed action. A cross-reference counts as recovered information only if its content is available and supports the interpretation. If essential context cannot be obtained, record its absence from the packet as unresolved access, not as an established omission in the source.

A bounded search begins with the local passage, then examines its section introduction, symbol list, relevant caption, and explicitly cited prerequisite location. Any extension beyond these locations is documented. "Not located" consequently means not found within a reported search boundary. It never means that the information is absent everywhere in an entire book. Source-level conclusions require a wider sampling plan and cannot be inferred from the vignettes in this article.

2.4. Construction of the Analytical Demonstration

Sixteen vignettes were deliberately written as eight contrasting pairs because the immediate purpose was to exercise the boundaries of an ordered decision procedure, not estimate the distribution of textbook defects. Each pair holds the general kind of instructional claim fixed while changing one focal factor: contextual definitions, relevance of a physical interaction, status of a velocity approximation, passive versus active interpretation of a coordinate transformation, system boundary, symbol definition, treatment of omitted series terms, or declared instructional purpose. Eight pairs therefore expose eight analytically distinct transitions, while sixteen cases allow the reasoning on both sides of each contrast to be inspected. The number is a design choice for conceptual coverage, not the result of sample-size calculation or claims of thematic saturation. The examples are author-constructed, not quotations from identified textbooks and not interventions with participants.

For each vignette, the record contains an excerpt, declared task, bounded context packet, focal criterion, applicability decision, information status, consistency judgment, physical rationale, and action. The annotations were stored in JSON and CSV to preserve exact inputs. The author examined dimensional coherence, referents, and the applicability conditions of the physical relations when assigning judgments; the velocity-transformation contradiction also has an explicit rational counterexample. These checks establish whether the stated rationale is inspectable, not whether independent experts would reach the same

conclusion. The cases intentionally span all five actions, and their frequencies must not be presented as evidence about authentic textbooks.

2.5. Logical Checks and Reproducibility

A Python script implements the declared precedence rules, verifies the eight expected pair transitions, reports the distribution of assigned actions, checks one rational counterexample to an asserted universal Galilean velocity relation, and generates Figure 1. It processes supplied labels and fixtures, not the source document's language. Agreement between script outputs and the recorded annotations is an implementation and reproducibility check. It is not independent evaluation of the physical judgments, inter-rater reliability, expert validation, or evidence of instructional effectiveness. The physical claims remain open to independent disciplinary examination.

This limitation is consequential: a program can reproduce an incorrectly justified annotation without detecting the underlying conceptual error. Guidance on reproducible research motivates preserving inputs, transformations, and outputs [28], but interpretive validity requires additional checks against disciplinary reasoning and authentic use. No agreement coefficient or expert-endorsement result is reported because no independent coding or expert panel was conducted. A future validity study should submit a preregistered set of authentic passages and a codebook to independent physics educators, compare initial ratings, investigate disagreements, and test revisions with learners. The supplement supplies the current codebook, vignettes, literature-role matrix, reference export, and executable output for such scrutiny.

3. RESULTS AND DISCUSSION

3.1. Seven Criteria Connecting a Model to Its Explanation

The first result is the set of seven criteria in Table 2. Their roles form a connected but non-identical chain rather than seven independent presence scores. C1 identifies the physical quantities, events, and frames to which a claim refers. C2 specifies the system whose energy or momentum is being discussed, and C3 records the interactions and physical assumptions under which the model applies. C4 marks mathematical approximations and their limiting conditions. C5 then tests whether the defined quantities and asserted operations are physically and mathematically consistent. C6 asks whether the claim is suitable for its declared comparison or task, and C7 examines how language, equations and figures convey that interpretation. A failure at C1 or C2 may prevent a defensible C5 or C6 judgment; it does not prove those downstream criteria false. The result is a sequence of questions whose relevance and supporting evidence must be recorded for each case.

Table 2. Seven criteria and the evidence each requires

Code and criterion	Evidence to record	Question guiding analysis
C1 Quantities and frames	Definitions of event identities, reference frame and time parameter	Can each quantity be attached to a specified measurement or state?
C2 System boundaries	Included bodies or fields and transfers crossing the boundary	Which system does an energy or momentum statement concern?
C3 Physical interactions and assumptions	Motion interaction idealisation and relevant exclusions	Are assumptions sufficient for the physical task being set?
C4 Mathematical approximations	Approximated expression retained terms and limiting conditions	Is a simplification identified and distinguished from an exact relation?
C5 Mathematical and physical consistency	Units, symbol definitions, reasoning steps, and relevant identities	Do the stated quantities and operations support the asserted relation?
C6 Applicability and comparison	Target observable comparison conditions, tolerance, and domain when needed	Does the claimed use follow under the stated conditions?
C7 Representation and interpretation	Links among prose, equations, diagrams and physical referents	What interpretation is licensed by the linked representations?

A practical reading of Table 2 is to ask, for each applicable criterion: what evidence is present, what interpretive issue would its absence create, and would that issue justify clarification, correction, or deferral? For example, when evaluating a time-dilation explanation, C1 locates the proper-time and coordinate-time definitions; C3 checks the relative-motion assumptions; C4 asks whether an approximation is being asserted; C5 checks the relation under those definitions; C6 links it to the task; and C7 evaluates whether the prose or diagram licenses the intended interpretation. C2 is included only if a system-boundary issue actually bears on that task. Thus, a lesson on coordinate transformations need not describe a microscopic absorption mechanism, whereas an energy-transfer problem may need an interaction model or an explicit statement of underdetermination. Relevance must be justified before an omission is marked.

3.2. Separating Information Status from Consistency

For each criterion, the framework records two different judgments. Information status describes where an explanation can be found: local, contextual, not located, unresolved, or outside scope. Local means present in the focal passage; contextual means recoverable at an identified location in the supplied packet. Not located requires a completed bounded search. Unresolved indicates that necessary context could not be obtained. Outside scope applies to a criterion that the declared purpose does not require.

Consistency describes the relationship between the identified claim and its stated interpretation: consistent, contradicted, undetermined, or not assessed. A consistent judgment applies to the bounded claim under specified conditions. A contradicted judgment requires a demonstrable conflict, such as incompatible dimensions or an exact universal claim refuted by an admissible counterexample. Undetermined means that essential referents or conditions have not been established. Not assessed is used when an omitted requirement is outside scope. None of these labels is a global quality rating for a whole resource.

The two records are independent in principle and must not be replaced by a single score. A correct relation may lack a derivation promised by the lesson; a defined symbol may be plainly wrong; or a prerequisite definition may be available through a nearby cross-reference. In the first situation, clarification may be needed; in the second, an explicit contradiction requires correction; and in the third, retaining the bounded claim may be justified. Each decision requires the analyst to report the relevant criterion and evidence location rather than infer the quality of an entire book from a fragment.

3.3. An Ordered Procedure for Proportionate Action

Five actions connect the judgments to revision. Retain means that the focal criterion is adequately addressed for the stated task. Clarify means that an identifiable explanatory or task-specification gap should be repaired without treating an otherwise consistent relation as false. Correct means that a demonstrated inconsistency requires revision. Defer means that a consistency judgment must wait for essential context, physical specification, or a warranted interpretation. Exempt means that an omitted requirement is outside the declared purpose. Exemption applies only to that requirement.

The order is important. First, if an explicit claim is demonstrably contradicted, correct it; a narrow educational scope cannot excuse the contradiction. Second, if the focal requirement is genuinely outside scope and there is no explicit contradiction, exempt that requirement. Third, if context is incomplete, relevance is uncertain, or the physical interpretation is undetermined, defer judgment and identify what must be obtained. Fourth, where the bounded claim is consistent but the task requires additional explanation, clarify it. Otherwise retain it. Figure 1 represents this order; the supplied script implements the same precedence.

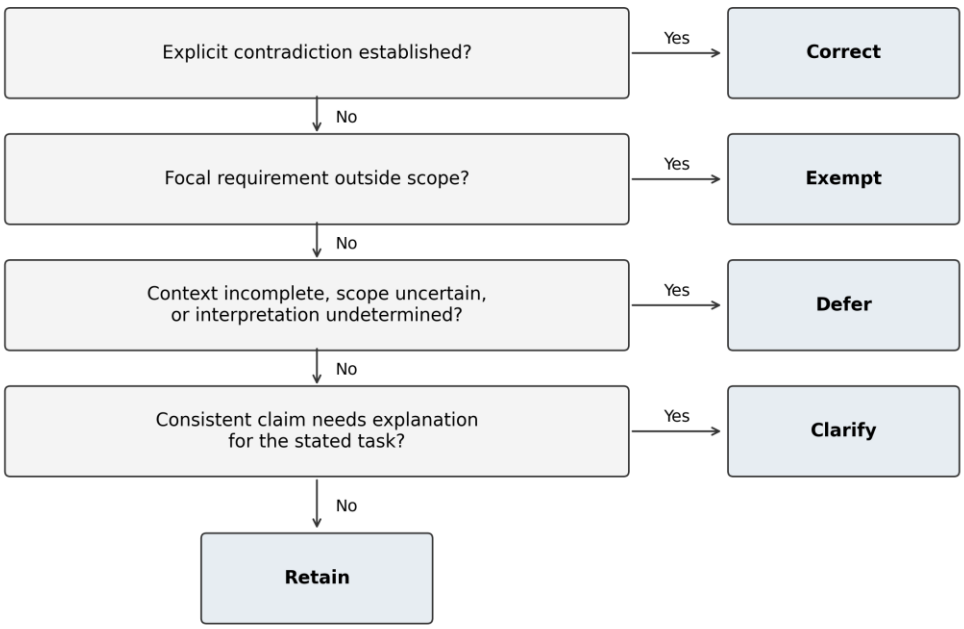


Figure 1. Ordered action procedure applied to analyst-supplied judgments. An explicit contradiction takes priority over an exemption.

The ordered procedure is a documentation aid, not an automatic truth detector. Its path can be checked against a concrete case: for V05 the Galilean relation is explicitly treated as a low-speed approximation, but the worksheet omits the tolerance required by its accuracy question, so the outcome is clarified under C4 and C6. For V06, the same expression is asserted as exact for all admissible speeds, and a counterexample establishes a contradiction under C5; correct therefore takes priority. In either case, a different answer would require a documented change in the passage, its purpose, or the physical justification. The diagram expresses this decision order; independent researchers must still assess the adequacy of the underlying judgments.

3.4. Contrasting Examples and What Their Decisions Establish

The demonstration comprises sixteen author-constructed cases in eight designed pairs. The annotated outputs were five retain, two clarify, four correct, three defer, and two exempt, and the script reproduced the prescribed transitions. These numbers describe only this deliberately balanced illustrative set; they do not measure error prevalence, effect size, student learning, or inter-rater agreement. Table 3 makes the focal contrast and its changed action visible, while the explanations below provide the accompanying physical or contextual reason.

Table 3. Outcomes of the eight constructed contrasts

Pair and cases	Focal change	First action	Second action
P1 V01–V02	Definitions become recoverable	Defer	Retain
P2 V03–V04	An interaction model becomes relevant	Exempt	Defer
P3 V05–V06	Approximate relation becomes a universal equality	Clarify	Correct
P4 V07–V08	Passive redescription becomes a claimed physical operation	Retain	Correct
P5 V09–V10	System boundary becomes unspecified	Retain	Defer
P6 V11–V12	Momentum is explicitly redefined as velocity	Retain	Correct
P7 V13–V14	Neglected terms are asserted to vanish	Retain	Correct
P8 V15–V16	A reference sheet becomes a promised derivation	Exempt	Clarify

3.4.1. Recoverable Context Changes the Time-Interval Judgment

V01 supplies only a time-dilation statement without the surrounding definitions. V02 supplies the same statement with definitions of the intervals and constant inertial relative motion. The applicable relations are

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{1}$$

$$\Delta t = \gamma \, \Delta\tau \tag{2}$$

Here c is the speed of light, v is the clock's constant velocity relative to the inertial laboratory, $\Delta\tau$ is proper time between two events on that clock, and Δt is the laboratory coordinate-time interval between the same events. Equation (2) in this form is restricted to

the stated constant velocity. Accelerated histories require the appropriate treatment of varying velocity rather than an unexplained single factor.

V01 receives defer because the isolated sentence does not establish those referents. V02 receives retain because its supplied preceding paragraph establishes them. This pair demonstrates why local omission and source-level omission must be distinguished. It also makes the retrieval burden explicit: the analyst records which paragraph supplies which definition. The framework does not presume that learners will necessarily find or understand that paragraph; that is a further usability question.

3.4.2. A Valid Approximation and a False Universal Equality Require Different Revisions

For collinear inertial frames, with frame S' moving at velocity V along the common positive axis relative to S, the exact transformation of a particle velocity u is

$$u' = \frac{u-V}{1-\frac{uV}{c^2}} \tag{3}$$

The Galilean expression is an approximation when the denominator correction is negligible for the task. V05 states that approximation and its limiting condition, but the worksheet asks for an accuracy decision without providing a tolerance. Its action is to clarify: the task needs a criterion for acceptable error. V06 instead states that the Galilean equality is exact for every allowed velocity. The choice $u = 0.6c$ and $V = 0.2c$ gives $u' = (5/11)c$ from equation (3), whereas the asserted equality gives $0.4c$. This is a direct counterexample, so V06 requires correction.

The pair distinguishes incomplete specification of a learning task from physical inconsistency. Merely adding a tolerance to V06 would not repair its universal exactness claim. Conversely, replacing V05 wholesale would discard an appropriately identified limiting approximation. The useful revision follows the reason for the judgment.

3.4.3. Explicit Notation Can Establish a Contradiction

For a timelike particle state, the energy–momentum relation is

$$E^2 = p^2c^2 + m^2c^4 \tag{4}$$

Here E is total energy, p is the magnitude of three-momentum, and m is rest mass; energy and momentum refer to the same state and inertial frame. V11 defines the symbols in this way and receives retention. V12 uses the same printed equation but explicitly defines p as velocity in SI units. Under that definition, the term containing p has dimensions incompatible with the other terms. V12 therefore receives a correction.

The comparison shows why the presence of a familiar equation is insufficient evidence of consistency. The equation and its definitions must be read together. It also shows why an analyst should not silently repair a symbol while evaluating the original passage. A charitable reconstruction can be proposed as the revision, but the evidence record must retain what the passage actually says.

3.4.4. Scope Protects Legitimate Brevity without Protecting False Claims

V03 excludes a variable-mass interaction model from a coordinate-transformation lesson; V04 needs an interaction description for an energy-transfer task. The actions are exempt and deferred, respectively. V04 is not assigned the dynamics of a particular absorption process by assumption. The analyst identifies the missing physical specification and requests it. This guards against imposing a preferred specialised model on an underdetermined situation.

V15 and V16 both omit an energy-relation derivation. V15 is a retrieval sheet whose objective excludes derivations. V16 is a complete standalone packet that promises a derivation but provides only a consistent final relation. For the derivation-traceability criterion, the outcomes are exempt and clear. The second case requires explanatory work, but its omission does not prove that the stated final relation is false. These examples make the educational purpose a documented condition of judgment, rather than a general excuse for incomplete writing.

3.4.5. Small Terms and Omitted Terms Are Not Identical Claims

V13 identifies a quadratic kinetic-energy approximation under unchanged physical assumptions. V14 asserts that all higher terms vanish exactly throughout a finite interval of nonzero speeds.

For positive fixed rest mass,

$$K = (\gamma - 1)mc^2 \approx \frac{1}{2}mv^2 \tag{5}$$

with γ given by Equation (1). The approximation in equation (5) neglects higher even powers in the small-speed expansion; it does not make them identically zero. V13 receives retain and V14 receives correct. The pair does not establish a universal numerical threshold for all low-speed uses. A quantitative application would still need a target observable, a comparison baseline, and an acceptable error.

The remaining pairs concern passive frame changes and system boundaries. V07 correctly treats a frame transformation as a redescription; V08 incorrectly presents that redescription alone as physically accelerating the body. V09 attaches energy conservation to an isolated combined system and an inertial frame. V10 leaves the referent of “the energy” unspecified despite mentioning transfer. The respective actions are retain, correct, retain, and defer. Their purpose is to demonstrate distinct analytical decisions, not to claim new results about Lorentz transformations or energy conservation.

3.5. What Is Added to Existing Approaches

The proposed contribution is the operational connection among criterion, bounded evidence, consistency judgment, and editorial action. Modelling studies motivate attention to model construction, purpose and limitations [6]–[10]; representation research explains why the link between notation and physical referents matters [11]–[20]; and document-analysis guidance motivates recording the exact passage and search boundary [25]–[27].

None of these neighboring strands is dismissed. The synthesis specifies a distinctive workflow for a different task: evaluating whether a particular instructional claim should be retained, clarified, corrected, deferred pending evidence, or exempted because the requirement is outside the resource's declared purpose.

Three features explain why the workflow is more discriminating than a single checklist. First, it does not conflate the location of an explanation with whether a stated relation is true under declared conditions. Second, it treats instructional scope as a relevance gate without letting brevity protect an explicitly false claim. Third, it treats deferral as an auditable conclusion when a frame, system boundary, prerequisite, or interaction model remains unresolved. These features are demonstrated by constructed contrasts, not by a comparison trial against established rubrics; claims of higher reliability or superior learning outcomes would therefore be premature.

The research object is the connection between an instructional claim and its justification. A standard physical relation can supply a consistency check, but the check alone does not determine whether an explanation fulfils its educational purpose. The velocity counterexample above supports a particular correction decision; it serves as an illustration. The educational product is the analytical protocol and its transparent demonstration.

3.6. Uses in Teaching and Resource Development

The protocol suggests three uses for teachers and resource developers: focused revision of lecture notes, guided comparison of alternative explanations, and structured discussion in teacher preparation. Table 4 lists the proposed activities and the evidence they would produce. A simple classroom exercise could give students two versions of a time-dilation explanation, ask them to identify the events and frames, and require a written justification for retaining, clarifying, or deferring. An instructor can then discuss disagreements before editing the resource. Such activities are proposed mechanisms for making scientific reasoning explicit, not interventions whose learning effects have been measured in this study.

Table 4. Proposed educational uses and observable products

Use	Activity	Product available for evaluation
Material revision	Inspect a passage and its bounded context before editing	A revision linked to a recorded physical or explanatory reason
Teacher education	Compare paired passages and justify different actions	Written identification of the changed condition and its consequence
Source comparison	Apply the same criterion within comparable purposes and levels	An evidence matrix with locators and scope decisions
Assessment design	Ask learners to explain conditions and test transfer to another case	Responses addressing assumptions and interpretation beyond formula recall

For resource authors, a short correction may be more effective than adding an entire derivation: specifying the clock and the events can remove ambiguity about proper time; drawing a system boundary can clarify a conservation statement; and naming the approximation and its use condition can prevent an exactness claim from being over-

generalised. Table 4 links each use to a product that could later be evaluated. The objective is not maximal explanation but evidence sufficient for the task and retrievable at a reasonable location.

Teachers may face practical difficulties when applying seven criteria: incomplete access to a textbook's prerequisites, disagreements over audience knowledge, uncertainty about the system boundary, and time constraints when reviewing many passages. A workable introduction would therefore start with one focal criterion and an explicit learning objective, require reviewers to cite the supporting lines, and allow a defer outcome rather than forcing an unsupported choice. In a teacher-education workshop, participants could independently annotate a pair of contrasting passages, compare their evidence records, and agree on the smallest justified edit. The disagreement log would inform later refinement of the codebook.

The same design is potentially transferable beyond special relativity because identifying a model's system, assumptions, representation, and domain is relevant elsewhere in physics. Candidate contexts include ideal-gas models, circuit approximations and energy-conservation explanations. Such transfer would require topic-specific operational definitions and physical counterexamples; the present criteria cannot be assumed valid unchanged. Physical warrant and novice accessibility must also be assessed separately: a definition may be recoverable for an expert while still hard for a student to find or interpret.

3.6.1. An Illustrative Application to an Open Textbook

As a bounded desk illustration using a publicly available teaching text, OpenStax University Physics, Volume 3, Section 5.3 defines proper time using two events that occur at the same position in the clock's rest frame, distinguishes the corresponding Earth-frame interval, and presents the time-dilation relation [29]. If the focal analytical task is to decide whether the time intervals in that relation are identifiable, C1 is applicable. The definitions are available in the section context and are consistent with the stated inertial comparison so that the limited C1 judgment can be retained. This judgment concerns a specified excerpt and criterion, not the quality of the book as a whole.

The example also shows why the source boundary matters. If a worksheet reproduces only the formula but does not define its intervals or cite the textbook section, the analyst should initially record an unresolved or unlocated definition in that worksheet's packet rather than transfer the original book's judgment automatically. Further retrieval may change the action. This example illustrates a practical application of the procedure to an authentic resource; it is neither a sampled textbook analysis nor independent validity evidence, and it has not been included in the constructed-case counts.

3.7. Limits and an Empirical Evaluation Pathway

The framework currently has conceptual and implementation checks, not independent construct validation or demonstrated classroom effectiveness. The paired cases were authored and annotated within the same development process, the published textbook illustration below is only a bounded desk application, and the literature synthesis

is selective. These choices limit any claim of generality, interpretive reliability, or frequency of shortcomings in educational resources. Because purposeful contrasts are designed to trigger different actions, their action distribution is not a sample statistic. The novelty claim therefore concerns the specified integration and its reproducible decision order, not an established empirical advantage over prior frameworks.

A stronger evaluation programme would first ask independent special-relativity and physics-education experts to judge the clarity, physical correctness, and coverage of each criterion, using documented disagreements to revise the codebook. A separate study would then select an authentic corpus with editions, pages and search boundaries recorded, have trained coders independently classify bounded passages, and report agreement with an appropriate design and uncertainty analysis. Challenging examples and unresolved judgments should be retained in the audit trail. Extension to other topics should be tested with topic-specific cases rather than inferred from terminology alone.

Only after interpretive validity is examined should classroom usefulness be tested. A classroom study could compare student work on original and revised passages, using tasks that require naming assumptions, interpreting the symbols, distinguishing approximation from exactness, and applying a relation to a new situation. Student interviews or written explanations could probe whether learners actually retrieve context and justify an inference. Such measures would assess a different outcome from code reproducibility. No student-learning benefit, expert agreement, or independent coding result is claimed by the present paper.

4. CONCLUSION

This study proposes an auditable claim-level framework for deciding how to revise assumptions and explanations in undergraduate special relativity resources. Its conceptual contribution is to integrate seven physically and educationally motivated criteria with two separate judgments: where relevant explanatory information can be recovered, and whether the specified claim is physically consistent. A declared task and bounded context determine which requirements are pertinent. An ordered decision rule then distinguishes clarifying an explanatory gap from correcting a demonstrated contradiction, while permitting justified retention, deferral, and exemption.

The contrasting cases and a limited public-textbook illustration show how the procedure can organise reasoned editorial decisions without treating brevity as error. For physics teachers and curriculum developers, the immediate recommendation is to record the focal claim, task, referents, evidence location and smallest justified action before revising a lesson or worksheet. These are proposed practices, not verified teaching improvements. Next steps are independent expert review of the criteria, systematic application to authentic resources, inter-analyst agreement studies and classroom research on conceptual interpretation and transfer. The framework remains open to refinement in light of those findings.

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ChatGPT (OpenAI, accessed October 2026) assisted with literature discovery and Python code generation. Its output is not an independent analyst assessment. The examples are explicitly author-constructed analytical demonstrations, and the accompanying code checks supplied annotations rather than autonomously evaluating learning resources. No empirical student outcomes or independent expert validation are reported.

DATA AND MATERIALS AVAILABILITY

The supplementary document contains the full codebook, sixteen vignette packets, a literature-role matrix, and the complete reproduction script. The accompanying reproducibility archive supplies machine-readable cases, the rule-checking and figure-generation script, reference metadata, an IEEE-ordered RIS export, execution results, and file checksums. The logical checks use the Python standard library; regenerating the figure additionally requires Matplotlib. No participant data are involved.

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