Analysis of the Geometric Thinking Level of Primary School Students with Moderate Academic Ability in Mataram City: Evidence from a Plane Geometry Task

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ABSTRACT

Geometric thinking is a crucial skill for solving geometry problems and navigating everyday life. This descriptive qualitative study aims to analyze the levels of geometric thinking regarding plane figures among primary school students with moderate academic ability. The study also describes the errors students make when solving problems involving plane geometry. Data were collected through interviews and an analysis of students' written answers to five open-ended questions designed to assess specific indicators of geometric thinking: 1) visual skills; 2) verbal skills; 3) drawing skills; 4) logical skills; and 5) applied skills. This study involved 531 fifth-grade students from 12 public primary schools in Mataram City; however, the analysis focused on 450 students classified as having moderate academic ability based on their obtained score intervals. The researchers interviewed five students who provided the most common answers for each question. The collected data were analyzed using thematic analysis, including familiarization, coding, searching for initial themes, reviewing and developing themes, defining and naming themes, and report writing. The results indicate that students with moderate academic ability demonstrated proficiency primarily in visual, drawing, and verbal skills, yet still frequently made errors regarding applied skills. The results also indicate that students with moderate mathematical ability have not yet fully grasped geometric concepts. These findings enable teachers to identify the capabilities and errors of students with moderate academic ability during geometry instruction, thereby improving their understanding of plane geometry.

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1. INTRODUCTION

Indonesian primary school students' ability to understand and solve mathematics problems currently remains poor. According to the Trends in International Mathematics and

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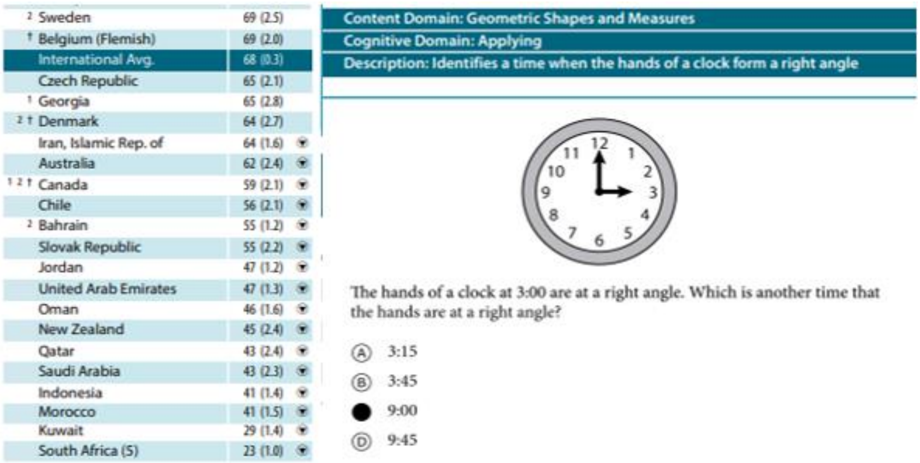
Science Study (TIMSS), which tests children's mathematics skills in participating countries, including numbers, statistics, and geometry, Indonesia ranks 44th and 49th among participating countries [1]. Furthermore, the report shows that Indonesia's mathematics score of 397 is below the average score of participating countries (the average score of TIMSS members is 500). Examples of TIMMS geometry problems and the results showing Indonesia's ranking compared to participating countries can be seen in Figure 1.



2 Sweden	69 (2.5)
† Belgium (Flemish)	69 (2.0)
International Avg.	68 (0.3)
Czech Republic	65 (2.1)
1 Georgia	65 (2.8)
2 † Denmark	64 (2.7)
Iran, Islamic Rep. of	64 (1.6)
Australia	62 (2.4)
1 2 † Canada	59 (2.1)
Chile	56 (2.1)
2 Bahrain	55 (1.2)
Slovak Republic	55 (2.2)
Jordan	47 (1.2)
United Arab Emirates	47 (1.3)
Oman	46 (1.6)
New Zealand	45 (2.4)
Qatar	43 (2.4)
Saudi Arabia	43 (2.3)
Indonesia	41 (1.4)
Morocco	41 (1.5)
Kuwait	29 (1.4)
South Africa (5)	23 (1.0)

Figure 1. Example of TIMSS Questions and Rankings of Indonesian Primary School Students

Geometry is one of the core subjects that must be taught at the primary school level. Geometry taught at the primary school level in Indonesia is divided into two parts, namely plane geometry (often called 2-dimensional) and geometry of solid shapes (often called 3-dimensional). Plane geometry includes the introduction of points, lines, and angles as well as the shapes and properties of squares, rectangles, rhombuses, kites, trapezoids, and circles, while geometry of solid shapes includes the introduction of shapes and properties of cubes, cuboids, prisms, cylinders, pyramids, cones, and spheres [2]. This geometry material is taught to students from lower to upper grades, starting with the introduction of geometric shapes and progressing to solving geometry-related problems.

Geometry is a material that plays a crucial role in building student understanding. Geometry is very useful in helping students visualize, think critically, solve problems, make arguments, and provide proof [3]. Through geometry, students must describe geometric shapes using given characteristics and solve geometry-related problems. Geometry can also help students develop ideas about shape and space and connect various mathematical concepts when solving problems [4]. There are various ways to learn geometry, including games and activities such as paper folding and drawing, which aim to increase knowledge of the shapes and characteristics of geometric objects.

Despite the long history of geometry instruction, students still struggle to master it. They perceive geometry as a highly formal and complex subject, making it difficult to master [5]. Many students remain at the knowledge (C1) and understanding (C2) levels. Students still struggle to answer questions related to geometric elements, geometric properties, constructing geometric shapes using planes, and manipulating geometric shapes [6], [7]. Students have difficulty understanding mathematical concepts [8], connecting various geometric concepts to solve problems [9], understanding geometric definitions and theorems, using visual skills, and demonstrating mathematical literacy [5]. Figure 2 shows a student answer illustrating the difficulty of answering geometry questions about geometric elements.

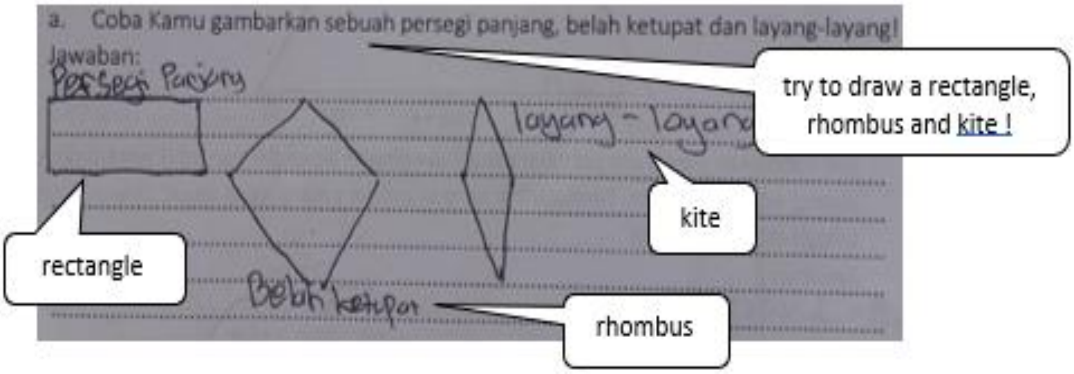


Figure 2. Student’s Answer for Drawing Skill Plane Figures

Based on Figure 2, students were indeed able to draw the plane figures and label them according to the problem statement; however, they failed to indicate geometric elements on the figures, such as marking parallel and equal-length sides, drawing diagonals for kites and rhombuses, identifying the types of angles, and labeling the vertices. These results indicate that geometry, specifically regarding plane figures, remains a challenge for students. The difficulties students encounter are closely linked to their geometric thinking abilities.

Geometric thinking is the ability to understand, reason about, and visualize shapes and patterns through mental activities such as recognizing shapes, analyzing properties, and solving problems [10]. It progresses from basic abilities, such as recognizing the shapes and properties of plane figures, to more complex aspects, including classification and determining geometric relationships [11]. Geometric thinking has several levels: first, visual skills; second, verbal skills; third, drawing skills; fourth, logical skills; and fifth, application skills [12]. Visual skills are students’ ability to recognize and interpret geometrically represented images, verbal skills are students’ ability to read, understand, and communicate geometric information using words, drawing skills are abilities that help students understand the relationship between shapes and their properties through direct practice, logical skills are students’ ability to identify similarities and differences between geometric shapes through logical reasoning, and application skills are students’ ability to solve geometric problems through understanding geometric concepts [12], [13].

The results of the study indicate that the level of geometric thinking is not uniform. [14] stated that students can reach the visualization and analysis level by constructing geometric shapes and identifying them using known data. Students can explain the similarities and differences between the geometric shapes presented, but they have not yet reached the abstraction stage. Students still have difficulty finding concepts from the geometric shapes presented and connecting various geometric concepts to solve problems. Likewise, the results of [15] research stated that when connected with other mathematical materials, students are no longer able to apply geometric concepts in solving non-routine problems, even at the most basic stages (visualization), such as making straight lines, drawing lines normally, or drawing the reflection results of a geometric shape. Students' inability to master geometric thinking can make it difficult to solve geometry problems, which can also affect their ability to solve related problems in everyday life.

Interviews conducted at several primary schools in Mataram City, attended by students from several sub-districts, also showed that students have varying levels of geometric thinking. These interviews revealed students' weaknesses in learning geometry, particularly planes. Generally, teachers stated that students could still create representations of planes, but some students could not provide complete characteristics. The teacher also stated that students did not understand the properties of plane shapes, especially data shapes that, in representation, have the same shape (square, rhombus, and kite), so it was difficult to distinguish the plane shapes. Other interview results also revealed that students largely did not understand the concept of area or circumference in real-world contexts (story problems), even though they could work with it when it was not presented in a story-problem format. The interview results show that students' geometric thinking levels differ, and their difficulties in solving these problems follow Hoffer's levels of geometric thinking.

Given this context, it is important to research students' levels of geometric thinking. Such research will yield comprehensive information regarding the geometric thinking levels of primary school students. In addition to identifying these levels, it is also necessary to analyze the errors students make when solving the assigned problems. Furthermore, the results of this analysis can serve as a basis for formulating appropriate measures or policies to help improve the geometric thinking abilities of primary school students in Mataram city.

Many studies have been conducted on geometric thinking, including on geometric proof and justification skills [16], representation, spatial structure, and measurement [17], geometric problem solving [18], errors and misconceptions in solving geometric problems [19], developing in-depth geometric thinking collaboratively [20], and 3D geometric thinking skills [7]. Furthermore, research on the level of geometric thinking has been conducted, namely the level of geometric thinking in the application of spatial functions [3], on the analysis of the level of geometric thinking through van Hiele's theory in middle school students [13], [16], and the level of geometric thinking of prospective teachers using van Hiele's theory [4]. While these studies applied the van Hiele framework, less research has used Hoffer's skill framework to interpret primary school students' written answers when solving geometry problems; thus, this study is warranted.

Research on Hoffer's levels of geometric thinking is worthwhile because the framework provides a detailed explanation of the characteristics of geometric thinking at each level, and these characteristics are well suited to primary school students. Based on the foregoing, this study aims to analyze the geometric thinking levels of primary school students with moderate academic ability in Mataram City and to describe their work at each level of geometric thinking.

2. METHOD

This study uses a qualitative case study method. Case study research aims to analyze a case in detail and depth from a single individual, group, or community to generalize it to several specific groups [21]. This case study examines primary school students' geometric thinking levels on the topic of planes, based on Hoffer's theory. Table 1 shows the indicators of geometric thinking levels according to Hoffer's theory.

Table 1. Indicators of Geometric Thinking Levels According to Hoffer's Theory

No	Levels	Indicator
1	Visual skills	Able to remember and recognize flat geometric shapes
2	Drawing skills	Able to draw geometric shapes accurately, even adding marks to the drawing to explain its properties, such as marking angles or sides of equal length
3	Verbal skills	Able to explain or describe geometric shapes, properties, relationships, and concepts
4	Logical skills	Able to compare and construct logical arguments about the relationships between flat geometric shapes
5	Applied skills	Able to recognize physical models of geometric shapes, apply the properties of geometric models to the properties of physical objects, and use these geometric models in problem solving

This study involved 531 fifth-grade primary school students in six districts of Mataram City. The schools were selected using purposive sampling based on their accreditation status in Mataram City, and the study focused specifically on students with moderate academic ability. To categorize students by academic ability, the study used the "three-box method" [22]. The "three-box method" is a technique used to divide a group of students into three subgroups based on the scores they achieved. The study involved five essay questions, each with a minimum score of zero (0) and a maximum score of 10, resulting in a total possible score range of 0 to 50. The formula for determining these three student groups is as follows:

$$I = \frac{X_{max} - X_{min}}{3}$$

Where:
$I =$ category interval length
X_{max} = maximum value
$X_{min} =$ minimum value
$3 =$ number of categories

Data collection was conducted through interviews and a set of geometry problems consisting of five open-ended questions, with each question addressing a specific indicator of geometric thinking. A single indicator was assigned to each question because Hoffer's criteria for geometric thinking are hierarchical in nature; this implies that students can only answer questions at a higher level after completing the preceding level. Two experts specializing in mathematics and mathematics education validated the problems used in this study. The validation process encompassed content validation and curriculum alignment. Interviews were conducted with five primary school students, selected for having the most common answer pattern within the "moderate academic ability" group, using open-ended questions that allowed the researcher to probe beyond the predetermined inquiries. The researcher conducted repeated interviews to elicit consistent answers, and this consistency

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served as the basis for concluding the thinking levels of primary school students with moderate academic ability.

Based on the students' answers, their levels of geometric thinking will be analyzed according to established indicators. This study will analyze the data using thematic analysis in six steps: familiarization with the data, coding, identifying initial themes, reviewing and developing themes, determining and naming themes, and writing a report [23]. This research presents the coding scheme for geometric thinking levels in Table 2.

Table 2. Coding of Geometric Thinking Level Data

Levels of thinking	Code	Theme
visual skills	writing the names of plane figures	Introduction to plane figures
verbal skills	writing the definitions of plane figures	Understanding concepts of plane figures
drawing skills	drawing plane figures	Using representations of plane figures
logical skills	writing the differences and similarities between plane figures	Introduction to relationships between plane figures
applied skills	solving real-life problems related to plane figures.	Using formulas and procedures

Through these six steps of thematic analysis, we will obtain a report on the level of flat plane geometric thinking of primary school students, specifically grade 5 students with moderate academic ability. Next, we will also describe the responses of five research subjects in solving geometry-level problems. The flow of research to be carried out can be seen in Figure 3.

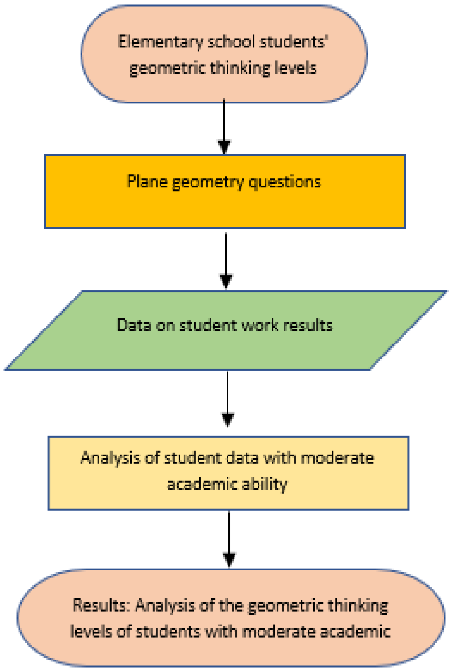


Figure 3. Research Flow Diagram

3. RESULTS AND DISCUSSION

Based on the results of tests conducted on 531 primary school students, various answers were found based on the level of geometric thinking in solving geometry problems. Based on the students' answers, the geometric thinking ability levels were divided into three groups based on the three-box method theory [22]: high, moderate, and low. These groups were determined by the highest scores, which were then divided into three parts. Table 3 shows the grouping of students' abilities based on these geometric thinking categories.

Table 3. Grouping of Student Abilities Based on Geometric Thinking Categories

No	Group	Score range	Number of students	Percentage
1	Low	0-15	65	12.25 %
2	Moderate	16-33	450	84.75 %
3	High	34-50	16	3.01 %

Based on Table 3, this study describes the answers of students with moderate academic ability when solving geometry problems, categorized according to indicators of geometric thinking levels. The study focuses on students with moderate academic ability because this group represents the largest segment compared to other academic groups. Each student's answer was classified into a specific answer category for each question, resulting in a total of 450 students per category. Table 4 presents the answers of students with moderate academic ability for each problem, classified by geometric thinking category.

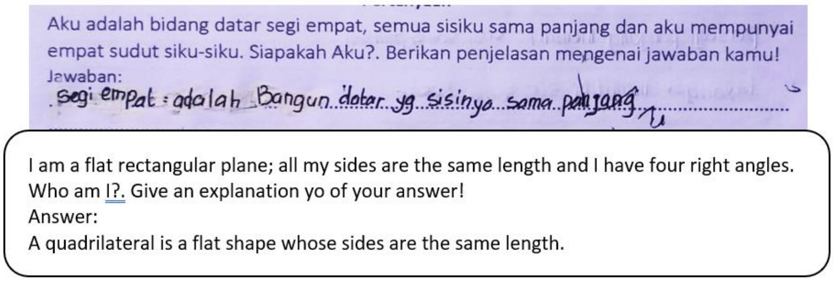
Table 4. Responses of Students with Moderate Academic Ability in Solving Problems Based on Geometric Thinking Categories

No	Geometric Thinking Categories	Indicator	Responses Category	Number of Students	Percentage
1	Visual skills	Able to remember and recognize flat geometric shapes	Students write the names of flat surfaces that do not match the characteristics given in the problem.	103	22.88 %
			Students describe a flat surface that matches the characteristics given in the problem.	3	0.66 %
			The student wrote the name of the flat surface correctly, but the reason given was wrong.	19	4.23 %
			The student wrote the name of the flat surface correctly but did not write the reason	101	22.45 %
			Students write the name of the plane correctly and give the correct reason	224	49.78 %
2	Drawing skills	Able to draw geometric shapes accurately, even adding marks to the drawing to explain its	Students cannot describe the specified flat plane	2	0.45 %
			Students can describe several specified flat surfaces but have not been given signs on the flat surfaces	4	0.89 %

		properties, such as marking angles or sides of equal length	Students can describe all the specified flat surfaces but have not been given signs on the flat surfaces	440	98 %
			Students can describe all the specified flat surfaces and have given signs on several of these flat surfaces	3	0.66 %
3	Verbal skills	Able to explain or describe geometric shapes, properties, relationships, and concepts	Students are wrong in defining the given flat plane	62	13,78 %
			Students use pictures to define flat surfaces	1	0,23 %
			Students use pictures to define a given plane	3	0,66 %
			Students explain the definition of a plane through analytical, genetic, and formulaic definitions	384	85.33 %
4	Logical skills	Able to compare and construct logical arguments about the relationships between flat geometric shapes	Students can write down the similarities and differences of the given flat surfaces	58	12.88 %
			Students make pictures to write down the similarities and differences of the given flat planes	1	0.22 %
			Students do not write answers on the worksheet	9	2 %
			Students make mistakes in writing the similarities and differences of the given flat surfaces	96	21.33 %
			Students only write down some of the similarities and differences between the given planes	2	0.45 %
			Students do not answer the questions given	99	22 %
			Students only write down the similarities or differences between the given planes correctly	184	41.12 %
5	Applied skills	Able to recognize physical models of geometric shapes, apply the properties of geometric models to the properties of physical objects, and use these geometric models in problem solving.	Students do not write answers on the worksheet	14	3.11 %
			Students wrote the wrong answer	405	90 %
			Students write the answers correctly but do not write the process.	21	4.67 %
			Students write the answers correctly and write the process correctly.	10	2.22 %

3.1. Visual Skills

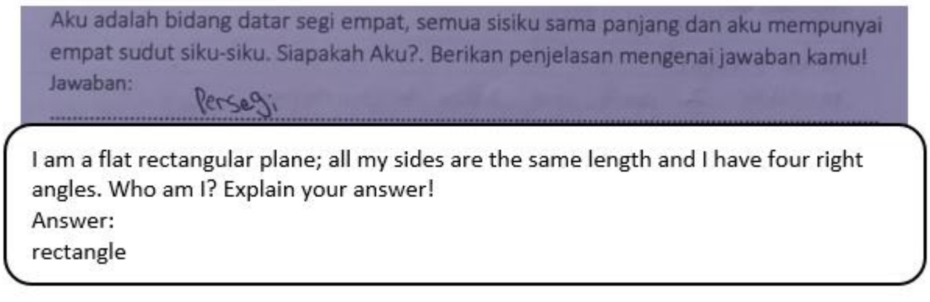
Based on Table 4, regarding visual skills, students were asked to recall and recognize two-dimensional geometric shapes. Based on the problem presented, the answers from students with moderate academic ability were dominated by three types of answers: 1) writing the name of a two-dimensional shape that did not match the given characteristics; 2) correctly identifying the shape but failing to provide a reason; and 3) correctly identifying the shape and providing a valid reason. For answers to writing the name of a data plane that does not match the characteristics given, students write the answer with the name of a flat plane that has a similar shape or words that are often spoken every day, such as rectangle, square, cube, and beam [24], [25]. This also shows that the students' concepts are not yet perfect because they cannot differentiate between examples of three-dimensional or two-dimensional geometry. Figure 4 shows an example of a student's written answer naming a flat plane that does not match the characteristics.


Aku adalah bidang datar segi empat, semua sisiku sama panjang dan aku mempunyai empat sudut siku-siku. Siapakah Aku?. Berikan penjelasan mengenai jawaban kamu!
Jawaban:
segi empat : adalah Bangun datar yg sisinya sama panjang

I am a flat rectangular plane; all my sides are the same length and I have four right angles. Who am I?. Give an explanation yo of your answer!
Answer:
A quadrilateral is a flat shape whose sides are the same length.

Figure 4. First Type of Answer from Student with Moderate Academic Ability for Visual Skills

In the second type of answer, students correctly identify the name of the plane figure but provide no justification; they state the correct name without noting the relevant geometric features [26], [27]. Ideally, students should have included the characteristics specified in the problem statement, but they did not. Interview findings reveal that students hesitated to list the characteristics provided in the problem because they believed other, more accurate characteristics existed beyond those explicitly stated. Figure 5 shows an example of this second answer type correctly naming the plane figure without justification.


Aku adalah bidang datar segi empat, semua sisiku sama panjang dan aku mempunyai empat sudut siku-siku. Siapakah Aku?. Berikan penjelasan mengenai jawaban kamu!
Jawaban:
Persegi

I am a flat rectangular plane; all my sides are the same length and I have four right angles. Who am I? Explain your answer!
Answer:
rectangle

Figure 5. Second Type of Answer from Student with Moderate Academic Ability for Visual Skills

For the third type of answer, namely writing the correct name of the plane accompanied by the appropriate reason, the student understood the problem. This also shows that the student can recognize and identify a plane based on the characteristics given. An example of an answer from a student with moderate ability in writing the third type of answer, namely writing the correct name of the plane accompanied by the appropriate reason, can be seen in Figure 6.

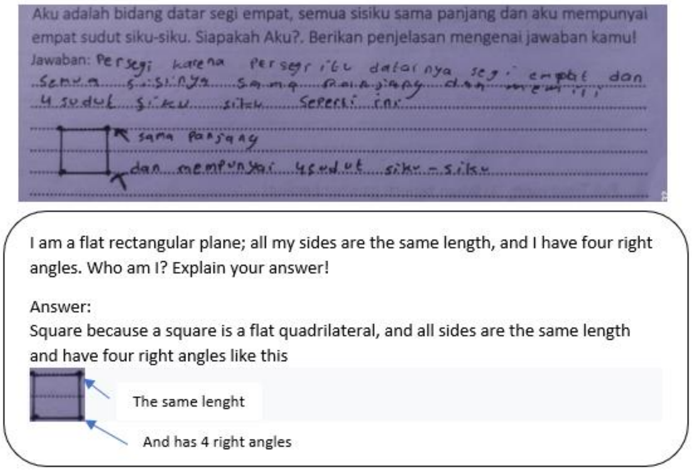


Figure 6. Third Type of Answer from Student with Moderate Academic Ability for Visual Skills

In visual skills, recognizing planes is a prerequisite skill that every student must master to understand more complex geometric concepts, such as understanding the properties of planes, understanding the relationships between planes, completing area and circumference calculations, and solving geometry problems. Students with moderate academic ability generally find it easy to recognize plane shapes, but they recognize plane shapes based on whole shapes rather than using geometric properties. The results of this study indicate that students with moderate academic ability more often write plane shapes based on everyday objects, for example, marking a square with a box. Students prioritize shapes without analyzing the properties that distinguish squares from boxes [28]. Furthermore, students with moderate academic ability often write that all plane surfaces with four sides are called quadrilaterals because they have a weak ability to understand the properties of plane surfaces, without recognizing the specific characteristics of these plane surfaces. However, students with moderate academic ability still write the properties of the given plane surfaces even though they do not write the symbols that mark the characteristics of the plane surfaces [29].

3.2. drawing Skills

Regarding drawing skills, students are required to draw geometric shapes accurately and add markings to illustrate their properties, such as indicating equal angles or sides. Based on the given problem, the dominant answer among students with moderate proficiency was the ability to draw all the specified plane figures, though they did not yet include markings on the figures they drew. In this type of answer, the student draws and labels the plane figure but does not include distinguishing features, such as markings indicating parallel sides, right angles, equal side lengths, or diagonal intersections [26], [27]. Figure 7 shows an example of a student's answer that describes all the specified planes but does not yet include signs on the drawn plane.

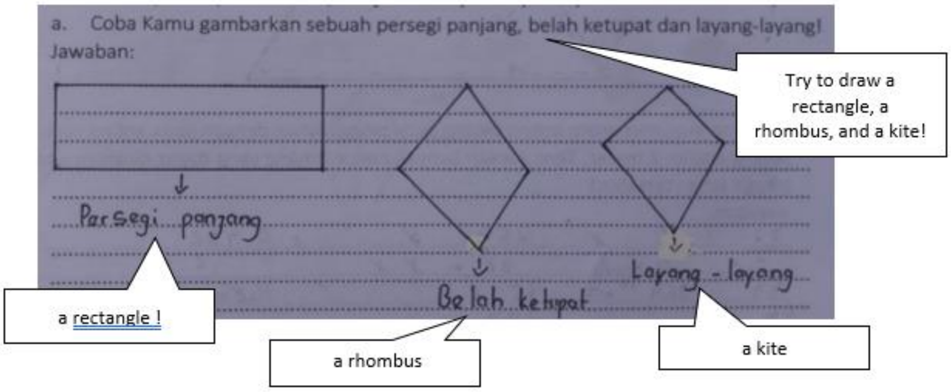
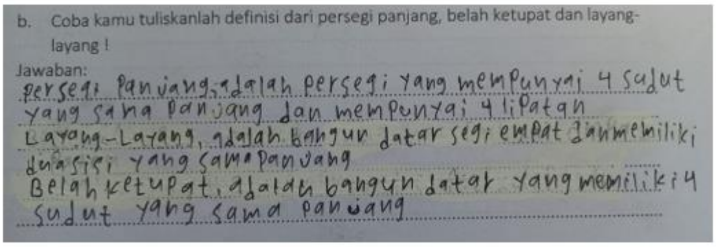


Figure 7. Example of Answer from Student with Moderate Academic Ability for Drawing Skills

In drawing skills, students with moderate academic ability could depict the requested plane but did not demonstrate the size, accuracy, or characteristics that indicate the plane's nature. Students with moderate academic ability also provided drawings of planes with incorrect proportions, such as determining the length of sides, the size of angles, symmetrical shapes, and size comparisons, which resulted in students with moderate academic ability drawing rhombuses and kites with almost the same shape. This indicates that students with moderate academic ability have not fully fulfilled the properties and characteristics of planes accurately [30]. Although the drawings were not perfect, students with moderate academic ability were still able to name the flat surfaces they drew. This indicates that students with moderate academic ability already possess intermediate mathematical representation skills, namely the ability to transform verbal information into simple visual forms [31], and can still be further trained with a proper conceptual understanding of the actual drawing.

3.3. Verbal Skills

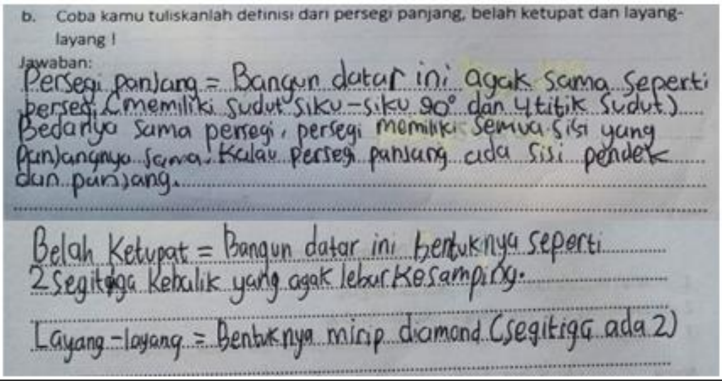
In the verbal skills, students were asked to explain or define a plane. Based on the given problem, the results of students with moderate academic ability were dominated by two types of answers: students who incorrectly defined a plane and students who explained the definition of a plane through conceptual, operational, and axiomatic definitions. For the first type of answer, namely students incorrectly defining a plane, students with moderate academic ability define a plane using characteristics that do not correspond to a plane, for example, sides and angles. Students write that a square is a plane figure with four sides of unequal length [32], whereas, regarding angles, students with moderate academic ability explain that a rhombus is a plane figure with four angles of equal measure [33], [34]. Figure 8 shows an example of a student's answer defining a plane figure.


b. Coba kamu tuliskanlah definisi dari persegi panjang, belah ketupat dan layang-layang !
Jawaban:
persegi panjang, adalah persegi yang mempunyai 4 sudut yang sama panjang dan mempunyai 4 lipatan
Layang-Layang, adalah bangun datar segi empat dan memiliki dua sisi yang sama panjang
Belah ketupat, adalah bangun datar yang memiliki 4 sudut yang sama panjang

Try writing the definitions of a rectangle, rhombus, and kite!
Answer:
A rectangle is a square that has 4 equal angles and 4 folds.
A kite is a quadrilateral with two equal sides.
A rhombus is a plane figure with 4 equal angles.

Figure 8. First Type of Answer from Student with Moderate Academic Ability for Verbal Skills

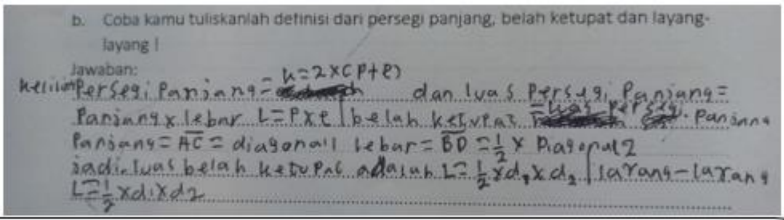
For the second type of answer, which explains the definition of a plane through analytical, genetic, and formulaic definitions, students with moderate academic abilities write the analytical definition of a plane using specific characteristics of a plane, such as sides, angles, and axes of symmetry. For the genetic definition, students connect the process of forming the plane requested in the problem with other planes. Figure 9 shows examples of student answers when solving problems using analytical and genetic definitions.


b. Coba kamu tuliskanlah definisi dari persegi panjang, belah ketupat dan layang-layang !
Jawaban:
Persegi panjang = Bangun datar ini agak sama seperti persegi (memiliki sudut siku-siku 90° dan 4 titik sudut) Bedanya sama persegi, persegi memiliki semua sisi yang panjangnya sama. Kalau persegi panjang ada sisi pendek dan panjang.
Belah Ketupat = Bangun datar ini bentuknya seperti 2 segitiga kebalik yang agak lebar kesamping.
Layang-layang = Bentuknya mirip diamond (segitiga ada 2)

Try writing the definitions of a rectangle, rhombus, and kite!
Answer:
Rectangle = This plane shape is somewhat similar to a square (it has 90^{0}-degree right angles and 4 vertices).
The difference is that a square has all sides of equal length. A rectangle has short and long sides.
Rhombus = This plane shape is shaped like 2 inverted triangles that are slightly wider at the sides.
Kite = This shape is similar to a diamond (there are 2 triangles).

Figure 9. Second Type of Answer from Student with Moderate Academic Ability for Verbal Skills

For the formula definition, students write the definition of a plane using the area formula for that plane. Figure 10 shows examples of student answers for the second type of answer.


b. Coba kamu tuliskanlah definisi dari persegi panjang, belah ketupat dan layang-layang !

Jawaban:

keliling persegi panjang = k = 2 x (p + l) dan luas persegi panjang = panjang x lebar L = p x l | belah ketupat = luas persegi panjang = AC = diagonal lebar = BD = 1/2 x diagonal 2 jadi luas belah ketupat adalah L = 1/2 x d1 x d2 | layang-layang L = 1/2 x d1 x d2

Try writing the definitions of a rectangle, rhombus, and kite!
Answer:
The perimeter of the rectangle = k = 2 x $(p + l)$, and the area of the rectangle = length x width L = pxl. Rhombus = area of the rectangle = $\overline{AC}$ = diagonal width = $\overline{BD}$ = $\frac{1}{2}$ x diagonal2. So the area of the rhombus is L = $\frac{1}{2}x\ d_1 x\ d_2$| Kite
L = $\frac{1}{2}x\ d_1 x\ d_2$

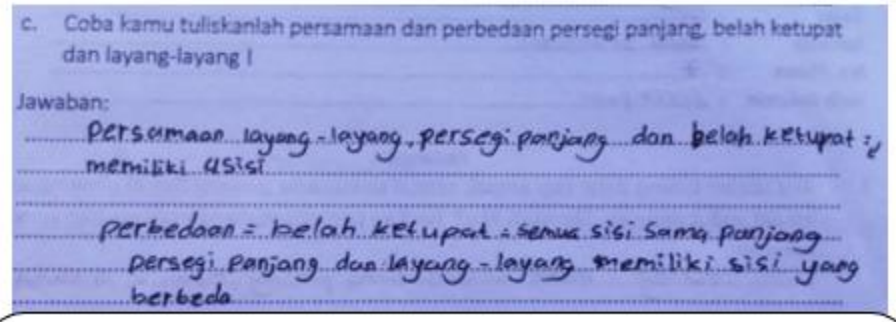
Figure 10. Third Type of Answer from Student with Moderate Academic Ability for Verbal Skills

For verbal skills, students with moderate academic ability still define planes in general. The definitions given are only based on the image of the plane, such as only based on the number of sides, the number of angles, and the axis of symmetry. However, they are not given completely, such as providing the relationship between the sides and angles [35], so that the definitions given for each plane have similarities without any distinctive characteristics that differentiate each definition of the plane. This shows that the visual abilities of students with moderate academic ability still dominate in terms of providing definitions of each plane [28]. Furthermore, students with moderate academic ability still use incorrect names, such as for angles with equal lengths. This indicates that students with moderate academic ability still have suboptimal geometric vocabulary and can still be developed academically. Students with moderate academic ability also appear to be more likely to use visual abilities and experiences gained in learning to define planes compared to formal definitions based on the plane's specific properties [36].

Interestingly, when defining a plane, students with moderate academic ability already use genetic definitions and formulas owned by students with high academic ability [37]. Medium academic students define a plane using other planes, such as rectangles with squares, kites with rhombuses, and rhombuses with triangles (genetic definitions). This shows that students with moderate academic ability can already understand that several planes can be arranged to form a new plane through an understanding of the hierarchical structure of geometric classification [35]. For formula definitions, students with moderate academic ability write the formulas for the area and perimeter of each given plane. The definition of a formula is a consequence of the analytical definition of any plane surface, for example, a rectangle. A rectangle is a plane surface that has 2 pairs of parallel sides, where the parallel sides are equal in length. The longer side is symbolized by p, and the shorter side by l; using these two sides, students with moderate academic ability define a rectangle by determining the area, namely *p x l*, and the perimeter, namely 2 (*p x l*). The ability to define a plane through the definition of a formula shows that students with moderate academic ability are able to connect properties, images, and formulas as an inseparable conceptual unit [38], [39].

3.4. Logical Skills

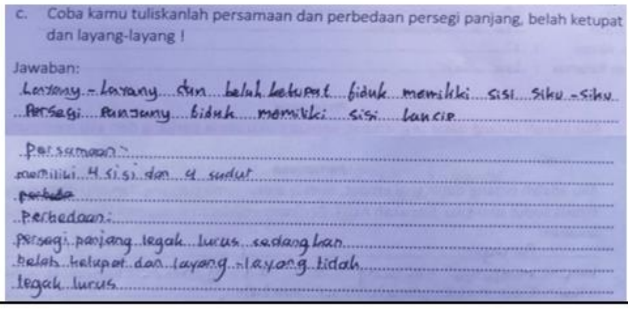
In logical skills, students are asked to compare and construct logical arguments about relationships in plane geometry. Based on Table 4, the answers of students with moderate abilities are dominant in four types of answers, namely 1) being able to write the similarities and differences of the given plane, 2) only writing the similarities or differences of the plane, 3) incorrectly writing the similarities and differences of the plane, and 4) not answering the questions given. In the first type of answer, students with moderate academic ability listed the similarities and differences between the given planes, although their answers were incomplete. Nevertheless, the answers provided demonstrated a grasp of the concepts of each plane presented in the problem. The first type of answer can be seen in Figure 11.



Try writing down the similarities and differences between rectangles, rhombuses, and kites!
Answer:
Similarities between kites, squares, and rhombuses: They have 4 sides. Differences between rhombuses: All sides are the same length. Rectangles and kites have different sides.

Figure 11. First Type of Answer from a Student with Moderate Academic Ability for Logical Skills

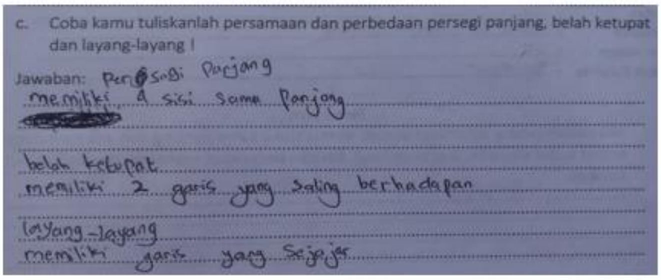
For the second type of answer, students can only write the similarities or differences of the given flat plane. In general, students with moderate academic abilities write down the similarities or differences of each flat plane based on the number of angles and side lengths they have. Students' ability to write down differences or similarities in flat planes shows that primary school students of moderate ability can compare and identify the characteristics of flat planes [24]. The second type of answer appears in Figure 12.



Try writing down the similarities and differences between rectangles, rhombuses, and kites!
Answer:
Kites and rhombuses do not have right angles. Rectangles do not have acute sides. Similarities:
They have 4 sides and 4 angles.
Differences:
Rectangles are perpendicular, while rhombuses and kites are not.

Figure 12. Second Type of Answer from Student with Moderate Academic Ability for Logical Skills

For the third type of response, students made errors in writing the similarities and differences between planes. Based on the interview results, students with moderate abilities stated that these errors occurred because they could not correctly remember the planes given, resulting in errors in writing the similarities and differences. The results of the third type of answer also indicate that students' conceptual understanding of the relationship between planes is still low because they cannot distinguish between examples and non-examples of planes and their properties. Figure 13 shows an example of the third type of answer.



Try writing down the similarities and differences between rectangles, rhombuses, and kites!
Answer:
A rectangle
has 4 sides of equal length.
A rhombus
has 2 lines that are opposite each other.
A kite
has parallel lines.

Figure 13. Third Type of Answer from Student with Moderate Academic Ability for Logical Skills

For the fourth type of answer, students were completely unable to list the similarities and differences between the given planes. Based on interviews, students with moderate academic ability said they could not identify the differences and similarities between the

 https://doi.org/10.58421/gehu.v5i4.1948

given planes. Another reason revealed that students with moderate academic abilities were still confused about distinguishing between the types of planes.

In logical skills, students with moderate academic ability more often connect planes through visual observation of the planes. Furthermore, students with moderate academic ability also write logical arguments using more geometric properties, such as explaining differences and similarities by connecting squares, kites, and rhombuses through the length of their sides, the types of angles the planes have, and the number of angles of each plane. This indicates that students with moderate academic ability have begun to integrate concepts about plane relationships, even though the relationships given are not completely accurate [40], [41].

3.5. Applied Skills

In applied skills, students are asked to recognize physical models of geometric shapes, apply the properties of geometric models to the properties of physical objects, and use these geometric models in problem solving. Based on Table 4, the dominant answer in this indicator is students writing incorrect answers. Students with moderate academic ability often write incorrect answers and do not write down the problem-solving steps commonly used to solve the problem, such as writing down the information known in the problem and the problem-solving process. Students with moderate academic ability have difficulty identifying the information provided and fail to document the problem-solving process [42]. Generally, students with moderate academic ability write down the answer to the given problem. Figure 14 shows an example answer from a student with moderate academic ability for applied skills.

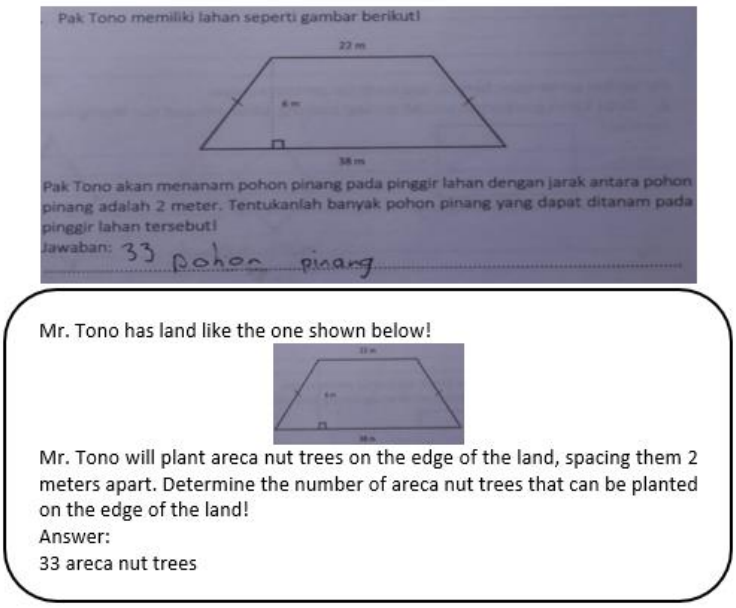


Figure 14. The Answer from Student with Moderate Academic Ability for Applied Skills

In applied skills, students with moderate academic ability were unable to solve the given problems using the process according to problem-solving standards. Although they were given a flat plane image representing the problem, students with moderate academic ability wrote only the answer. These results are inconsistent with Weigand et al. [41], which found that images can make it easier for students to solve problems by helping them identify sides, angles, dimensions, and relationships between parts. Students with moderate academic ability also show that their ability to apply mathematical concepts to solve everyday problems remains limited [37], [43].

4. CONCLUSION

The research findings indicate that fifth-grade primary school students with moderate academic ability in Mataram City show high accuracy in answering questions related to visual skills; however, this accuracy declines when addressing questions at the applied skill level. These students possess limited ability to solve problems involving geometric concepts applied to everyday life. This suggests that students with moderate mathematical ability in Mataram City have an uneven or incomplete understanding of the subject. These fifth-grade students' capabilities require immediate improvement, as knowledge and understanding of geometry are essential to daily life. These findings reflect the interpretations of fifth-grade students with moderate academic ability in Mataram, based on a single assessment item for each geometric thinking skill. Given the importance of geometry education, teachers should use concrete objects, such as traditional foods or the shapes of traditional houses, as instructional aids to support student learning. Future research should employ multiple tasks for each geometric thinking skill, measure students' academic achievement, and investigate how instructional interventions influence the development of logical and applied geometric skills. The results of this study offer deeper insight into the characteristics of geometric skills among fifth-grade primary students of moderate academic ability, as well as the errors they make when solving geometry problems.

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