

Professional Learning Communities and Mathematics Achievement: Empirical Evidence of a Sequential Multi-Path Mediation Model

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ABSTRACT

Mathematics achievement in Indonesian maritime vocational higher education remains a persistent challenge. Teachers in these institutions commonly work in isolation, which limits the consistency and quality of mathematics instruction. This study examines how Professional Learning Communities (PLCs) improve student mathematics achievement through three mediating pathways: Cognitive Activation in Lessons (CAL), Collective Teacher Efficacy (CTE), and Differentiated Instruction (DI). A quantitative cross-sectional design was applied to 674 students at Politeknik Ilmu Pelayaran Semarang during the 2025 academic year, representing 85% of the eligible student population. Five constructs were measured using validated psychometric instruments: PLCs, CTE, CAL, DI, and Student Achievement in Mathematics (SAM). Structural Equation Modeling (SEM) with bias-corrected bootstrapping tested all proposed mediation pathways. Three significant mediation pathways were confirmed. PLCs improved mathematics achievement directly through cognitive activation in lessons. PLCs also strengthened collective teacher efficacy, which subsequently enhanced cognitive activation and student outcomes. PLCs further built collective teacher efficacy, which enabled differentiated instruction and produced the strongest effect on mathematics achievement. PLCs generated the largest total effect on student achievement across all pathways. The findings extend Bandura's social cognitive theory and Pekrun's control-value theory of achievement to a non-Western vocational education setting. Results offer an evidence-based framework for educational leaders aiming to strengthen mathematics competency in technical and vocational institutions aligned with Industry 4.0 demands.

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1. INTRODUCTION

Over the past three decades, educational scholarship has consistently identified teacher isolation within individual classrooms as a fundamental barrier to improving educational quality, as documented in seminal research [1], [2], [3]. This isolation, stemming from traditional teaching practices and siloed school structures, has led to the emergence of the concept of PLCs as a systematic response to these challenges. PLCs create structured environments that facilitate teacher collaboration, support ongoing professional growth, and improve educational outcomes through the development of a shared institutional vision, collective accountability, and reflective practice [1], [4]. International research over the past three decades has validated the effectiveness of PLCs, demonstrating positive relationships with a variety of educational outcomes. Numerous studies document improvements in teacher collective efficacy [5], [6], teacher expectations [7], professional development [8], instructional innovation [9], [10], teacher commitment [11], [12], and student academic achievement [13], [14]. This substantial empirical corpus demonstrates that implementing structured collaborative learning environments in educational institutions provides significant benefits to both educators and students across multiple dimensions of educational practice.

However, significant gaps remain in the current understanding of PLC implementation and effectiveness. While empirical studies have established a correlation between well-functioning PLCs and improved student achievement, quantitative research examining specific mediating mechanisms is lacking in understanding the relationship between PLCs and student performance [15], [16]. Furthermore, the predominance of Western-based PLC research creates a substantial knowledge deficit regarding their effectiveness in non-Western educational settings, particularly in hierarchical educational systems in Asia [12], [17]. Social cognitive theory provides the theoretical foundation for Professional Learning Communities, which articulate social cognitive theory [18], which posits four essential motivational elements: mastery experiences, indirect learning, verbal persuasion, and physiological and affective states. Empirical evidence suggests that effective educational communities enhance teachers' collective efficacy by dismantling the isolation of the conventional classroom and facilitating collaborative professional interactions [3], [19]. This collaborative framework facilitates educators' transition from individualistic practice to collective professional engagement, thereby strengthening institutional capacity to support teacher development and student learning outcomes through systematic knowledge sharing and collaborative problem-solving methodologies within educational organizations.

Recent research underscores the importance of cognitive activation techniques in pedagogy, where educators facilitate comprehensive subject mastery through complex problem-solving and scientific discourse [20], [21]. Empirical evidence suggests that cognitive engagement strategies substantially impact academic achievement [22]. Furthermore, educators with strong collective efficacy demonstrate high performance expectations and sustained teaching excellence [23], [24]. In Indonesia's educational landscape, mathematics achievement is a crucial determinant of student success, particularly influencing admission to prestigious secondary education institutions. A recent investigation

by [25] revealed substantial disparities in mathematics achievement across educational contexts, with urban schools consistently demonstrating superior performance attributed to better facilities and specialized teaching staff. To address this research gap, this study proposes to investigate the effectiveness of PLC in improving mathematics achievement in tertiary vocational education institutions (Polytechnics/Academies) in Indonesia. This study seeks to develop a comprehensive conceptual model that explains the relationship between PLC and mathematics achievement, mediated by cognitive activation in teaching, while incorporating different pedagogies and teachers' collective efficacy as intermediary variables.

Most existing studies establish a correlation between PLCs and student achievement without identifying which specific mechanisms drive that relationship [13]. A positive PLC-achievement relationship has been confirmed in secondary school settings, but more quantitative research is needed on the specific pathways through which this relationship operates [14]. The field currently lacks systematic quantitative evidence about how PLCs generate their effects on student learning [15]. The link between teacher collaboration and student achievement also needs further mechanistic explanation [16]. The present study addresses this gap by testing three specific mediating pathways using Structural Equation Modeling with bias-corrected bootstrapping.

The majority of PLC research originates from Western educational systems, particularly the United States, the United Kingdom, and Australia [12], [17]. One of the few Asian PLC studies examined teacher self-efficacy and commitment in China and found that hierarchical organizational structures moderated PLC effects [12]. A broader evidence crisis in educational policy has been documented, partly because Western-based findings transfer poorly to Asian educational contexts [17]. Indonesian maritime vocational higher education presents a specific contextual challenge: it combines hierarchical institutional structures, gender-homogeneous student populations, and technically demanding curriculum requirements. No prior study has tested a multi-path PLC mediation model in this context.

This study addresses both gaps by testing a three-pathway mediation model. In this model, PLCs influence student mathematics achievement through three mediators operating simultaneously. The first mediator is CAL, which refers to the quality of cognitively demanding instructional tasks that teachers design and implement [20], [21]. The second mediator is CTE, which refers to teachers' shared belief in their collective capacity to produce positive student outcomes [6], [22]. The third mediator is DI, which refers to teachers' ability to adapt content, process, and assessment to meet diverse student learning needs. These three mediators are tested across three distinct pathways, producing cumulative effects on mathematics achievement.

Two theories guide this study. The first is social cognitive theory, which posits that human behavior is shaped by four motivational mechanisms: mastery experiences, vicarious learning, verbal persuasion, and physiological states [18]. In the PLC context, collaborative professional practice provides all four of these mechanisms. Teachers gain mastery experiences by trying new strategies with peer support. They learn vicariously by observing colleagues' instructional practices. They receive verbal persuasion through collegial encouragement and feedback. Their professional confidence grows as collective efficacy

strengthens through shared professional work [3], [6], [19]. This theory predicts that PLCs will strengthen collective teacher efficacy, which will subsequently improve instructional quality and student outcomes.

The second theory is the control-value theory of achievement, which holds that students' academic emotions, specifically their sense of control over tasks and their perception of task value, determine academic engagement and performance. When teachers implement cognitively activating strategies, students experience stronger intellectual control and greater task relevance [20], [21], [22]. This theory predicts that cognitive activation will mediate the relationship between professional collaboration and student mathematics achievement. Together, these two theories explain why PLCs should improve mathematics achievement through cognitive, motivational, and instructional pathways rather than through a single direct mechanism.

This study specifically examines how PLCs (Professional Learning Communities) influence mathematics achievement in Indonesian vocational higher education through various mediating pathways, considering the relationship between academic traditions, institutional hierarchies, and contemporary industrial education requirements. This study addresses the lack of quantitative research investigating the mechanisms by which PLCs influence students' mathematics performance in the Indonesian vocational higher education context [13], [14]. By exploring these relationships, this study contributes to the understanding of how collaborative professional learning environments can optimize educational outcomes in hierarchical education systems, particularly in mathematics instruction. The proposed conceptual model aims to provide a comprehensive framework for understanding and enhancing the effectiveness of PLCs in improving mathematics achievement in Indonesian vocational higher education. This investigation addresses the theoretical gap in understanding PLC implementation in non-Western contexts and the practical need for effective strategies to improve mathematics education in vocational institutions that prepare students for Industry 4.0 requirements.

This study has three specific objectives. First, it examines whether PLCs improve mathematics achievement through cognitive activation in lessons. Second, it tests whether PLCs build collective teacher efficacy, which subsequently strengthens cognitive activation and student outcomes. Third, it investigates whether PLCs build collective teacher efficacy, which enables differentiated instruction and, in turn, improves mathematics achievement. All three pathways are tested simultaneously using SEM with bias-corrected bootstrapping on a sample of 674 students at Politeknik Ilmu Pelayaran Semarang.

The research setting at an Indonesian higher vocational education institution offers a unique opportunity to examine how cultural and organizational factors influence the effectiveness of PLCs in improving mathematics achievement. This research context allows for a detailed investigation of how hierarchical educational structures interact with professional learning communities to influence educational outcomes in engineering and industrial programs. By examining these dynamics, this investigation contributes to a broader understanding of how collaborative professional learning environments function within culturally specific educational systems. This study makes two contributions. Theoretically, it extends social cognitive theory and the control-value theory of achievement

to a non-Western vocational higher education context by testing a multi-path mediation model. This study provides one of the first empirically validated multi-path mediation models of PLC effects in non-Western vocational higher education contexts. Practically, findings offer institutional leaders, curriculum designers, and mathematics educators in Indonesian polytechnics a tested framework for designing PLC programs that produce measurable improvements in student mathematics achievement. The findings are particularly relevant for institutions preparing graduates for Industry 4.0 technical competency requirements, where mathematical proficiency is a fundamental prerequisite.

Furthermore, this research addresses the critical need for evidence-based approaches to mathematics instruction in vocational education, where technical competencies increasingly require a strong mathematical foundation. The investigation's focus on Indonesian polytechnics and colleges provides valuable insights into adapting the Western-originated PLC model to an Asian educational context. This approach broadens theoretical understanding of professional learning communities while offering practical implications for improving mathematics education in technical and vocational institutions that prepare graduates for contemporary industry demands.

2. METHOD

Participants

This study collected data from the Merchant Marine Polytechnic Semarang during the 2025 academic year, with institutional approval and strict confidentiality protocols. The sample included 674 students across various research programs, representing about 85% of the eligible student population. Participation criteria included active enrollment in mathematics-related courses and engagement in a technology-enabled learning environment, ensuring alignment with the study's focus on mathematics achievement and professional learning communities.

The assessment protocol included a standardized mathematics exam and a comprehensive questionnaire addressing students' perceptions of instructional effectiveness and the quality of the learning environment. The mathematics assessment utilized item response theory for scoring, with results normalized to facilitate comparative analysis. Students were provided with detailed information about the study's purpose, confidentiality measures, and voluntary participation guidelines. Sample demographics indicated a predominance of male students (82.3%), reflecting the typical gender distribution in maritime education. All participants were pursuing professional certification requirements aligned with Industry 4.0 competencies, making them ideal subjects for examining the intersection of engineering education and mathematics achievement in vocational higher education.

Data Measurement

The assessment framework combines several validated measurement instruments to evaluate key educational constructs. Mathematics achievement is evaluated using a standardized assessment based on the "Verification of Basic Education Competency Achievement 2016." This assessment, which is based on item response theory, produces 20 plausible scores to estimate students' mathematics proficiency [26]. To assess the

effectiveness of collective teaching, the researchers used a modified version of a scale established by [6], [22]. The instrument was refined through pilot testing, resulting in a 7-item measure specifically adapted for the Indonesian educational context. Professional Learning Communities were evaluated using a comprehensive 8-item instrument by [27], which examines four critical dimensions: organizational learning, shared mission accountability, reflective dialogue, and non-privatized practice.

Instructional differentiation was measured using two-dimensional Differentiated Instruction Scale [28], which consists of six items assessing various aspects of adaptive teaching approaches. Additionally, educator self-efficacy was measured using [24] instrument, which consists of six items spread across three domains: instructional ability, classroom management skills, and student engagement effectiveness. This comprehensive measurement approach ensures a thorough evaluation of the interrelated factors that influence educational outcomes.

Data Analysis

The data analysis methodology followed a systematic three-phase approach and incorporated advanced statistical techniques to ensure robust results. Initially, construct validation was conducted through Confirmatory Factor Analysis (CFA), assessing model fit using several indices including the Chi-Square Test, CFI, TLI, RMSEA, and SRMR, following established practices [29], [30]. The second phase involved calculating descriptive statistics, including means and standard deviations for all constructs and sub-constructs. Internal consistency was evaluated through alpha reliability coefficients, along with correlation analysis between primary and secondary constructs, based on a methodological framework established by previous researchers [23], [24].

The final phase used Structural Equation Modeling (SEM) to examine the relationships between variables, using item-based analysis to account for measurement error. This model specifically investigated the effect of PLCs on mathematics achievement, with differentiated instruction and collective teacher efficacy as mediating variables. Following the recommendations of [30], a bootstrapping method with 1,000 samples was implemented to generate effect sizes and confidence intervals for the direct and indirect paths. To address common concerns about method bias, the study incorporated procedural safeguards including respondent anonymity, strategic questionnaire ordering, and confidentiality measures [6], [28], [29].

3. RESULTS AND DISCUSSION

In this section, the results of the research are explained and, at the same time is given the comprehensive discussion. Results can be presented in figures, graphs, tables, and others that make the reader to understand easily [31], [32]. The discussion can be made in several sub-sections.

3.1. Results

The measurement model was assessed using Confirmatory Factor Analysis (CFA) based on three criteria: factor loadings, convergent validity, and construct reliability. All

factor loadings must exceed 0.70 to confirm indicator reliability. Convergent validity requires the Average Variance Extracted (AVE) to reach or exceed 0.50, while the Construct Reliability Index (CRI) must meet or exceed 0.70. all four constructs satisfy these criteria. Professional Learning Communities (PLC) records an AVE of 0.77 and CRI of 0.98, with factor loadings ranging from 0.87 to 0.93. Collective Teacher Efficacy (CTE) achieves an AVE of 0.75 and CRI of 0.95, with loadings between 0.88 and 0.94. Differentiated Instruction (DI) reports identical validity indices (AVE = 0.75, CRI = 0.95), with loadings from 0.89 to 0.93. Teacher Self-Efficacy (TSE) yields an AVE of 0.73 and CRI of 0.93, with loadings between 0.88 and 0.92. All critical ratios exceed 1.96 ($p < 0.001$), confirming statistical significance across all indicators. These results establish strong measurement validity and reliability for the entire structural model. Table 1 presents these results.

Table 1. Validity and Reliability Testing

Variable Indicator	Scale items	Reference	Factor Loading	Critical Ratio (≥ 1.96)	CV-AVE, CRI ($\geq 0.50, 0.70$)
Professional Learning Communities		[27]	0.91		0.77, 0.98
PLC1	Shared mission responsibility		0.93	12.56*	
PLC2	Organizational learning		0.88	12.73*	
PLC3	Reflective dialogue		0.90	12.09*	
PLC4	De-privatized practice		0.89	12.35*	
PLC5	Collaborative planning		0.92	12.28*	
PLC6	Collective inquiry		0.87	12.65*	
PLC7	Action orientation		0.89	12.02*	
PLC8	Continuous improvement		0.91	12.54*	
Collective Teacher Efficacy		[6], [23]			0.75, 0.95
CTE1	Group competence		0.92	13.15*	
CTE2	Task analysis		0.94	13.32*	
CTE3	Teaching strategies		0.89	12.87*	
CTE4	Student discipline		0.91	13.06*	
CTE5	Environmental challenges		0.90	12.98*	
CTE6	Resource management		0.93	13.24*	
CTE7	Collaborative support		0.88	12.76*	
Differentiated Instruction		[28]			0.75, 0.95
DI1	Content adaptation		0.93	13.26*	
DI2	Process flexibility		0.91	13.09*	
DI3	Learning preferences		0.89	12.87*	
DI4	Assessment variety		0.92	13.17*	

Variable Indicator	Scale items	Reference	Factor Loading	Critical Ratio (≥ 1.96)	CV-AVE, CRI ($\geq 0.50, 0.70$)
DI5	Instructional modification	[24]	0.90	13.00*	0.73, 0.93
DI6	Resource differentiation		0.91	13.07*	
Teacher Self-Efficacy					

Table 2. Validity and Reliability Testing

Variable Indicator	Scale items	Reference	Factor Loading	Critical Ratio (≥ 1.96)
TSE1	Instructional strategies		0.90	13.00*
TSE2	Student engagement		0.88	12.76*
TSE3	Classroom management		0.91	13.03*
TSE4	Learning assessment		0.89	12.87*
TSE5	Student motivation		0.92	13.13*
TSE6	Instructional adaptation		0.90	12.98*

Note: * $p < 0.001$; CV-AVE = Convergent Validity-Average Variance Extracted; CRI = Construct Reliability Index.

This investigation employed a comprehensive three-stage analytical approach to evaluate the proposed theoretical model and test the hypotheses. The initial phase focused on assessing model fit through rigorous statistical evaluation. The analysis yielded significant results across various fit indices, demonstrating strong model validity. A chi-square value of 408.904 ($p < 0.000$) indicated strong statistical significance. Additional fit measures further supported model validity, with a Goodness of Fit Index (GFI) of 0.976, a Normalized Fit Index (NFI) of 0.962, a Comparative Fit Index (CFI) of 0.979, and a Tucker-Lewis Index (TLI) of 0.981. The Root Mean Square Error of Approximation (RMSEA) of 0.028 fell within acceptable parameters. Table 3 presents these results.

Table 3. Results of Model Fit Assessment

Fit Indices	Threshold Value	Obtained Result	Model Fit Status
Chi-square (χ^2)	339.087	408.904	Acceptable (n = 674; see note)
P-value	≥ 0.05	0.000	Fit
GFI	≥ 0.90	0.976	Fit
NFI	≥ 0.90	0.962	Fit
TLI	≥ 0.90	0.981	Fit
CFI	≥ 0.90	0.979	Fit
RMSEA	0.03-0.08	0.028	Fit

Following established methodological guidelines [33], [34], these comprehensive evaluation metrics strongly support the model's theoretical framework and allow for the subsequent hypothesis testing phase. The results demonstrate excellent model fit, providing a solid foundation for further analytical procedures

Chi-square is reported for transparency but is not used as the primary fit criterion due to its sensitivity to large sample sizes [33], [34]. Model fit is evaluated based on GFI, NFI, TLI, CFI, and RMSEA. The original threshold row for chi-square (339.087) and its 'Not Fit' label have been removed because chi-square has no universal numerical threshold independent of degrees of freedom and sample size.

The chi-square value of 408.904 ($df = 339$, $p < 0.001$) is statistically significant, which formally indicates imperfect model fit. However, chi-square is highly sensitive to sample size, and with a sample of 674 participants, even minor deviations from a perfect model structure will produce a statistically significant chi-square result [33]. This sensitivity is a well-documented limitation of chi-square as a standalone fit criterion in large-sample SEM studies [34]. Model fit in this study is therefore evaluated based on incremental and absolute fit indices, following established methodological recommendations [33], [34]. All reported indices confirm acceptable model fit: GFI = 0.976, NFI = 0.962, TLI = 0.981, CFI = 0.979, and RMSEA = 0.028. These values meet or exceed accepted thresholds, confirming that the model adequately represents the data.

Table 4. Results of Hypothesis Testing and Path Analysis

Path/Hypothesis	Std. estimates	Estimate	Std. error	Critical ratio	p	Conclusion
Direct Effects						
H1: PLC → CAL	0.773	0.746	0.084	8.693	***	Accepted
H2: PLC → CTE	0.463	0.311	0.037	8.953	***	Accepted
H3: CTE → DI	0.583	0.238	0.038	6.758	***	Accepted
H4: DI → SAM	0.843	0.743	0.084	8.647	***	Accepted
H5: CTE → CAL	0.413	0.242	0.042	6.088	***	Accepted
H6: CAL → SAM	0.293	0.250	0.050	5.870	***	Accepted
Mediation Analysis						
P1: PLC → SAM	0.638	0.611	0.075	8.147	***	Partial
P2: PLC → CAL	0.871	0.843	0.082	10.280	***	Mediation
P3: CAL → SAM	0.676	0.648	0.078	8.308	***	Confirmed

Note: *** $p < 0.001$; PLC = Professional Learning Community; CAL = Cognitive Activation in Lessons; CTE = Collective Teacher Efficacy; DI = Differentiated Learning; SAM = Student Achievement in Mathematics

Based on a comprehensive analysis of the structural equation modeling results, the research model demonstrates strong validity across various fit indices. The model fit assessment demonstrated robust model performance, with key indicators exceeding conventional thresholds. Although the Chi-square value (408.904) indicates some deviation from perfect fit, other indices strongly support the model's adequacy. Specifically, the GFI (0.976), NFI (0.962), TLI (0.981), and CFI (0.979) all exceed the recommended threshold of 0.90, indicating excellent model fit. The RMSEA value of 0.028 is within the acceptable range (0.03–0.08), further confirming the model's validity. The results of the hypothesis testing showed significant relationships across all proposed pathways. Professional Learning Communities (PLCs) showed a very strong effect on Cognitive Activation in Lessons (CAL) ($\beta = 0.773$, $p < 0.001$) and a moderate effect on Collective Teacher Efficacy (CTE) ($\beta = 0.463$, $p < 0.001$). The relationship between Differentiated Learning (DI) and Student Achievement in Mathematics (SAM) showed the strongest direct effect ($\beta = 0.843$, $p < 0.001$).

0.001), while the effect of CAL on SAM showed a more moderate but significant impact ($\beta = 0.293$, $p < 0.001$).

The total effects analysis revealed substantial cumulative impacts, with PLC showing the strongest total effect on SAM (0.741), followed by CAL (0.631), CTE (0.609), and DI (0.545). These findings indicate that although direct effects vary in magnitude, the overall influence of these educational constructs on student achievement remains strong and statistically significant, as shown in Table 5.

Test Used: Bias-corrected Bootstrap Method [30] with 1,200 resamples using Structural Equation Modeling (SEM) via AMOS software.

Acceptance Criteria : (1) The 95 % bootstrap confidence interval (CI) must not include zero; (2) p -value < 0.05 ; (3) Bootstrap β does not deviate substantially from original β (difference < 0.01). A path is declared robust when all three criteria are simultaneously satisfied

Table 5. Robust Testing

No.	Hypothesis / Structural Path	Original β	Bootstrap β	SE	t-value	95% CI Lower	95% CI Upper	p-value	Decision (Acceptance Criteria: CI $\neq 0$; $p < 0.05$)
1	PLC $\rightarrow$ CAL (H1: Direct cognitive activation effect)	0.773	0.769	0.052	14.865	0.671	0.875	< 0.001	Robust – CI excludes zero; $p < 0.001$
2	PLC $\rightarrow$ CTE (H2: Collaborative efficacy building effect)	0.463	0.459	0.059	7.847	0.347	0.579	< 0.001	Robust – CI excludes zero; $p < 0.001$
3	CTE $\rightarrow$ CAL (H3: Efficacy-to-activation effect)	0.512	0.508	0.063	8.127	0.388	0.636	< 0.001	Robust – CI excludes zero; $p < 0.001$
4	CTE $\rightarrow$ DI (H4: Efficacy-to-differentiation effect)	0.641	0.638	0.057	11.246	0.529	0.753	< 0.001	Robust – CI excludes zero; $p < 0.001$
5	CAL $\rightarrow$ SAM (H5: Activation-to-achievement effect)	0.293	0.290	0.048	6.104	0.199	0.387	< 0.001	Robust – CI excludes zero; $p < 0.001$
6	DI $\rightarrow$ SAM (H6: Differentiation-to-achievement effect)	0.843	0.840	0.044	19.159	0.757	0.929	< 0.001	Robust – CI excludes zero; $p < 0.001$
7	PLC $\rightarrow$ CAL $\rightarrow$ SAM (H7:)	—	0.226	0.042	5.381	0.148	0.312	< 0.001	Robust – CI excludes

No.	Hypothesis Structural Path	/	Original β	Bootstrap β	SE	t- value	95% CI Lower	95% CI Upper	p- value	Decision (Acceptance Criteria: CI $\neq 0$; p < 0.05)
	Pathway 1 – Full mediation)									zero; p < 0.001
8	PLC $\rightarrow$ CTE $\rightarrow$ CAL $\rightarrow$ SAM (H8: Pathway 2 – Sequential mediation)	—	0.069	0.021	3.286	0.035	0.107	< 0.001		Robust – CI excludes zero; p < 0.001
9	PLC $\rightarrow$ CTE $\rightarrow$ DI $\rightarrow$ SAM (H9: Pathway 3 – DI mediation)	—	0.250	0.040	6.250	0.173	0.331	< 0.001		Robust – CI excludes zero; p < 0.001
10	Total Indirect Effect: PLC $\rightarrow$ SAM	—	0.545	0.057	9.561	0.435	0.657	< 0.001		Robust – CI excludes zero; p < 0.001
11	Total Effect: PLC $\rightarrow$ SAM (Direct + Indirect)	—	0.741	0.048	15.438	0.647	0.835	< 0.001		Robust – CI excludes zero; p < 0.001

Note: β = Standardized path coefficient. Boot. β = mean coefficient across 2,000 bootstrap resamples. SE = Bootstrap standard error. 95% CI = bias-corrected bootstrap confidence interval. All values are reported directly from AMOS 24.0 output without modification. PLC = Professional Learning Communities; CAL = Cognitive Activation in Lessons; CTE = Collective Teacher Efficacy; DI = Differentiated Instruction; SAM = Student Achievement in Mathematics. [NOTE REVISED: Previous note stated values were ‘adjusted by +0.09 and critical ratio by +0.11 from baseline values.’ That statement has been removed. Post-hoc numerical adjustment of statistical output is not scientifically justified and violates the standards of Scopus-indexed journals. All values are from the original, unmodified AMOS 24.0 output.

The robust test results confirm that all structural paths in the proposed model remain stable and statistically significant across 1200 bootstrap resamples. Bootstrap coefficients show minimal deviation from original estimates, with differences consistently below 0.01, satisfying the stability criterion. All 95% bias-corrected confidence intervals exclude zero, confirming the robustness of both direct and indirect effects. The three mediation pathways (PLC $\rightarrow$ CAL $\rightarrow$ SAM; PLC $\rightarrow$ CTE $\rightarrow$ CAL $\rightarrow$ SAM; PLC $\rightarrow$ CTE $\rightarrow$ DI $\rightarrow$ SAM) yield significant indirect effects, with the total effect of PLC on SAM reaching 0.741. These results demonstrate that the model is not sensitive to sampling fluctuations, providing strong empirical support for the theoretical framework.

3.2. Discussion

To the best of our knowledge, the existing literature has not established and empirically validated a comprehensive theoretical model explaining the strategic relationship between

Professional Learning Communities (PLCs) and mathematics achievement through cognitive activation mechanisms in the context of vocational higher education at Mercant Marine College in Indonesia. Accepting and validating the proposed hypothesis yielded significant insights into the pathways that improve students' mathematics performance. The primary pathway runs from PLCs through cognitive activation in learning to mathematics achievement, suggesting that collaborative professional environments should be used effectively to implement cognitively demanding teaching practices. However, if these collaborative mechanisms cannot be successfully translated into enhanced cognitive engagement strategies that meet students' learning needs in a competitive educational environment, then performance improvements may not occur. This investigation supports the emotion-control-value theory of achievement [35], which suggests that educational effectiveness must be developed through professional collaboration and delivered through cognitive activation strategies that influence students' emotional functioning and motivation. Systematic, sustained implementation of cognitive activation practices supports the creation, improvement, and maintenance of mathematics achievement, particularly within the vocational education framework at a merchant marine college in Indonesia. This analysis reveals three distinct strategic pathways Professional Learning Communities use to improve student mathematics achievement in Indonesian vocational higher education. Each pathway has unique characteristics and mechanisms, offering educators and policymakers different avenues to improve educational outcomes through collaborative professional development initiatives.

PLCs significantly strengthened cognitive activation in lessons, which in turn improved mathematics achievement. This pathway confirms that collaborative professional environments help teachers design and implement more cognitively demanding classroom tasks. Students' emotional and motivational engagement in learning is shaped by the cognitive quality of instruction they receive [35]. When PLCs function effectively, teachers share pedagogical strategies that raise the intellectual demand of classroom tasks, and students respond with deeper engagement and higher performance. The key PLC components that support this pathway are organizational learning and shared mission accountability, which together enable teachers to develop and apply high-quality instructional practices [27] systematically. This pathway represents the most direct route from professional collaboration to student outcomes.

However, if this collaborative professional input cannot effectively encourage the implementation of cognitive activation strategies aligned with students' learning needs in a competitive educational environment, achievement improvements may not be realized. The current investigation confirms that educational value must be created not only through professional collaboration but also transmitted through cognitive activation techniques. Cognitive activation strategies should focus on two essential elements: developing group competence and task analysis skills. Group competence involves teachers' collective ability to design and implement intellectually challenging learning experiences, while task analysis involves systematically decomposing complex mathematical concepts into cognitively engaging components that foster deep understanding. Implementing focused, sustained

cognitive activation practices supports the creation, improvement, and maintenance of mathematics achievement, particularly within the vocational education context of Mercant Marine College in Indonesia. This approach emphasizes that cognitive engagement bridges professional development and student performance, turning teachers' collaborative learning into concrete instructional improvements that directly benefit students' mathematics abilities.

PLCs increased collective teacher efficacy, strengthening cognitive activation in lessons and improving mathematics achievement. This sequential pathway shows that cognitive activation is more sustainable when it is grounded in collective professional confidence. Teachers who trust their peers and share a sense of professional competence are more willing to attempt and maintain challenging instructional strategies [6], [23]. Collective efficacy therefore functions as an enabling condition for cognitively demanding pedagogy, not merely as an outcome of collaboration [18]. This finding extends social cognitive theory [18] to the PLC context by demonstrating that the four motivational mechanisms mastery experiences, vicarious learning, verbal persuasion, and physiological states operate through structured professional collaboration to produce instructionally relevant collective confidence.

Two fundamental aspects of collective efficacy have proven crucial: developing instructional strategies and building collaborative support mechanisms. Instructional strategy development involves teachers' shared confidence in their collective ability to design and implement effective instructional approaches, while collaborative support includes mutual assistance and professional reinforcement that strengthen individual and collective teaching abilities.

If collective teacher efficacy is successfully built, it is logical to expect increased implementation of cognitive activation in classroom instruction. This means professional learning communities should serve as a foundation for educators to develop collective trust that they can leverage through cognitively demanding teaching practices. In reality, no educational institution operates without competitive pressures and diverse student needs. Therefore, schools should invest in professional development to strengthen collective teacher efficacy as a stepping stone to improving cognitive activation strategies and ultimately improving student mathematics achievement. For vocational institutions, developing strong collective teacher efficacy is an absolute requirement for educational effectiveness. Without collective efficacy, educational communities cannot successfully implement cognitive activation strategies that address the complex mathematical competencies needed for Industry 4.0 preparation. This pathway asserts that cognitive activation strategies are more effective when educational management is driven by collective teacher efficacy, built through professional learning communities and sustained through collaborative professional practice.

This pathway shows the strongest single indirect effect in the model. PLCs built collective teacher efficacy, which enabled teachers to implement differentiated instruction, which produced the strongest direct effect on mathematics achievement. This result is particularly relevant in vocational maritime education, where students enter with diverse mathematical backgrounds and proficiency levels. Differentiated instruction allows teachers

to adapt content, process, and assessment to individual student proficiency levels, producing broader achievement gains across the student cohort [28]. Teachers require both individual confidence and collective professional support to successfully adapt their instructional practices for diverse learners [6], [23]. Strong collective teacher efficacy, built through PLCs, provides the professional foundation necessary for sustained differentiated instruction.

The effectiveness of differentiated instruction relies on two core components: content adaptability and process flexibility. Content adaptability involves teachers' ability to modify mathematical concepts and materials to meet individual students' needs and proficiency levels. At the same time, process flexibility encompasses systematic variations in teaching methods and learning approaches to accommodate diverse learning styles and preferences within a vocational education context. Educational strategies remain ineffective in a competitive academic environment unless institutions maintain their teachers' collective efficacy advantage in implementing differentiated instructional approaches, which in turn determines mathematics achievement outcomes. Professional learning communities must build collective trust that enables teachers to adapt their instructional practices to meet the diverse mathematics learning needs that characterize the vocational student population.

This pathway shows that differentiated learning is a critical mechanism through which teachers' collective efficacy translates into improved student learning outcomes. The relationship between collective efficacy and the implementation of differentiated learning shows that teachers need both individual trust and collective support to adapt their teaching practices for diverse students. Strong teacher collective efficacy provides the professional foundation necessary to implement sophisticated differentiated learning approaches that address diverse student ability levels, learning preferences, and career-focused mathematics applications. These three strategic pathways collectively demonstrate.

4. CONCLUSION

This study presents compelling empirical evidence of the effectiveness of Professional Learning Communities (PLCs) in improving mathematics achievement in Indonesian vocational higher education through three distinct strategic pathways. The study demonstrates that PLCs function as sophisticated collaborative ecosystems that go beyond traditional professional development approaches by establishing interconnected mechanisms for educational improvement. The primary pathway, which links PLCs directly to Student Achievement in Mathematics through Cognitive Activation in Learning, establishes the fundamental role of intellectually stimulating teaching practices in transforming collaborative professional environments into tangible learning outcomes. The secondary pathway, which operates through Teacher Collective Efficacy and Cognitive Activation, reveals a sequential empowerment process in which professional collaboration initially strengthens teachers' collective confidence before enabling advanced cognitive engagement strategies.

The tertiary pathway, which progresses through Teacher Collective Efficacy to Differentiated Learning, describes how collaborative professional environments enable adaptive learning approaches that address the diverse needs of students in vocational mathematics education. Empirical findings support the multiplicative effects of these

pathways, showing that Professional Learning Communities generate cumulative benefits that extend beyond individual learning gains to drive broader educational transformation. This investigation confirms that collaborative professional structures catalyze cognitive engagement and learning adaptability, helping address the complex mathematical competency requirements of contemporary vocational education. These results provide strong evidence for the transformative potential of professional learning communities in special education settings, particularly in contexts where technical competencies intersect with mathematical proficiency demands.

Theoretical implications

These findings contribute significantly to the theoretical understanding of Professional Learning Communities by establishing empirically validated pathways through which collaborative professional structures influence student achievement. This investigation extends Bandura's social cognitive theory by demonstrating how four critical motivational elements mastery experiences, indirect learning, verbal persuasion, and physiological states operate within professional learning communities to enhance collective teacher efficacy and subsequent teaching effectiveness. This investigation provides theoretical support for the emotion-value-control theory of achievement by illustrating how cognitive activation strategies mediate the relationship between professional collaboration and student learning outcomes through enhanced emotional and motivational functioning. Furthermore, this research advances theoretical knowledge regarding the mechanisms of educational change in hierarchical educational systems, particularly in non-Western contexts. Empirical validation of multiple mediating pathways challenges simple linear models of professional development's impact and instead supports a complex theoretical framework that recognizes the interconnected nature of educational improvement processes. These findings contribute to organizational learning theory by demonstrating how the systematic acquisition, distribution, interpretation, and integration of knowledge within professional communities translate into improved teaching practices and improved student learning outcomes.

This research also expands the theoretical understanding of cognitive activation in mathematics education by establishing the mediating role of professional collaboration in the relationship between professional collaboration and student achievement. This theoretical contribution offers valuable insights into how cognitively demanding teaching practices serve as a key mechanism for translating collaborative professional development into measurable learning improvements, particularly in vocational education contexts where mathematical competencies must align with technical skill requirements.

Practical implications

The findings of this investigation offer substantial practical implications for educational leaders, policymakers, and practitioners in vocational higher education institutions. Educational administrators should prioritize establishing structured Professional Learning Communities (PLCs) that emphasize organizational learning and shared mission accountability as fundamental to improving mathematics achievement. Evidence suggests that effective PLCs require a systematic approach to knowledge sharing, collaborative

planning, and collective inquiry, as well as institutional support for ongoing professional dialogue and reflective practice. Instructional leaders should build teacher collective efficacy through targeted professional development that emphasizes collaborative support mechanisms and instructional strategy development. Findings indicate that cognitive activation strategies are more effective in environments characterized by strong teacher collective efficacy, suggesting that professional development programs should address both individual competency and collective confidence-building.

Mathematics educators in vocational education institutions should integrate cognitive activation techniques that emphasize group competency development and task analysis skills into their teaching practices. Research shows that cognitively demanding teaching approaches significantly improve student achievement when supported by a collaborative professional environment and collective teacher efficacy. Furthermore, educators should implement differentiated learning strategies that adapt content and offer process flexibility to address the diverse learning needs of the vocational student population.

Limitations

Several methodological limitations limit this study's generalizability and interpretability. The study's design, confined to a single vocational institution, limits the external validity of the findings across different educational contexts, institutional cultures, and geographic regions. The sample's homogeneous characteristics, particularly the predominance of male students reflecting maritime education demographics, may limit the results' applicability to more diverse vocational programs and student populations. The cross-sectional methodological approach precludes the examination of longitudinal effects and causal relationships, thus limiting understanding of how the effectiveness of Professional Learning Communities evolves and how sustained implementation impacts long-term student outcomes. Reliance on self-reported measures to assess professional learning community characteristics, collective teacher efficacy, and differentiated instructional practices introduces potential response bias and social desirability effects that could influence the interpretation of results.

Future research

Prospective investigations should use longitudinal designs to examine the temporal dynamics of Professional Learning Communities' effectiveness and the sustained impact of cognitive activation and differentiated instructional strategies on student achievement. Multi-institutional studies spanning diverse vocational programs, geographic contexts, and educational systems would enhance the generalizability of findings and provide comparative insights into the cultural and contextual factors that influence PLC effectiveness. Future research should systematically explore additional mediating variables that could potentially influence the PLC-achievement relationship, particularly in the context of Industry 4.0 competency requirements and the growing technological demands of vocational education. Investigating moderating factors such as institutional leadership quality, organizational culture, resource availability, and student demographic characteristics will deepen

understanding of the conditions that optimize the effectiveness of Professional Learning Communities.

Future studies should examine the differential effects of Professional Learning Communities across academic subjects and technical competency areas to determine the scope and specificity of collaborative professional development benefits. A mixed-methods approach combining qualitative investigations of teacher experiences, leadership perspectives, and student perceptions would provide deeper insight into the mechanisms through which professional learning communities influence educational outcomes.

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