16% Overall Similarity

The combined total of all matches, including overlapping sources, for each database.

Filtered from the Report

- Bibliography

Match Groups

53 Not Cited or Quoted 14%
Matches with neither in-text citation nor quotation marks

7 Missing Quotations 2%
Matches that are still very similar to source material

0 Missing Citation 0%
Matches that have quotation marks, but no in-text citation

0 Cited and Quoted 0%
Matches with in-text citation present, but no quotation marks

Top Sources

11% Internet sources

11% Publications

8% Submitted works (Student Papers)

Match Groups

53 Not Cited or Quoted 14%
Matches with neither in-text citation nor quotation marks

7 Missing Quotations 2%
Matches that are still very similar to source material

0 Missing Citation 0%
Matches that have quotation marks, but no in-text citation

0 Cited and Quoted 0%
Matches with in-text citation present, but no quotation marks

Top Sources

11% Internet sources
11% Publications
8% Submitted works (Student Papers)

Top Sources

The sources with the highest number of matches within the submission. Overlapping sources will not be displayed.

1	Publication	Zane Sheeran, Anna Sutton, Helena Dorothy Cooper-Thomas. "Environmental sus...	1%
2	Internet	journal-gehu.com	1%
3	Internet	www2.mdpi.com	<1%
4	Student papers	Varsity College on 2018-04-16	<1%
5	Student papers	Fresno City College on 2024-06-02	<1%
6	Internet	www.atlantis-press.com	<1%
7	Internet	fkip.ummetro.ac.id	<1%
8	Publication	Muhtarom Muhtarom, Sutrisno Sutrisno, Nizaruddin Nizaruddin, Yanuar Hery Mu...	<1%
9	Internet	futurity-education.com	<1%
10	Internet	www.bio-conferences.org	<1%

11 Internet

e-journal.hamzanwadi.ac.id <1%

12 Publication

Özlem Engin, Alaattin Pusmaz. "An Analysis of High School Student's Understandi... <1%

13 Internet

www.assessmentcentrehq.com <1%

14 Publication

Rohati Rohati, Yaya S. Kusumah, Kusnandi Kusnandi. "Exploring Students' Mathe... <1%

15 Student papers

A.B. Paterson College on 2026-02-27 <1%

16 Internet

media.neliti.com <1%

17 Internet

www.researchgate.net <1%

18 Internet

jelmar.wisnuwardhana.ac.id <1%

19 Internet

journal2.uad.ac.id <1%

20 Internet

jst-ud.vn <1%

21 Internet

www.grafiati.com <1%

22 Publication

D Herawati, D Ardianto. "Socioscientific Argumentation of Pre-Service Teachers a... <1%

23 Student papers

Heriot-Watt University on 2024-10-24 <1%

24 Publication

Ira Mutiaraningrum, Sri Wuli Fitriati, Issy Yuliasri, Mursid Saleh. "Indonesian voca... <1%

25 Publication

Ristati Ristati, Bahing Bahing, Olga Dona Retsi, Tutik Haryani, Novika Amalia. "Ch... <1%

26 Student papers

Universitas Pendidikan Indonesia on 2019-07-30 <1%

27 Internet

jurnal.unsur.ac.id <1%

28 Student papers

Graduate School of Human Sciences (English) on 2026-01-19 <1%

29 Publication

Scott Calvin. "XAFS for Everyone", CRC Press, 2024 <1%

30 Student papers

University of the Western Cape on 2013-10-01 <1%

31 Publication

W A Pertiwi, S Kusairi, A D Kusumawati, A P K Shanthie, L D Haeruman. "Profile of ... <1%

32 Internet

data.nysed.gov <1%

33 Internet

dergipark.org.tr <1%

34 Internet

ijere.iaescore.com <1%

35 Internet

journal.unesa.ac.id <1%

36 Internet

link.springer.com <1%

37 Internet

ojs.umrah.ac.id <1%

38 Internet

www.igpme.org <1%

39 Internet

www.up.ac.za <1%

40 Publication

M. Iqbal Maulana Halisna, Sunismi Sunismi, Anies Fuady. "Mathematical resilienc... <1%

41 Publication

Demiray, Esra. "An Investigation of Prospective Middle School Mathematics Teac... <1%

42 Student papers

Georgia Southern University on 2024-10-21 <1%

43 Publication

Siti Nabila, Ratu Ilma Indra Putri, Zulkardi Zulkardi. "Development of learning vid... <1%

44 Student papers

UIN Syarif Hidayatullah Jakarta on 2025-01-08 <1%

Journal of General Education and Humanities
Vol. 5, No. 2, April 2026, pp. 2457 – 2468, https://doi.org/10.58421/gehu.v5i2.1175
ISSN 2963-7147

Levels of Mathematical Argumentation: An Analysis of Junior High School Students' Reasoning in Number Pattern Tasks

Marlina Marlina[1], Rohati Rohati[2], Ade Kumalasari[3], Sri Winarni[4]
[1,2,3,4]Jambi University, Jambi, Indonesia

Article Info

Article history:

Received 2026-02-09
Revised 2026-03-07
Accepted 2026-03-18

Keywords:

Junior High School Students
Mathematical Argumentation
Mathematical Reasoning
Number Patterns
Toulmin Argumentation Pattern

ABSTRACT

Mathematical argumentation is important for developing students' reasoning abilities. This study aims to analyze junior high school students' written and oral argumentation abilities when solving reasoning tasks. This research employed a descriptive qualitative approach, involving 31 ninth-grade students from SMP Negeri 7 Muaro Jambi. The study was conducted in the odd semester of the 2024/2025 academic year. The research instruments consisted of a mathematical reasoning test, interview protocols, and documentation. The mathematical reasoning test included three questions. The results of the tests and interviews were analyzed using the main components of Toulmin's argumentation model, which consist of claims, data, warrants, backing, qualifiers, and rebuttals. Based on the test and interview results, students' argumentation abilities were categorized into five levels, from Level 1 to Level 5. The findings show that 31 students (100%) reached Level 1, 15 (48.38%) reached Level 2, and 9 (29.03%) reached Level 3. No students reached Level 4 or Level 5. These findings indicate that students can construct claims, data, and warrants when solving mathematical reasoning tasks. However, students still need to be trained to develop backing, qualifiers, and rebuttals in order to complete the argumentation process appropriately in both written and oral forms.

Corresponding Author:

Rohati Rohati
Faculty of Teacher Training and Education, Mathematics Education, Universitas Jambi
Email: rohati.fkip@unja.ac.id

1. INTRODUCTION

Mathematical reasoning is an essential skill that students need to develop in school [1], [2], [3], [4]. One effective way to improve students' mathematical reasoning is by engaging them in constructing appropriate mathematical arguments [5], [6], [7]. Many researchers emphasize the importance of argumentation in mathematics learning. Mathematical argumentation is closely related to the formulation of valid conjectures that lead to the reasoning or proof process, particularly in developing generalizations [8]. The ability to express logical thinking through well-structured arguments can contribute to the development of sound mathematical reasoning [9].

The process of argumentation involves proposing assumptions, testing them, providing refutations, revising ideas, retesting, and finally justifying the conclusions. Through this process, students can develop a deeper understanding and reasoning abilities in mathematics. Therefore, integrating argumentation activities into mathematics instruction can significantly improve students' mathematical reasoning skills and overall mathematical competence [10], [11].

Several studies have highlighted the importance of engaging students in constructing written mathematical arguments [12], [13], [14], [15], [16], [17], [18], [19] and communicating their arguments orally [20], [21], [22]. Classroom discussions that encourage students to express different opinions can expose them to uncertainty and complex problem situations, thereby supporting the development of mathematical argumentation skills [23].

However, in practice, teachers often struggle in validate and analyze students' arguments [24]. In addition, students' ability to construct and develop arguments, both in written and oral forms, still needs improvement [25]. Previous research also shows that engaging students in written argumentation activities in mathematics learning can improve students' conceptual understanding and content knowledge [26]. Investigating students' arguments can therefore provide valuable insights into the quality of their mathematical reasoning and the challenges they encounter during the learning process.

In learning number pattern material, students are required to use strong mathematical reasoning skills [27]. One of the indicators of mathematical reasoning ability proposed by many researchers is the ability to construct valid arguments [6], [17], [18], [19], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38]. According to Jureczko, patterns and generalizations are among the most fundamental topics in mathematics education [39]. Many mathematical tasks presented in schools, including numerical and graphical problems, involve identifying and generalizing patterns [39]. However, generalization can only be achieved through appropriate mathematical argumentation [40], [41].

Klock also emphasizes that observing students' argumentation in early classroom tasks is important for understanding how students construct and develop more complex mathematical arguments [34]. Therefore, number pattern material is considered appropriate for exploring students' argumentation abilities, both in written and oral forms.

The components of argumentation used in this study refer to Toulmin's Argumentation Pattern (TAP). Toulmin's framework identifies several key components in constructing arguments, including claims, data, warrants, backing, qualifiers, and rebuttals [36], [42], [43], [44], [45]. These components help describe how arguments are structured and justified in reasoning processes.

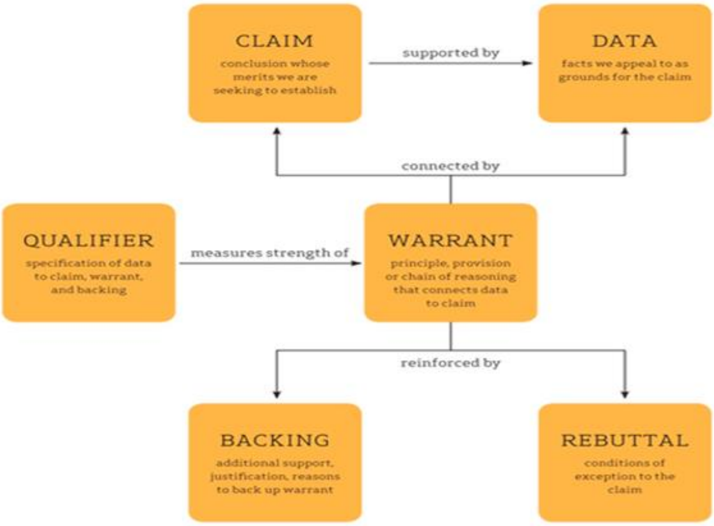

Figure 1. Main Component of Toulmin's Argumentation

Based on Toulmin's framework, students' argumentation abilities can be categorized into several levels. Evagorou et al. proposed a framework for evaluating students' written argumentation skills that classifies argumentation into five levels, from the lowest to the highest[46]. Level 1 represents simple claims or counterclaims. Level 2 includes claims supported by data. Level 3 consists of claims supported by data and warrants, sometimes accompanied by weak rebuttals. Level 4 includes arguments that clearly present both claims and counterarguments. Level 5 represents extended arguments that include multiple rebuttals.

Many previous studies have examined students' argumentation abilities using Toulmin's framework. For example, one study found that students' argumentation abilities in solving numeracy problems were mostly at Levels 1 and 2 [47]. However, previous studies have not extensively examined students' argumentation abilities when solving mathematical reasoning tasks specifically designed to develop argumentation skills. Therefore, this study focuses on analyzing the levels of junior high school students' written and oral argumentation when solving mathematical reasoning tasks related to number patterns.

16

2. METHOD

This study employed a descriptive qualitative research design. The research was conducted at SMP Negeri 7 Muaro Jambi, involving 31 ninth-grade students. The research

20

was carried out during the first semester of the 2024/2025 academic year. The research instruments used in this study consisted of a mathematical reasoning test, interview protocols, and documentation. The mathematical reasoning test consisted of three problems on number patterns.

26

Before being administered to students, the test instrument was validated by experts across three aspects: the construction of the questions, the clarity of the language, and the appropriateness of the content. The validation results indicated that the mathematical reasoning test met the required criteria and was suitable for use with several minor revisions. Similarly, the interview protocol was validated based on three aspects: the structure of the interview guide, language clarity, and the relevance of the interview content. The validation results showed that the interview protocol was also appropriate for use after minor revisions.

This study aimed to analyze students' written and oral argumentation when solving mathematical reasoning tasks. The units of analysis consisted of students' written responses to the reasoning test and the interview transcripts obtained after the students completed the tasks. All arguments identified during these activities were coded to facilitate analysis.

The data analysis process was conducted through several stages. First, each student's argument was coded inductively. The coded arguments were then grouped into categories based on the relevant argumentation components. Students' written responses and interview data were subsequently reviewed and labeled according to these categories. To ensure the trustworthiness of the findings, the study applied criteria of credibility, confirmability, and transferability.

The students' reasoning tasks were analyzed based on the main indicators of the Toulmin Argumentation Pattern (TAP). According to Toulmin's framework, argumentation consists of six main components:

1. Claim – a statement or conclusion presented to an audience.
2. Data – facts or evidence used to support the claim.
3. Warrant – reasoning that connects the data to the claim.
4. Backing – additional support that strengthens the warrant.
5. Qualifier – statements that indicate the strength or limitation of a claim.
6. Rebuttal – statements that acknowledge possible counterarguments or exceptions.

After identifying the components of students' arguments using the Toulmin Argumentation Pattern, students' argumentation abilities were categorized into five levels. The criteria used to determine students' argumentation levels are presented in Table 1.

Table 1. Argumentation Level Criteria

Argumentation Level	Description of Argumentation Components
Level 5	Students present claims, data, warrants, backing, and rebuttals.
Level 4	Students present claims, data, warrants, and backing.
Level 3	Students present claims, data, and warrants.
Level 2	Students present claims supported by data.
Level 1	Students present claims without supporting data.

3. RESULTS AND DISCUSSION

3.1 RESULT

Based on the results of the mathematical reasoning test and interview data, the levels of students' argumentation abilities were identified using the Toulmin Argumentation Pattern framework. The analysis showed that all 31 students reached Level 1 argumentation ability (100%). Furthermore, 15 students reached Level 2 argumentation ability (48.38%), while 9 students reached Level 3 argumentation ability (29.03%). None of the students achieved Level 4 or Level 5 argumentation ability. In this study, S1, S2, and S3 were selected as representative examples of students demonstrating Level 1, Level 2, and Level 3 argumentation, respectively. The percentage distribution of students' argumentation levels is presented in Figure 2.

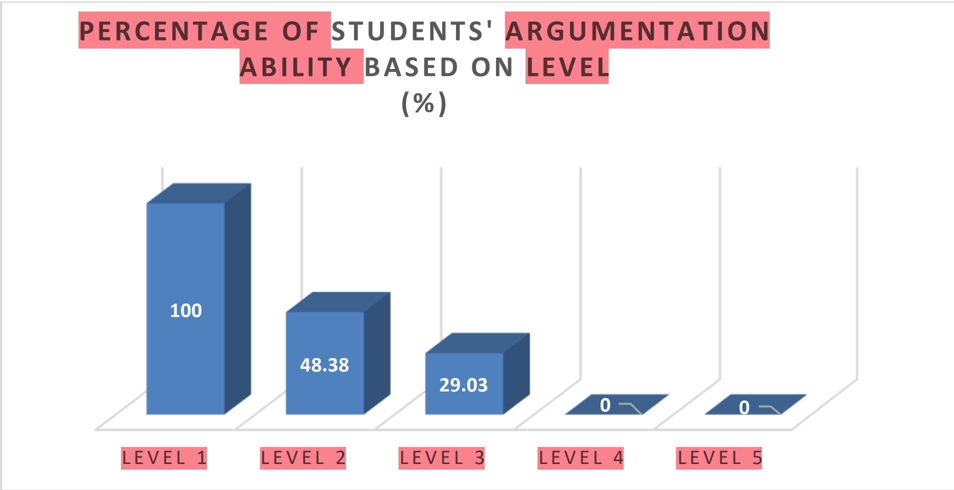

Figure 2. Percentage Achievement at Each Level of Argumentation Students

Level 1 Argumentation Ability

The analysis results show that 100% of students reached Level 1 argumentation ability, indicating that all students were able to provide claims when answering the questions. However, most students were not able to provide supporting data or explanations to justify their claims. Figure 3 presents Question 1, which was used to identify students' Level 1 argumentation ability.

> A parade drum band of 49 students forms a line formation. The front is 11 students, then behind them 2 more, then next 2 more and so on. Determine the number of students in the last line and explain how to get it.!

Figure 3. Question Number 1

Based on students' answers, all 31 students were categorized as having Level 1 argumentation ability for this task. According to the Level 1 criteria, students were able to present a claim but were unable to support it with appropriate data or reasoning. Figure 4 shows an example of a student's answer from Student S1.

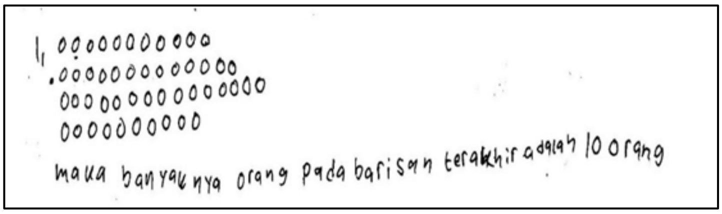

Figure 4. Answer Students S1 on Question Number 1

Student S1 only provided a claim in the answer. S1 stated that the number of students in the last row was 10, but no explanation or method was provided to justify this answer. In the written response, S1 represented the number of students using circle symbols. The first

 https://doi.org/10.58421/gehu.v5i2.1175

row contained 11 circles, the second 13, the third 15, and the final 10. However, S1 did not explain how the number 10 was obtained as the answer.

To further explore the student's reasoning, an interview was conducted with S1. The interview results indicate that S1 was unable to justify their answer.

Interview Excerpt with Student S1

Researcher : Based on Question 1, do you understand the question?
S1 : Yes, I understand.
Researcher : What is asked in the question?
S1 : The number of students in the last row.
Researcher : How many students are in the last row?
S1 : Ten students.
Researcher : Why did you answer ten students?
S1 : (Silent)… I do not know.
Researcher : Why did you think there were 10 students in the last row?
S1 : (Silent).

Based on the interview results, it can be concluded that S1 presented only a claim, without providing data or reasoning to support it. Therefore, S1's argumentation was categorized as Level 1 argumentation ability.

Level 2 Argumentation Ability

The analysis results show that 15 students met the criteria for Level 2 argumentation ability, representing 48.38% of the participants. Students at Level 2 were able to present claims supported by appropriate data. Based on the students' responses to Question 2, Student S2 demonstrated Level 2 argumentation ability. Figure 5 shows an example of a student's answer from student S2.

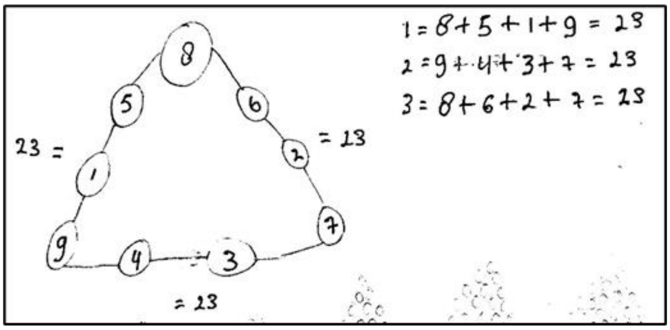

Figure 5. S2's answer for question number 2

Based on the written response, S2 correctly presented a claim by arranging the numbers 1 to 9 so that each row produced a total of 23. In addition, S2 provided supporting data by showing the calculations for each row:

- Row 1: 8 + 5 + 1 + 9 = 23
- Row 2: 9 + 4 + 3 + 7 = 23
- Row 3: 8 + 6 + 2 + 7 = 23

These calculations serve as evidence supporting the claim that the arrangement satisfies the required condition. The interview results also confirmed that S2 understood the reasoning behind the answer.

Interview Excerpt with Student S2

Researcher : Do you understand Question 2?
S2 : Yes, I understand.
Researcher : What information is given in the question?
S2 : We must arrange the numbers 1 to 9 in a circle so that each row totals 23.
Researcher : How did you determine the arrangement?
S2 : I tried several possibilities until I found the correct one.
Researcher : Researcher: How do you know that your answer is correct?
S2 : Because the sum of each row is 23.

Based on both the written response and interview results, it can be concluded that S2 provided a claim supported by data, meeting the criteria for Level 2 argumentation ability.

Level 3 Argumentation Ability

Students were categorized as having Level 3 argumentation ability if they could present claims, supporting data, and warrants. In this study, 9 students met the criteria for Level 3 argumentation ability, representing 29.03% of the participants. Student S3 demonstrated an example of Level 3 argumentation ability in Question 3.

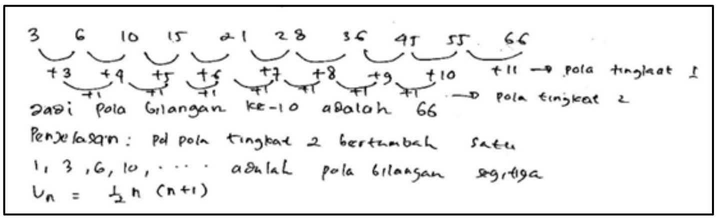

Figure 6 Answer Students S3's Answer to Question 3

In the response, S3 claimed that the 10th pattern contains 66 circles. This claim was supported by identifying the pattern of the sequence. S3 observed that the first few terms of the sequence were 3, 6, 10, 15, and continued in a similar pattern. S3 then analyzed the differences between consecutive terms:

- $6 - 3 = 3$
- $10 - 6 = 4$
- $15 - 10 = 5$

Based on this observation, S3 concluded that the differences increase by one unit at each step. Therefore, the difference between the 9th and the 10th terms is 11, resulting in the calculation 55 + 11 = 66. Hence, the 10th term of the sequence is 66.

S3 also provided a warrant by recognizing that the sequence corresponds to triangular numbers, which follow a known mathematical pattern. Using this reasoning, S3 justified the claim that the 10th pattern contains 66 circles.
The interview results confirmed that S3 understood the reasoning process used to determine the answer. Based on the written and interview data, S3 demonstrated the ability to present

claims, supporting data, and warrants, which meet the criteria for Level 3 argumentation ability.

Interview Excerpt with Student S3

Researcher	:	Do you understand Question 3?
S3	:	Yes, I understand, Ma'am.
Researcher	:	What information is given in the question?
S3	:	In the first picture, there are 3 circles. In the second picture, there are 6 circles. In the third picture, there are 10 circles. In the fourth picture, there are 15 circles. The question asks how many circles there are in the 10th pattern.
Researcher	:	According to you, what is the answer?
S3	:	The answer is 66, Ma'am.
Researcher	:	Why?
S3	:	Because the pattern forms the sequence 3, 6, 10, 15, and so on.
Researcher	:	How did you get the answer 66?
S3	:	The difference between Pattern 1 and Pattern 2 is 3. The difference between Pattern 2 and Pattern 3 is 4, and the difference between Pattern 3 and Pattern 4 is 5. This means the difference increases by one each time. If the pattern continues, the difference between the 9th and the 10th patterns is 11. Therefore, the 10th pattern equals the 9th pattern plus the difference, which is 55 + 11 = 66. The sequence becomes 3, 6, 10, 15, 21, 28, 36, 45, 55, and 66.
Researcher	:	Are you certain about your answer?
S3	:	Yes, I am sure, Ma'am, because the numbers in the pattern correspond to triangular numbers, namely 1, 3, 6, 10, 15, 21, and so on.

3.2 DISCUSSION

Based on students' written responses to the reasoning questions and interview results, 31 students (100%) achieved Level 1 argumentation. This finding indicates that all students were able to make claims when solving Questions 1, 2, and 3. The process of building an argument begins when students formulate claims. Claims are statements or assertions based on facts [48].

In Question 1, students were asked to determine the number of students in the last row. To solve this problem, students first needed to understand the question by identifying the pattern in Rows 1, 2, and 3, which increased by 2 from Row 1 to Row 2 and from Row 2 to Row 3. Once students identified the pattern, they were able to solve the problem using addition and subtraction rules. Most students claimed there were 10 students in the last row. This claim was demonstrated in various ways, such as drawing circles or pictures representing the number of students in each row. However, some students were unable to provide sufficient evidence or appropriate reasoning to support their claims.

Claims made by students should be supported by evidence, especially when students have a good understanding of the underlying concepts [49]. Students who have a good conceptual understanding can connect different concepts and apply formulas or algorithms flexibly, accurately, efficiently, and appropriately to solve problems [50]. Students who are only able to make claims demonstrate that they can apply relevant concepts and algorithms to solve the problem, but they may not yet be able to justify their reasoning fully.

The results also showed that 15 students reached Level 2 argumentation, representing 48.38% of the participants. Students who reach Level 2 can make claims and provide supporting evidence. In Question 2, students were asked to place the numbers in the circle arrangement. To solve this problem, students first needed to understand that the numbers 1–9 had to be placed in the circles such that each row totaled 23. Students who achieved Level 2 could claim that the numbers 1–9 could be arranged so that each row sums to 23. They also provided correct data to support their claims. For example, the first row was 8 + 5 + 1 + 9 = 23, the second row was 9 + 4 + 3 + 7 = 23, and the third row was 8 + 6 + 2 + 7 = 23.

Providing data as evidence in solving mathematical problems helps students understand mathematical facts, procedures, and concepts. Students' ability to understand concepts not only helps them memorize formulas but also enables them to grasp the deeper meaning of mathematical learning [51]. Mathematics is a subject that requires argumentation in problem-solving. Argumentation is a process in which claims are supported by data and justified by warrants; it is essential for solving problems effectively [51].

Students reach Level 3 argumentation when they can construct claims, data, and warrants. Based on the results of this study, 9 students (29.03%) achieved Level 3 argumentation. This finding indicates that these students were able to solve the problems correctly by providing appropriate claims, supporting data, and logical warrants. In Question 3, students were asked to determine the 10th pattern in the circle pattern. Students solved this problem by first identifying the difference between consecutive patterns. After claiming that the 10th pattern was 66, students provided data by identifying the increasing differences between the patterns. Based on this data, students were able to justify their reasoning by applying the triangular number formula, $U_n = \frac{1}{2}n(n+1)$, to determine the result.

Students who can construct claims, data, and warrants demonstrate that they understand the problem and can determine appropriate strategies to solve it. However, no students reached Level 4 or Level 5 argumentation in this study. This finding indicates the need for teachers to train students to develop backing, qualifiers, and rebuttals in their arguments, both in written and oral forms. These results are consistent with previous research, which found that students' argumentation abilities were mostly at Level 1 (58.10%), followed by Level 2 (30.95%), and Level 3 (10.95%), while no students reached Level 4 or Level 5 in learning [52].

4. CONCLUSION

The results of this study indicate that students' abilities in mathematical argumentation were distributed across three levels. A total of 31 students (100%) achieved Level 1 argumentation, demonstrating their ability to formulate claims when solving reasoning problems. Furthermore, 15 students (48.38%) reached Level 2 argumentation, indicating that they were able to construct claims supported by appropriate data, while 9 students (29.03%) achieved Level 3 argumentation by providing claims, data, and warrants in their reasoning. However, no students reached Level 4 or Level 5 argumentation, which involves the use of backing, qualifiers, and rebuttals. These findings suggest that although students have begun to develop basic components of mathematical argumentation, their ability to construct more complete arguments remains limited. Therefore, students need to be trained to develop backing, qualifiers, and rebuttals so they can construct more comprehensive arguments in both written and oral forms. The findings also imply that teachers play an important role in fostering students' mathematical argumentation in classroom learning, particularly through activities that encourage reasoning, explanation, and justification when solving mathematical problems.

ACKNOWLEDGEMENT

The authors would like to express their sincere gratitude to the students and mathematics teachers at SMP Negeri 7 Muaro Jambi who participated in this study with enthusiasm and dedication throughout the research process. We also extend our appreciation to the school administration for facilitating access to the learning infrastructure and providing the necessary technical support. The authors gratefully acknowledge the financial support provided by the Research and Community Service Institute of Universitas Jambi through the 2024 Faculty Research Grant Scheme (376/UN21.11/PT.01.05/SPK/2024). In addition, the authors would like to thank the anonymous reviewers for their constructive feedback, which significantly improved the quality of this manuscript.

REFERENCES

[1] A. Mukuka, S. Balimuttajjo, and V. Mutarutinya, "Teacher Efforts Towards the Development of Students' Mathematical Reasoning Skills," *Heliyon*, vol. 9, no. 4, 2023, doi: 10.1016/j.heliyon.2023.e14789.

[2] O. E. Onoshakpokaiye, "an Overview of Reasoning Ability in Mathematics and Mathematics Achievement of Students in Tertiary Institution," *IJIET (International Journal of Indonesian Education and Teaching)*, vol. 7, no. 2, pp. 309–318, 2023, doi: 10.24071/ijiet.v7i2.5988.

[3] I. Dana Wahyudi and Walid, "Mathematical Reasoning Ability Of Students Based On Learning Style Using Missouri Mathematics Project Learning Model," *Unnes Journal of Mathematics Education*, vol. 9, no. 3, pp. 206–210, 2020, doi: 10.15294/ujme.v9i3.44538.

[4] R. Rohati, Y. S. Kusumah, and K. Kusnandi, "Exploring Students' Mathematical Reasoning Behavior in Junior High Schools: A Grounded Theory," *Educ. Sci. (Basel).*, vol. 13, no. 3, p. 252, 2023, doi: 10.3390/educsci13030252.

[5] E. Golla and A. Reyes, "Pisa 2022 Mathematics Framework," no. November 2018, 2022.

[6] B. Nergård, "Preschool Children's Mathematical Arguments in Play-Based Activities," *Mathematics Education Research Journal*, vol. 35, no. 1, pp. 193–216, 2023, doi: 10.1007/s13394-021-00395-6.

[7] Rohati, Y. S. Kusumah, K. Kusnandi, and M. Marlina, "How Teachers Encourage Students' Mathematical Reasoning during the Covid-19 Pandemic?," *Jurnal Pendidikan Indonesia*, vol. 11, no. 4, pp. 715–726, 2022, doi: https://ejournal.undiksha.ac.id/index.php/JPI/article/view/52756.

[8] P. J. Lin, "the Development of Students' Mathematical Argumentation in a Primary Classroom," *Educacao and Realidade*, vol. 43, no. 3, pp. 1171–1192, 2018, doi: 10.1590/2175-623676887.

[9] V. Durand-Guerrier, P. Boero, N. Douek, S. S. Epp, and D. Tanguay, *Examining the Role of Logic in Teaching Proof*, vol. 15. 2012. doi: 10.1007/978-94-007-2129-6_16.

[10] W. D. Butler, D. Corcoran, T. Farrell, S. Nicmhuirí, M. O. Connor, and J. Travers, "Mathematics in Early Childhood and Primary Education (3-8 years): Executive Summaries," no. 18, p. 31, 2014.

[11] C. Björklund, M. Van Den Heuvel-Panhuizen, and A. Kullberg, "Research on Early Childhood Mathematics Teaching and Learning," *ZDM - Mathematics Education*, vol. 52, no. 4, pp. 607–619, 2020, doi: 10.1007/s11858-020-01177-3.

[12] D. Muhtadi, Sukirwan, R. Hermanto, Warsito, and A. Sunendar, "How do Students Promote Mathematical Argumentation Through Guide-Redirecting Warrant Construction?" *J. Phys. Conf. Ser.*, vol. 1613, no. 1, 2020, doi: 10.1088/1742-6596/1613/1/012031.

[13] K. Kruse, “Can Argumentative Writing Improve Math Knowledge for Elementary Students with a Mathematics Learning Disability?: a Single-Case Classroom Intervention Investigation 1”.

[14] K. W. Kosko, “View of Making Students’ Mathematical Arguments Explicit.pdf.”

[15] K. Wilkie and M. Ayalon, “Learning to Argue While Arguing to Learn: Students’ Emotional Experiences During Argumentation for Graphing Real-Life Functions,” *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 19, no. 8, 2023, doi: 10.29333/ejmste/13435.

[16] T. G. Campbell, S. King, and J. Zelkowski, “Comparing Middle Grade Students’ Oral and Written Arguments,” *Research in Mathematics Education*, vol. 23, no. 1, pp. 21–38, 2021, doi: 10.1080/14794802.2020.1722960.

[17] K. N. Bieda, D. A. Bowers, and V. A. B. Küchle, “The Genre(s) of Argumentation in School Mathematics,” *Michigan Reading Journal*, vol. 51, no. 2, pp. 41–52, 2019.

[18] S. P. Yee, J. D. Boyle, Y. Y. (Winnie) Ko, and S. K. Bleiler-Baxter, “Effects of Constructing, Critiquing, and Revising Arguments Within University Classrooms,” *Journal of Mathematical Behavior*, vol. 49, no. May 2016, pp. 145–162, 2018, doi: 10.1016/j.jmathb.2017.11.009.

[19] A. J. Stylianides, “the Role of Mode of Representation in Students’ Argument Constructions,” *CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education*, pp. 213–220, 2015.

[20] B. Wilson, “Digital Commons @ University of Nebraska - Lincoln Mathematical Communication through Written and Oral Expression,” 2009.

[21] A. J. Stylianides, “Secondary Students’ Proof Constuctions in Mathematics: The Role of Written Versus Oral Mode of Argument Representation,” *Review of Education*, vol. 7, no. 1, pp. 156–182, 2019.

[22] S. Jablonski and M. Ludwig, “Mathematical Arguments in the Context of Mathematical Giftedness – Analysis of Oral Argumentations with Toulmin to cite this Version : HAL Id : hal-02398107,” *HAL open science*, pp. 1–10, 2019.

[23] K. H. Hauge and R. Barwell, “Post-Normal Science and Mathematics Education in Uncertain Times: Educating Future Citizens for Extended Peer Communities,” *Futures*, vol. 91, pp. 25–34, 2017, doi: 10.1016/j.futures.2016.11.013.

[24] J. Van De Pol, M. Volman, and J. Beishuizen, “Scaffolding in Teacher-Student Interaction: a Decade Of Research,” *Educ. Psychol. Rev.*, vol. 22, no. 3, pp. 271–296, 2010, doi: 10.1007/s10648-010-9127-6.

[25] O. T. Kaufmann and A. Ryve, “Teachers’ Framing of Students’ Difficulties In Mathematics Learning In Collegial Discussions,” *Scandinavian Journal of Educational Research*, vol. 67, no. 7, pp. 1069–1085, 2023, doi: 10.1080/00313831.2022.2115134.

[26] S. A. Kiuhara, J. R. Levin, M. Tolbert, B. V. O’Keeffe, R. E. O’Neill, and J. M. Jameson, “Teaching Argument Writing in Math Class: Challenges And Solutions to Improve the Performance of 4th And 5th Graders with Disabilities,” *Read. Writ.*, no. 0123456789, 2023, doi: 10.1007/s11145-023-10459-7.

[27] S. Nabila and R. I. I. Putri, “Students’ Mathematical Reasoning Skills on Number Pattern Using PMRI And Collaborative Learning Approach,” *Jurnal Elemen*, vol. 8, no. 1, pp. 290–307, 2022, doi: 10.29408/jel.v8i1.4733.

[28] V. Svoboda and J. Peregrin, “Logically Incorrect Arguments,” *Argumentation*, vol. 30, no. 3, pp. 263–287, 2016, doi: 10.1007/s10503-015-9375-1.

[29] A. Glassner, “Evaluating Arguments In Instruction: Theoretical and Practical Directions,” *Think. Skills Creat.*, vol. 24, pp. 95–103, 2017, doi: 10.1016/j.tsc.2017.02.013.

[30] A. T. Cahyono, S. Susiswo, and T. D. Chandra, “Condition of Students’ Mathematical Reasoning Abilities Based on Their Ability to Argue,” *Journal of Disruptive Learning Innovation (JODLI)*, vol. 2, no. 2, p. 98, 2021, doi: 10.17977/um072v2i22021p98-112.

[31] D. Godden and J. Grey, *Reasoning by Grounded Analogy*, vol. 199, no. 3–4. Springer Netherlands, 2021. doi: 10.1007/s11229-020-02974-9.

[32] L. Linda and I. Asyura, “Students’ Mathematical Reasoning Ability in Solving Post-Covid-19 PISA Model Math Problems,” *Jurnal Riset Pendidikan Matematika*, vol. 8, no. 2, pp. 140–152, 2021, doi: 10.21831/jrpm.v8i2.44739.

[33] M. Meyer and S. Schnell, “What Counts as a ‘Good’ Argument in School?—How Teachers Grade Students’ Mathematical Arguments,” *Educational Studies in Mathematics*, vol. 105, no. 1, pp. 35–51, 2020, doi: 10.1007/s10649-020-09974-z.

[34] S. I. Klock, “Arguments and Group Discussions: Middle-School Students’ Initial Work with Mathematical Arguments Initial Work with Mathematical Arguments,” *Int. J. Math. Educ. Sci. Technol.*, 2024, doi: 10.1080/0020739X.2024.2328347.

[35] C. Rumsey and C. W. Langrall, "Promoting Mathematical Argumentation," *Teach. Child. Math.*, vol. 22, no. 7, pp. 412–419, 2016.

[36] F. H. van Eemeren, B. Garssen, E. C. W. Krabbe, A. F. Snoeck Henkemans, B. Verheij, and J. H. M. Wagemans, *Toulmin's Model of Argumentation*. 2014. doi: 10.1007/978-90-481-9473-5_4.

[37] M. Ayalon and R. Even, "Factors Shaping Students' Opportunities To Engage In Argumentative Activity," no. September 2014, pp. 575–601, 2016.

[38] J. T. Shemwell, K. R. Gwarjanski, D. K. Capps, S. Avargil, and J. L. Meyer, "Supporting Teachers to Attend to Generalisation in Science Classroom Argumentation," *Int. J. Sci. Educ.*, vol. 37, no. 4, pp. 599–628, 2015, doi: 10.1080/09500693.2014.1000428.

[39] J. Joanna, "the Strategies of Using a Special Kind of Number Patterns in Different Stages of Education," *Educational Research and Reviews*, vol. 12, no. 12, pp. 643–652, 2017, doi: 10.5897/err2017.3244.

[40] C. Dahlman, "Unacceptable Generalizations in Arguments on Legal Evidence," *argumentation*, vol. 31, no. 1, pp. 83–99, 2017, doi: 10.1007/s10503-016-9399-1.

[41] F. Zenker, C. Dahlman, S. Sikström, L. Wahlberg, and F. Sarwar, "Generalization in Legal Argumentation," *J. Forensic Psychol. Res. Pract.*, vol. 20, no. 1, pp. 80–99, 2020, doi: 10.1080/24732850.2019.1689782.

[42] D. Hitchcock, "Good Reasoning on the Toulmin Model," *Argumentation*, vol. 19, no. 3, pp. 373–391, 2005, doi: 10.1007/s10503-005-4422-y.

[43] S. Erduran, "Toulmin's Argument Pattern as a 'Horizon of Possibilities' in the Study of Argumentation in Science Education," *Cult. Stud. Sci. Educ.*, vol. 13, no. 4, pp. 1091–1099, 2018, doi: 10.1007/s11422-017-9847-8.

[44] E. Erdem and R. Gurbuz, "an Analysis of Seventh-Grade Students' Mathematical Reasoning.," *Cukurova University Faculty of Education*, vol. 45, no. 1, pp. 123–142, 2015, doi: 10.14812/cufej.2015.007.

[45] V. M. Cabello, P. M. Moreira, and P. G. Morales, "Elementary Students' Reasoning in Drawn Explanations Based on a Scientific Theory," *educ. Sci. (Basel).*, vol. 11, no. 10, 2021, doi: 10.3390/educsci11100581.

[46] M. Evagorou, E. Papanastasiou, and M. Vrikki, "What do We Really Know About Students' Written Arguments? Evaluating Written Argumentation Skills," *European Journal of Science and Mathematics Education*, vol. 11, no. 4, pp. 615–634, 2023, doi: 10.30935/scimath/13284.

[47] B. K. Wardhani, Lady Agustina, and C. K. Galatea, "Analisis Kemampuan Argumentasi Siswa dalam Menyelesaikan Soal Numerasi Berdasarkan Teori Toulmin (Level 1 dan Level 2)," *MATHEdunesa*, vol. 12, no. 1, pp. 166–175, Jun. 2023, doi: 10.26740/mathedunesa.v12n1.p166-175.

[48] A. R. Hakim, W. Widodo, and T. Sunarti, "Profile of Toulmin's Scientific Arguments Students and Technological Utilities in Global Warming Topic," *JPPS (Jurnal Penelitian Pendidikan Sains)*, vol. 12, no. 1, 2022, doi: 10.26740/jpps.v12n1.p85-99.

[49] L. B. Tristanti, A. Sutawidjaja, A. R. As'ari, and M. Muskar, "The Construction of Deductive Warrant Derived from Inductive Warrant in Preservice-Teacher Mathematical Argumentations.," *Educational Research and Reviews*, vol. 11, no. 17, pp. 1696–1708, 2016.

[50] V. Aledya, "Kemampuan Pemahaman Konsep Matematika pada Siswa," *Kemampuan Pemahaman Konsep Matematika Pada Siswa*, vol. 2, no. May, 2019.

[51] H. D. Puspithasari and H. Pujiastuti, "Analysis of Students' Understanding of Mathematical Concepts in Solving Comparative Problems," *Journal of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, vol. 5, no. 1, p. 181, Jan. 2021, doi: 10.31331/medivesveteran.v5i1.1173.

[52] S. Gökçe, A. Aydoğan Yenmez, and T. Çelik, "Argumentation-Based Learning: an Example of Mathematical Questions Through Online Interactions Among Prospective Teachers," *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi*, no. 53, 2020, doi: 10.21764/maeuefd.527779.

[53] L. Triananda, E. A. P, N. Andriani, and P. Fisika, Kemampuan Berargumentasi Siswa Menggunakan Model Toulmin Pada Materi Hukum Newton di SMP Negeri 57 Palembang," "*JIFP* (*Jurnal Ilmu Fisika dan Pembelajarannya*), vol. VII, pp. 33–39, http://jurnal.radenfatah.ac.id/index.php/jifp/.