

Levels of Mathematical Argumentation: An Analysis of Junior High School Students' Reasoning in Number Pattern Tasks

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ABSTRACT

Mathematical argumentation is important for developing students' reasoning abilities. This study aims to analyze junior high school students' written and oral argumentation abilities when solving reasoning tasks. This research employed a descriptive qualitative approach, involving 31 ninth-grade students from SMP Negeri 7 Muaro Jambi. The study was conducted in the odd semester of the 2024/2025 academic year. The research instruments consisted of a mathematical reasoning test, interview protocols, and documentation. The mathematical reasoning test included three questions. The results of the tests and interviews were analyzed using the main components of Toulmin's argumentation model, which consist of claims, data, warrants, backing, qualifiers, and rebuttals. Based on the test and interview results, students' argumentation abilities were categorized into five levels, from Level 1 to Level 5. The findings show that 31 students (100%) reached Level 1, 15 (48.38%) reached Level 2, and 9 (29.03%) reached Level 3. No students reached Level 4 or Level 5. These findings indicate that students can construct claims, data, and warrants when solving mathematical reasoning tasks. However, students still need to be trained to develop backing, qualifiers, and rebuttals in order to complete the argumentation process appropriately in both written and oral forms.

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1. INTRODUCTION

Mathematical reasoning is an essential skill that students need to develop in school [1], [2], [3], [4]. One effective way to improve students' mathematical reasoning is by engaging them in constructing appropriate mathematical arguments [5], [6], [7]. Many researchers emphasize the importance of argumentation in mathematics learning. Mathematical argumentation is closely related to the formulation of valid conjectures that lead to the reasoning or proof process, particularly in developing generalizations [8]. The

ability to express logical thinking through well-structured arguments can contribute to the development of sound mathematical reasoning [9].

The process of argumentation involves proposing assumptions, testing them, providing refutations, revising ideas, retesting, and finally justifying the conclusions. Through this process, students can develop a deeper understanding and reasoning abilities in mathematics. Therefore, integrating argumentation activities into mathematics instruction can significantly improve students' mathematical reasoning skills and overall mathematical competence [10], [11].

Several studies have highlighted the importance of engaging students in constructing written mathematical arguments [12], [13], [14], [15], [16], [17], [18], [19] and communicating their arguments orally [20], [21], [22]. Classroom discussions that encourage students to express different opinions can expose them to uncertainty and complex problem situations, thereby supporting the development of mathematical argumentation skills [23].

However, in practice, teachers often struggle to validate and analyze students' arguments [24]. In addition, students' ability to construct and develop arguments, both in written and oral forms, still needs improvement [25]. Previous research also shows that engaging students in written argumentation activities in mathematics learning can improve students' conceptual understanding and content knowledge [26]. Investigating students' arguments can therefore provide valuable insights into the quality of their mathematical reasoning and the challenges they encounter during the learning process.

In learning number pattern material, students are required to use strong mathematical reasoning skills [27]. One of the indicators of mathematical reasoning ability proposed by many researchers is the ability to construct valid arguments [6], [17], [18], [19], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38]. According to Jureczko, patterns and generalizations are among the most fundamental topics in mathematics education [39]. Many mathematical tasks presented in schools, including numerical and graphical problems, involve identifying and generalizing patterns [39]. However, generalization can only be achieved through appropriate mathematical argumentation [40], [41].

Klock also emphasizes that observing students' argumentation in early classroom tasks is important for understanding how students construct and develop more complex mathematical arguments [34]. Therefore, number pattern material is considered appropriate for exploring students' argumentation abilities, both in written and oral forms.

The components of argumentation used in this study refer to Toulmin's Argumentation Pattern (TAP). Toulmin's framework identifies several key components in constructing arguments, including claims, data, warrants, backing, qualifiers, and rebuttals [36], [42], [43], [44], [45]. These components help describe how arguments are structured and justified in reasoning processes.



Figure 1. Main Component of Toulmin’s Argumentation

Based on Toulmin’s framework, students’ argumentation abilities can be categorized into several levels. Evagorou et al. proposed a framework for evaluating students’ written argumentation skills that classifies argumentation into five levels, from the lowest to the highest[46]. Level 1 represents simple claims or counterclaims. Level 2 includes claims supported by data. Level 3 consists of claims supported by data and warrants, sometimes accompanied by weak rebuttals. Level 4 includes arguments that clearly present both claims and counterarguments. Level 5 represents extended arguments that include multiple rebuttals.

Many previous studies have examined students’ argumentation abilities using Toulmin’s framework. For example, one study found that students’ argumentation abilities in solving numeracy problems were mostly at Levels 1 and 2 [47]. However, previous studies have not extensively examined students’ argumentation abilities when solving mathematical reasoning tasks specifically designed to develop argumentation skills. Therefore, this study focuses on analyzing the levels of junior high school students’ written and oral argumentation when solving mathematical reasoning tasks related to number patterns.

2. METHOD

This study employed a descriptive qualitative research design. The research was conducted at SMP Negeri 7 Muaro Jambi, involving 31 ninth-grade students. The research was carried out during the first semester of the 2024/2025 academic year. The research instruments used in this study consisted of a mathematical reasoning test, interview protocols, and documentation. The mathematical reasoning test consisted of three problems on number patterns.

Before being administered to students, the test instrument was validated by experts across three aspects: the construction of the questions, the clarity of the language, and the appropriateness of the content. The validation results indicated that the mathematical reasoning test met the required criteria and was suitable for use with several minor revisions. Similarly, the interview protocol was validated based on three aspects: the structure of the

interview guide, language clarity, and the relevance of the interview content. The validation results showed that the interview protocol was also appropriate for use after minor revisions.

This study aimed to analyze students' written and oral argumentation when solving mathematical reasoning tasks. The units of analysis consisted of students' written responses to the reasoning test and the interview transcripts obtained after the students completed the tasks. All arguments identified during these activities were coded to facilitate analysis.

The data analysis process was conducted through several stages. First, each student's argument was coded inductively. The coded arguments were then grouped into categories based on the relevant argumentation components. Students' written responses and interview data were subsequently reviewed and labeled according to these categories. To ensure the trustworthiness of the findings, the study applied criteria of credibility, confirmability, and transferability.

The students' reasoning tasks were analyzed based on the main indicators of the Toulmin Argumentation Pattern (TAP). According to Toulmin's framework, argumentation consists of six main components:

1. Claim – a statement or conclusion presented to an audience.
2. Data – facts or evidence used to support the claim.
3. Warrant – reasoning that connects the data to the claim.
4. Backing – additional support that strengthens the warrant.
5. Qualifier – statements that indicate the strength or limitation of a claim.
6. Rebuttal – statements that acknowledge possible counterarguments or exceptions.

After identifying the components of students' arguments using the Toulmin Argumentation Pattern, students' argumentation abilities were categorized into five levels. The criteria used to determine students' argumentation levels are presented in Table 1.

Table 1. Argumentation Level Criteria

Argumentation Level	Description of Argumentation Components
Level 5	Students present claims, data, warrants, backing, and rebuttals.
Level 4	Students present claims, data, warrants, and backing.
Level 3	Students present claims, data, and warrants.
Level 2	Students present claims supported by data.
Level 1	Students present claims without supporting data.

3. RESULTS AND DISCUSSION

3.1 RESULT

Based on the results of the mathematical reasoning test and interview data, the levels of students' argumentation abilities were identified using the Toulmin Argumentation Pattern framework. The analysis showed that all 31 students reached Level 1 argumentation ability (100%). Furthermore, 15 students reached Level 2 argumentation ability (48.38%), while 9 students reached Level 3 argumentation ability (29.03%). None of the students achieved Level 4 or Level 5 argumentation ability. In this study, S1, S2, and S3 were selected as representative examples of students demonstrating Level 1, Level 2, and Level 3

argumentation, respectively. The percentage distribution of students' argumentation levels is presented in Figure 2.

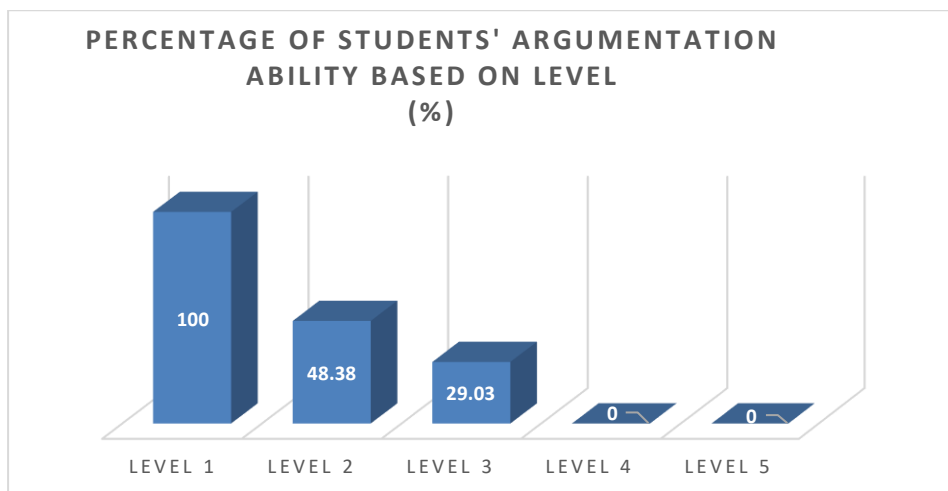


Figure 2. Percentage Achievement at Each Level of Argumentation Students

Level 1 Argumentation Ability

The analysis results show that 100% of students reached Level 1 argumentation ability, indicating that all students were able to provide claims when answering the questions. However, most students were not able to provide supporting data or explanations to justify their claims. Figure 3 presents Question 1, which was used to identify students' Level 1 argumentation ability.

A parade drum band of 49 students forms a line formation. The front is 11 students, then behind them 2 more, then next 2 more and so on. Determine the number of students in the last line and explain how to get it!

Figure 3. Question Number 1

Based on students' answers, all 31 students were categorized as having Level 1 argumentation ability for this task. According to the Level 1 criteria, students were able to present a claim but were unable to support it with appropriate data or reasoning. Figure 4 shows an example of a student's answer from Student S1.

11 0000000000
 .000000000000
 00000000000000
 0000000000
 maka banyaknya orang pada barisan terakhir adalah 10 orang

Figure 4. Answer Students S1 on Question Number 1

Student S1 only provided a claim in the answer. S1 stated that the number of students in the last row was 10, but no explanation or method was provided to justify this answer. In the written response, S1 represented the number of students using circle symbols. The first

row contained 11 circles, the second 13, the third 15, and the final 10. However, S1 did not explain how the number 10 was obtained as the answer.

To further explore the student's reasoning, an interview was conducted with S1. The interview results indicate that S1 was unable to justify their answer.

Interview Excerpt with Student S1

- Researcher : Based on Question 1, do you understand the question?
 S1 : Yes, I understand.
 Researcher : What is asked in the question?
 S1 : The number of students in the last row.
 Researcher : How many students are in the last row?
 S1 : Ten students.
 Researcher : Why did you answer ten students?
 S1 : (Silent)... I do not know.
 Researcher : Why did you think there were 10 students in the last row?
 S1 : (Silent).

Based on the interview results, it can be concluded that S1 presented only a claim, without providing data or reasoning to support it. Therefore, S1's argumentation was categorized as Level 1 argumentation ability.

Level 2 Argumentation Ability

The analysis results show that 15 students met the criteria for Level 2 argumentation ability, representing 48.38% of the participants. Students at Level 2 were able to present claims supported by appropriate data. Based on the students' responses to Question 2, Student S2 demonstrated Level 2 argumentation ability. Figure 5 shows an example of a student's answer from student S2.

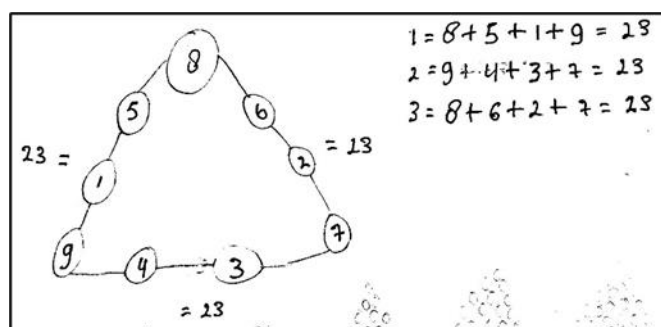


Figure 5. S2's answer for question number 2

Based on the written response, S2 correctly presented a claim by arranging the numbers 1 to 9 so that each row produced a total of 23. In addition, S2 provided supporting data by showing the calculations for each row:

- Row 1: $8 + 5 + 1 + 9 = 23$
- Row 2: $9 + 4 + 3 + 7 = 23$
- Row 3: $8 + 6 + 2 + 7 = 23$

These calculations serve as evidence supporting the claim that the arrangement satisfies the required condition. The interview results also confirmed that S2 understood the reasoning behind the answer.

Interview Excerpt with Student S2

- Researcher : Do you understand Question 2?
- S2 : Yes, I understand.
- Researcher : What information is given in the question?
- S2 : We must arrange the numbers 1 to 9 in a circle so that each row totals 23.
- Researcher : How did you determine the arrangement?
- S2 : I tried several possibilities until I found the correct one.
- Researcher : Researcher: How do you know that your answer is correct?
- S2 : Because the sum of each row is 23.

Based on both the written response and interview results, it can be concluded that S2 provided a claim supported by data, meeting the criteria for Level 2 argumentation ability.

Level 3 Argumentation Ability

Students were categorized as having Level 3 argumentation ability if they could present claims, supporting data, and warrants. In this study, 9 students met the criteria for Level 3 argumentation ability, representing 29.03% of the participants. Student S3 demonstrated an example of Level 3 argumentation ability in Question 3.

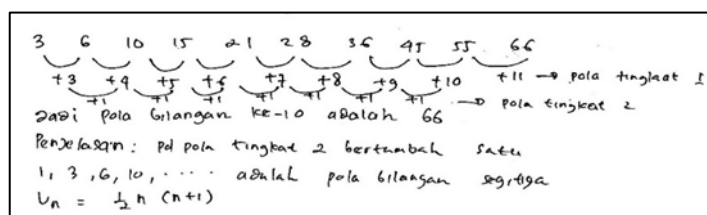


Figure 6 Answer Students S3's Answer to Question 3

In the response, S3 claimed that the 10th pattern contains 66 circles. This claim was supported by identifying the pattern of the sequence. S3 observed that the first few terms of the sequence were 3, 6, 10, 15, and continued in a similar pattern. S3 then analyzed the differences between consecutive terms:

- $6 - 3 = 3$
- $10 - 6 = 4$
- $15 - 10 = 5$

Based on this observation, S3 concluded that the differences increase by one unit at each step. Therefore, the difference between the 9th and the 10th terms is 11, resulting in the calculation $55 + 11 = 66$. Hence, the 10th term of the sequence is 66.

S3 also provided a warrant by recognizing that the sequence corresponds to triangular numbers, which follow a known mathematical pattern. Using this reasoning, S3 justified the claim that the 10th pattern contains 66 circles.

The interview results confirmed that S3 understood the reasoning process used to determine the answer. Based on the written and interview data, S3 demonstrated the ability to present

claims, supporting data, and warrants, which meet the criteria for Level 3 argumentation ability.

Interview Excerpt with Student S3

- Researcher : Do you understand Question 3?
S3 : Yes, I understand, Ma'am.
Researcher : What information is given in the question?
S3 : In the first picture, there are 3 circles. In the second picture, there are 6 circles. In the third picture, there are 10 circles. In the fourth picture, there are 15 circles. The question asks how many circles there are in the 10th pattern.
Researcher : According to you, what is the answer?
S3 : The answer is 66, Ma'am.
Researcher : Why?
S3 : Because the pattern forms the sequence 3, 6, 10, 15, and so on.
Researcher : How did you get the answer 66?
S3 : The difference between Pattern 1 and Pattern 2 is 3. The difference between Pattern 2 and Pattern 3 is 4, and the difference between Pattern 3 and Pattern 4 is 5. This means the difference increases by one each time. If the pattern continues, the difference between the 9th and the 10th patterns is 11. Therefore, the 10th pattern equals the 9th pattern plus the difference, which is $55 + 11 = 66$. The sequence becomes 3, 6, 10, 15, 21, 28, 36, 45, 55, and 66.
Researcher : Are you certain about your answer?
S3 : Yes, I am sure, Ma'am, because the numbers in the pattern correspond to triangular numbers, namely 1, 3, 6, 10, 15, 21, and so on.

3.2 DISCUSSION

Based on students' written responses to the reasoning questions and interview results, 31 students (100%) achieved Level 1 argumentation. This finding indicates that all students were able to make claims when solving Questions 1, 2, and 3. The process of building an argument begins when students formulate claims. Claims are statements or assertions based on facts [48].

In Question 1, students were asked to determine the number of students in the last row. To solve this problem, students first needed to understand the question by identifying the pattern in Rows 1, 2, and 3, which increased by 2 from Row 1 to Row 2 and from Row 2 to Row 3. Once students identified the pattern, they were able to solve the problem using addition and subtraction rules. Most students claimed there were 10 students in the last row. This claim was demonstrated in various ways, such as drawing circles or pictures representing the number of students in each row. However, some students were unable to provide sufficient evidence or appropriate reasoning to support their claims.

Claims made by students should be supported by evidence, especially when students have a good understanding of the underlying concepts [49]. Students who have a good conceptual understanding can connect different concepts and apply formulas or algorithms

flexibly, accurately, efficiently, and appropriately to solve problems [50]. Students who are only able to make claims demonstrate that they can apply relevant concepts and algorithms to solve the problem, but they may not yet be able to justify their reasoning fully.

The results also showed that 15 students reached Level 2 argumentation, representing 48.38% of the participants. Students who reach Level 2 can make claims and provide supporting evidence. In Question 2, students were asked to place the numbers in the circle arrangement. To solve this problem, students first needed to understand that the numbers 1–9 had to be placed in the circles such that each row totaled 23. Students who achieved Level 2 could claim that the numbers 1–9 could be arranged so that each row sums to 23. They also provided correct data to support their claims. For example, the first row was $8 + 5 + 1 + 9 = 23$, the second row was $9 + 4 + 3 + 7 = 23$, and the third row was $8 + 6 + 2 + 7 = 23$.

Providing data as evidence in solving mathematical problems helps students understand mathematical facts, procedures, and concepts. Students' ability to understand concepts not only helps them memorize formulas but also enables them to grasp the deeper meaning of mathematical learning [51]. Mathematics is a subject that requires argumentation in problem-solving. Argumentation is a process in which claims are supported by data and justified by warrants; it is essential for solving problems effectively [51].

Students reach Level 3 argumentation when they can construct claims, data, and warrants. Based on the results of this study, 9 students (29.03%) achieved Level 3 argumentation. This finding indicates that these students were able to solve the problems correctly by providing appropriate claims, supporting data, and logical warrants. In Question 3, students were asked to determine the 10th pattern in the circle pattern. Students solved this problem by first identifying the difference between consecutive patterns. After claiming that the 10th pattern was 66, students provided data by identifying the increasing differences between the patterns. Based on this data, students were able to justify their reasoning by applying the triangular number formula, $U_n = \frac{1}{2}n(n + 1)$, to determine the result.

Students who can construct claims, data, and warrants demonstrate that they understand the problem and can determine appropriate strategies to solve it. However, no students reached Level 4 or Level 5 argumentation in this study. This finding indicates the need for teachers to train students to develop backing, qualifiers, and rebuttals in their arguments, both in written and oral forms. These results are consistent with previous research, which found that students' argumentation abilities were mostly at Level 1 (58.10%), followed by Level 2 (30.95%), and Level 3 (10.95%), while no students reached Level 4 or Level 5 in learning [52].

4. CONCLUSION

The results of this study indicate that students' abilities in mathematical argumentation were distributed across three levels. A total of 31 students (100%) achieved Level 1 argumentation, demonstrating their ability to formulate claims when solving reasoning problems. Furthermore, 15 students (48.38%) reached Level 2 argumentation, indicating that they were able to construct claims supported by appropriate data, while 9 students (29.03%) achieved Level 3 argumentation by providing claims, data, and warrants in their reasoning. However, no students reached Level 4 or Level 5 argumentation, which

involves the use of backing, qualifiers, and rebuttals. These findings suggest that although students have begun to develop basic components of mathematical argumentation, their ability to construct more complete arguments remains limited. Therefore, students need to be trained to develop backing, qualifiers, and rebuttals so they can construct more comprehensive arguments in both written and oral forms. The findings also imply that teachers play an important role in fostering students' mathematical argumentation in classroom learning, particularly through activities that encourage reasoning, explanation, and justification when solving mathematical problems.

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